Jestr

Journal of Engineering Science and Technology Review 4 (1) (2011) 101-104

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

Research Article

Application of the homotopy perturbation method to seven-order Sawada–Kotara equations

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Received 5 January 2011; Accepted 17 March 2011

Abstract

In this letter we implemented the homotopy perturbation method (HPM) for approximating the solution of the seventh-order Sawada–Kotera (sSK) and a Laxs seventh-order KdV (LsKdV) equations. Comparing the results with exact solutions has led us to significant consequences. The results reveal that the HPM is very effective, simple and convenient to systems of nonlinear equations. It is predicted that the HPM can be found widely applicable in engineering.

Keywords: seventh-order Sawada–Kotera equations; HPM, partial differential equations, nonlinear

1. Introduction

Most scientific problems such as seventh-order Sawada–Koteraequations are inherently of nonlinearity. A limited number of these problems haveexact analytical solution. The other nonlinear equations should be solved using other methods. Some of them are solved using numerical techniques and some are solved by using perturbation method. In the numerical method, stability should be considered to avoid inappropriate results. In the perturbation method, we should exert the small parameter in the equation. Therefore, finding the small parameter and exerting it into the equation are the difficulties of this method. These limitations with the common perturbation method, makes it difficult for developing different application.

Many different methods have recently been introduced to eliminate the small parameter, such as artificial parameter method introduced by He [1], the homotopy perturbation method by He [2,3] and the variation iteration method by He[4–6]. One of the semi-exact methods is the homotopy perturbation method (HPM) [7–20].

In this work we consider to implement the HPM to the sSK and LsKdV equations which can be shown in the form

$$u_t + (63u^4 + 63(2u^2u_{xx} + uu_x^2) + \\ + 21(uu_{xxxx} + u_{xx}^2 + u_xu_{xxx}) + u_{xxxxxx})_x = 0, \quad (1)$$

$$u_t + (35u^4 + 70(u^2u_{xx} + uu_x^2) + \\ 7(2uu_{xxxx} + 3u_{xx}^2 + 4u_xu_{xxx}) + u_{xxxxxx})_x = 0, \quad (2)$$

Respectively Eq. (1) is known as the seventh-order Sawada–Kotera equation [1] and Eq. (2) is known as the Lax_s seventh-order KdV equation [2].

2. Basic idea of He's homotopy perturbation method

To illustrate the basic idea of this method, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \ r \in \Omega \quad (3)$$

We consider the boundary conditions of:

$$B(u, \partial u / \partial n) = 0, \ r \in \Gamma \quad (4)$$

Where A is a general differential operator, B a boundary operator, $f(r)$ is an analytical function and Γ is the boundary of the domain Ω.

The operator A can be divided into two parts of L and N, where L is linear part, while N is nonlinear. Eq. (3) can be rewritten as:

$$L(u) + N(u) - f(r) = 0 \quad (5)$$

By the use of homotopy technique we construct it as $v(r, p) : \Omega \times [0,1] \rightarrow \Re$ which satisfies Eq. (5):

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ISSN: 1791-2377

$$H(v,p)=(1-p)[L(v)-L(u_0)]+ + p[A(v)-f(r)]=0,\ P\in[0,1],\ r\in\Omega \tag{6}$$

Where $p\in[0,1]$ is an embedding parameter and u_0 is an initial approximation of Eq.(5),which satisfy the boundary conditions. By considering Eq. (5) we will have:

$$\begin{cases} H(v,0)=L(v)-l(u_0)=0 \\ H(v,1)=A(v)-f(r)=0 \end{cases} \tag{7}$$

By changing p from zero to unity $v(r,p)$ changes from $u_0(r)$ to $u(r)$ .This method is called deformation. $L(v)-L(u_0)$ and $A(v)-f(r)$ are called homotopy. According to HPM, we can first use the embedding parameter p as "small parameter", and assume that the solution of Eq. (6) can be written as a series of the power of p :

$$v=v_0+pv_1+p^2v_2+... \tag{8}$$

By setting $p=1$ the approximate solution of Eq. (6) result:

$$u=\lim_{p\to 1} v=v_0+v_1+v_2+... \tag{9}$$

The combination of the perturbation method and the homotopy method is called HPM, which compensate the lacks of limitations in the traditional perturbation methods so this technique can have full advantage of the traditional perturbation techniques. The series (9) is convergent for most cases. However the convergence rate depends on the nonlinear operator.The following opinions are suggested by He [7]:

(1) The second derivative of *N(V)* with respect to *V* must be small because the parameter may be relatively large, i.e., *p*→1.
(2) The norm of *L*−1∂*N/*∂*V* must be smaller than one so that the series converges.

Meanwhile, D.D. Ganji et al.[23] could have found a new method for converge HPM. The basic idea in this method is to keep the inherent stability of nonlinear dynamic, even the selected linear part is not. They transformed a nonlinear complex differential equation to a series of linear and nonlinear parts, almost simpler differential equations. These sets of equations are then solved iteratively.

3. Implementation of the method

3.1 Equation (1)

We will begin with sSk equation (1) with the initial condition by:

$$u(x,0)=\frac{4k^2}{3}(2-3\tanh^2(kx)). \tag{10}$$

where k is arbitrary constants.

First, we construct a homotopy in the form:

$$(1-p)(\dot{v}-\dot{v}_o)+ + p\left(\dot{v}+\begin{pmatrix}63v^4+63\left(2v^2v_{xx}+vv^2{}_x\right)\\ +21\left(vv_{xxxx}+v_{xx}^2+v_xv_{xxx}\right)+v_{xxxxxx}\end{pmatrix}_x\right) \tag{11}$$

Where "dot" shows differential with respect to *t*, and the initial approximations are as following:

$$v_0(x,t)=u_0(x,t)=u(x,0) \tag{12}$$

And

$$v=v_0+pv_1+p^2v_2+... \tag{13}$$

By substituting Eqs.(13) into Eqs. (11) and arranging the coefficients of "*p*" powers, we have:

$$(252v_ov_{ox}+v_{0xxxxxxxx}+\dot{v}_1+...+21v_0v_{0xxxxx})p+ +(252v_0v_{1x}+21v_1v_{0xxxxx}+v_2+...21v_0v_{1xxxxx})p^2+... \tag{14}$$

For solving Eqs.(14) We only use two approximation of*p*even though results show fast convergence to the exact result. In order to obtain the unknown of $v_{i,}\, i=1,2$, we must construct and solve equations with twounknowns, considering the initial conditions of $v_i(x,0)=0$, *i*= 1, ,and having the initial approximations of Eqs.(10):

$$\begin{aligned} v_1(x,t)=&-1451520k^{11}\tanh(kx)^{11}t+5322240k^{11}\tanh(kx)^9t\\ &+129024k^9\tanh(kx)^9t-387072k^9\tanh(kx)^7t\\ &-7547904k^{11}\tanh(kx)^7t+408576k^9\tanh(kx)^5t\\ &-\frac{514048}{3}k^9\tanh(kx)^3t+204288k^{11}\tanh(kx)t\\ &+\frac{62464}{3}k^9\tanh(kx)t \end{aligned} \tag{15}$$

$$v_2(x,t)= -\frac{8192}{9}k^{16}t^2\begin{pmatrix}1363351-81003981\tanh(kx)^2\\ +344246124474k^2\tanh(kx)^8-123591794382k^2\tanh(kx)^6\\ -362054561\tanh(kx)^6+856900170\tanh(kx)+7638080247\tanh(kx)^8...\end{pmatrix} \left(\tanh(kx)^2-1\right)$$

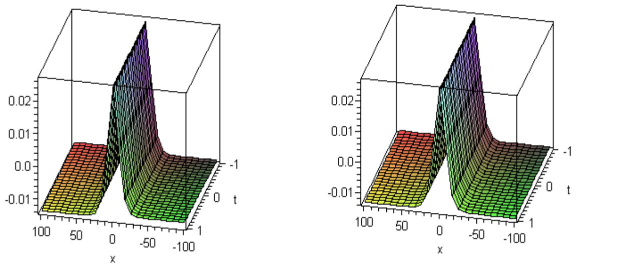
From Eqs.(7), and assume two approximation of *p*we will obtain:

$$u(x,t)=\lim_{p\to 1}\sum_{i=0}^{2}p^iv_i(x,t)=\sum_{i=0}^{2}v_i(x,t) \tag{16}$$

Exact solution of sSK equation (1) as follows:

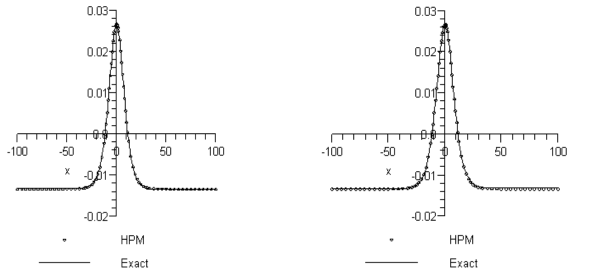
$$u(x,t)=\frac{4k^2}{3}\left(2-3\tanh^2\left(k\left(x-\frac{256k^6}{3}t\right)\right)\right) \tag{17}$$

By the drawing of 3-D and 2-D figures of exact solution and HPM solution (solved with two approximation), the figures are similar to each other.



1a 1b

Fig.1 HPM result (solved with two approximation) for $u(x,t)$ (1a), and exact solution for $u(x,t)$ (1b), where *k=0.1*.



2a 2b

Fig. 2 The comparison of the results of the HPM and exact solution, at $t = 0.25$ (2a) and $t=-.25$ (2b), where *k=0.1*

3.2 Equation(2)

For further comparison of the HPM, we consider the LsKdV equation [2] Eq. (2) with initial condition is given by

$$u(x,0) = 2k^2 \operatorname{sec}h^2(kx) \tag{18}$$

Where k is arbitrary constants.
First, we construct a homotopy in the form:

$$(1-p)(\dot{v}-\dot{v}_o)+ + p\left(\dot{v}+\begin{pmatrix}35v^4+70(v^2v_{xx}+vv^2{}_x)+\\ 7(2vv_{xxxx}+3v_{xx}^2+4v_xv_{xxx})+v_{xxxxxx}\end{pmatrix}_x\right) \tag{19}$$

The initial approximations are as following:

$$v_0(x,t) = u_0(x,t) = u(x,0) \tag{20}$$

and

$$v = v_0 + pv_1 + p^2v_2 + \ldots \tag{21}$$

By substituting Eqs.(21) in to Eqs. (19) and arranging the coefficients of "*p*" powers, we have:

$$(252v_ov_{ox}+v_{0xxxxxxx}+\dot{v}_1+\ldots+21v_0v_{0xxxxx})p + (252v_0v_{1x}+21v_1v_{0xxxxx}+v_2+\ldots 21v_0v_{1xxxxx})p^2+\ldots \tag{22}$$

For solving Eqs. (22) We only use two approximation of *p* even though results show fast convergence to the exact result. In order to obtain the unknown of, $v_{i,} i=1,2$ we must construct and solve equations with two unknowns, considering the initial conditions off $v_i(x,0)=0$, *i*= 1 and having the initial approximations of Eqs.(18):

$$v_1(x,t)=1024k^9t\begin{pmatrix}-43680k^2\sinh(3kx)+\\+154560k^2\sinh(kx)+3360k^2\sinh(5kx)\\+\sinh(9kx)+7\sinh(7kx)-\\-820\sinh(5kx)+1988\sinh(3kx)+\\+2814\sinh(kx)\end{pmatrix}\Big/ \begin{pmatrix}\cosh(11kx)+11\cosh(9kx)+55\cosh(7kx)+\\+165\cosh(5kx)+330\cosh(3kx)+\\+462\cosh(kx)\end{pmatrix} \tag{23}$$

$$v_2(x,t)= =512k^{19}t^2\begin{pmatrix}121280k^4\sinh(6kx)-\\-3309310k^2\sinh(2kx)\\-60320\sinh(kx)+8740k^5\sinh(13kx)-\\-7780k^3\sinh(11kx)\\+940k^2\sinh(9kx)+90k\sinh(7kx)\\+280\sinh(5kx)........\end{pmatrix}\Big/ (55\cosh(15kx)+331\cosh(13kx)+\ldots)+\ldots$$

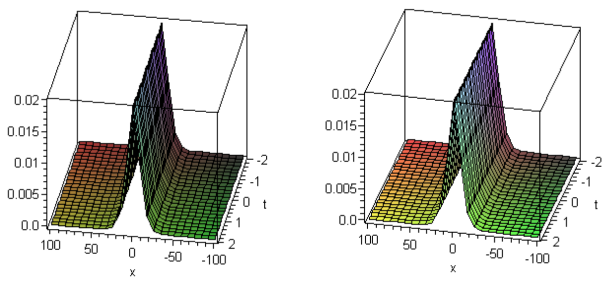
From Eqs.(7), and assume two approximation of *p* we will obtain:

$$u(x,t)=\lim_{p\to1}\sum_{i=0}^{2}p^iv_i(x,t)=\sum_{i=0}^{2}v_i(x,t) \tag{24}$$

Exact solution of sSK equation (2) as follows:

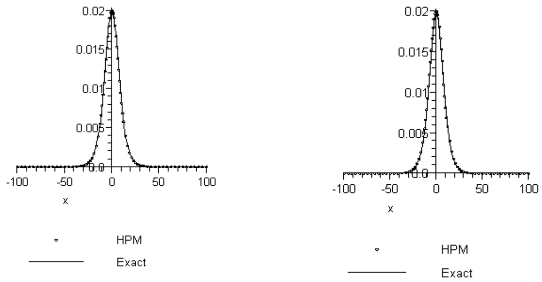
$$u(x,t)=\frac{4k^2}{3}\left(2-3\tanh^2\left(k\left(x-\frac{256k^6}{3}t\right)\right)\right) \tag{25}$$

By the drawing of 3-D and 2-D figures of exact solution and HPM solution (solved with two approximation), the figures are similar to each other.



3a 3b

Fig. 3 HPM result (solved with two approximation) for $u(x,t)$ (3a), and exact solution for $u(x,t)$ (3b), where *k=0.1*.



4a 4b

Fig. 4 The comparison of the results of the HPM and exact solution, at *t* = 0.*25*(4a) and*t=-.25*(4b), where*k=0.1*

4. Conclusions

In this work, homotopy perturbation method has been successfully applied to find the solution of nonlinear sSK and LsKdV equations. All the examples show that the results of the present method are in excellent agreement with the exact solutions. In our work, we use the Maple Package to calculate the functions obtained from the Homotopy perturbation method.The results show that this method provides excellent approximations to the solution of this nonlinear system with high accuracy. Finally,it has been attempted to show the capabilities and wide-range applications of the HPM.

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