Galvanomagnetic Effects "Sensors based on Hall Effect"

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Abstract
The Hall effect is the generation of a transverse electromotive force in a sample carrying an electric current and exposed to perpendicular magnetic field. Depending on the sample geometry, this electromotive force may cause the appearance of a transverse voltage across the sample, or a current deflection in the sample. The generation of this transverse voltage, called Hall voltage, is the generally known way for the appearance of the Hall effect.

The resistance of this sample increasing under influence of the magnetic field, this called magnetoresistance effect. Both the Hall effect and the magnetoresistance effect belong to the more general class of phenomena called galvanomagnetic effects. Galvamomagnetic effects are the manifestations of charge transport phenomena in condensed matter in the presence of a magnetic field.

The sensor applications of Hall effect became important only with the development of semiconductor technology. For one thing, the Hall effect is only strong enough for this propose in some semiconductors. Therefore, the first Hall effect magnetic sensor became commercially available in the mid 1950s, a few year after the discovery of high-mobility compound semiconductors. Our goal in this paper is to understand the physically background of the Hall and the magnetoresistance effects. We are going to discuss the effect of parameters in those phenomena and how we can make better our technology to improve better efficiency.

Keywords: Hall Effect, Galvanomagnetic Effect, Hall Sensors, Wheel Hall Sensor, MR Effects.

1.1 Approximate analysis of Hall effect

In this section we will see the Hall effect. That give us an idea for the Hall effect and for magnetocencentration effect and help us to understand much more about solid state physics.

To simplify the analysis, we also assume equilibrium carrier concentrations in samples, and thus neglect diffusion currents.

All galvanomagnetic effects come about as a manifestation of the action of the Lorentz force on quasi-free charge carriers in condensed matter. The Lorentz force is the force acting on a charged particle in a electromagnetic field. Moreover, the existence of the Lorentz force is fundamental indication of a very presence of an electric and/or magnetic fields. It is give by

$$\vec{F} = e\vec{E} + e(\vec{v} \times \vec{B})$$
(1.1)

Here e denotes the particle charge (for electrons \(e=-q\) and for holes \(e=q\) where q is he magnitude of an electron charge) \(\vec{E}\) is the electric field, \(\vec{v}\) is the carrier velocity and \(\vec{B}\) the magnetic induction. The first term on the right hand side of (1.1) is often referred to as electrostatic force, and the second term as the Lorentz force.

In our analysis we use Smooth-drift approximation, this approximation consist of assumption that charge carriers move uniformly as a result of an electric field or other driving forces, the velocity of the movement being the same for all carriers and equal to the appropriate drift velocity. So we can replace the carrier velocity of each individual carrier, in \(\vec{v}\) (Equation (1.1)), by the average drift velocity of all carriers. We neglect completely the thermal motion of the carriers and the energy dissipation effect of scattering by a smooth friction.

1.2 The (original) Hall effect

We shall first study the Hall effect in the form in which Hall[1] discovered. Hall’s experimental device was a long gold leaf. We shall also consider a long flat sample; but in order to quickly come modern applications of Hall effect, we shall study it in semiconductor samples.

Let us consider the transport of carriers in a long and thin semiconductor strips, such as those shown in figures 1.1a and 1.1b.(Long strip means that the length of a strip l is much larger than w).

Suppose that in the strips, along the x direction, an external electric field \(E_y(Ex,0,0)\) is established, and the magnetic field is zero. Then in the Lorentz force (Equation (1.1)) only the first term, which is the electrical force exists. As we know the electrical force causes the drift of charge carriers. In the present case, charge carriers drift along the...
strip in two opposite directions. The drift velocities are given by:

\[ \vec{v}_{dp} = \mu_p \vec{E}, \quad \vec{v}_{dn} = \mu_n \vec{E} \]  \hspace{1cm} (1.2)

where \( \mu_p \) and \( \mu_n \) are the mobilities of holes and electrons, respectively. The associated current densities are given by:

\[ j_p = q \mu_p n \vec{E}, \quad j_n = q \mu_n n \vec{E} \]  \hspace{1cm} (1.3)

where \( p \) and \( n \) denote the hole and the electron density in p-type and n-type strip, respectively.

**1.3 Hall electric field**

Let us now expose the current carrying strip to a magnetic field. Let the magnetic induction \( B \) be collinear with the y-axis (figure 1.1). Now on each charge carrier in the samples both parts of Lorentz force (Equation (1.1)) act. Since we assumed a uniform directional velocity of all carriers in the sample, the magnetic forces acting on a carriers are given by:

\[ \vec{F}_p = e \vec{v}_{dp} \times \vec{B}, \quad \vec{F}_n = e \vec{v}_{dn} \times \vec{B} \]  \hspace{1cm} (1.4)

These forces push carriers towards the upper edges of the strips. Consequently the carrier concentration at these upper edges of the strips start to increase, while the carrier concentration at the lower edges start to decrease.

**1.4 Hall voltage**

A more tangible effect with associated with Hall field is the appearance of transverse voltage between the edges of a strip. This voltage is known as Hall voltage. Let us choose two points M and N at the opposite edges of the strip, under the condition that both points lie in the same equipotential plane when \( B=0 \) (figure 1.1). Then the Hall voltage is given by

\[ V_H = \int_M^N E_H dz \]  \hspace{1cm} (1.9)

and

\[ V_{Hp} = \mu_p E_x B_y w, \quad V_{Hn} = \mu_n E_x B_y w \]  \hspace{1cm} (1.10)

for our p-type and n-type strips, respectively. Here the negative sign has been omitted and \( w \) denotes the width of the strips. The generation of the transverse Hall electric field and the associated Hall Voltage under the experimental conditions similar to that shown in figure 1.1 are usually referred to as the Hall effect. However, there is another, more fundamental feature of the Hall effect: the Hall angle.

**1.5 Hall angle**

In the presence of a magnetic field, the total electric field in the sample \( E=E_e+EH \) is not collinear with the external electric field \( E_e \). In the present case the current in the sample is confined to the direction of the external electric field, and the current density is collinear with the external electric field (figure 1.1). Hence the total electric field is not collinear with the current density either. Therefore, the Hall effect in a long sample shows up through the tilting of the total electric field relative to the external electric field and the current density in the sample. With the aid of figure 1.2, the magnitude of the Hall angle is given by

\[ \tan \Theta_H = \frac{|E_H|}{E_e} \]  \hspace{1cm} (1.11)

Fig. 1.2: The vector diagrams of electric fields and current densities in long samples: (p), in a p-type material; (n), in an n-type material \( J \) is the current density, \( E_e \) the external electric field, \( E_H \) the Hall electric field, \( E \) the total electric field and \( \Theta_H \) the Hall angle.
For the reason which will become clear later, it is convenient to measure the Hall angle with respect to the direction of the total electric field, as shown in figure 1.2. Then we may define the Hall angle as the angle of inclination of the current density $\mathbf{J}$ with respect to the total electric field $\mathbf{E}$. According to this definition and (1.11), we obtain for the two particular cases from figures 1.1 and 1.2

$$\tan \Theta_{hp} = \mu_p B_y, \quad \tan \Theta_{hn} = -\mu_n B_y$$

(1.12)

For $p$-type and $n$-type semiconductor strip, respectively. The value of the Hall angle depends only on the applied magnetic induction and the mobility of the charge carriers. The sign of the Hall angle coincides with the sign of the charge carriers.

1.6 Hall coefficient

From equations (1.3) and (1.8) we can find the relationship between the current density and the Hall field:

$$\vec{E}_{hp} = -\frac{1}{q_p}[\vec{J} \times \vec{B}] , \quad \vec{E}_{hn} = \frac{1}{q_n}[\vec{J} \times \vec{B}]$$

(1.13)

The equations can be rewritten as

$$\vec{E}_H = -R_H[\vec{J} \times \vec{B}]$$

(1.14)

where $R_H$ is a parameter called the Hall coefficient. Equation (1.14) is a modern representation of the conclusions made by Hall after his experimental findings.

The Hall coefficient is a material parameter that characterizes the intensity and the sign of the Hall effect in a particular material. The unit of the Hall coefficient is VmA$^{-1}$T$^{-1}$, which is sometimes expressed in a more compact form as $\Omega$ m$^{-1}$ or, equivalently, m$^3$C$^{-1}$.

From a comparison of equations (1.13) and (1.14), we can find the Hall coefficients of strongly extrinsic semiconductors

$$R_{hp} = \frac{1}{q_p}, \quad R_{hn} = \frac{1}{q_n}$$

(1.15)

for $p$-type and $n$-type, respectively. The sign of the Hall coefficient coincides with the sign of the majority carriers, and the magnitude of the Hall coefficient is inversely proportional to the majority carriers concentration. In the practical applications of the Hall effect it is convenient to operate with pure macroscopic and integral quantities characterizing a Hall device. To this end, for a long Hall device we obtain from equations (1.11) and (3.14) the Hall voltage:

$$V_H = \frac{R_H IB}{t}$$

(1.16)

Here $t$ is the thickness of the strip, $I$ is the device current given by $I = J_w t$ and $B_\bot$ the component of the magnetic induction perpendicular to the device plane. We omit the negative sign.

Equation (1.16) shows why a plate is a preferential shape of the Hall device with voltage output: for a given biasing current and magnetic induction, the thinner the sample, the higher the resulting Hall voltage.

1.7 The current deflection effect

Consider now the Hall effect in a very short Hall plate. A sample for a Hall effect experiment is called short if its dimension in the current direction is much smaller than that in the direction of the magnetic force acting on carriers. As example of a short sample is the short strip shown in figure 1.3, the basic shape of this device, its position relative to the coordinate system, and the notation are the same as in case of the long strips in figure 1.3. Now, however, the strip is short, $l << w$, and it is laterally sandwiched between two large current contacts. In such short samples the Hall effect takes a form which is sometimes called current deflection effect.

Let us now find a quantitative relationship between the electric field and the current density in a short sample. We shall again use the approximation which we introduced above: we neglect the thermal motion of the carriers, and assume that carriers move uniformly under the action of the Lorentz force.

$$\vec{F}_p = q\vec{E} + q[\vec{v}_p \times \vec{B}] , \quad \vec{F}_n = -q\vec{E} - q[\vec{v}_n \times \vec{B}]$$

(1.17)

for holes and electrons, respectively. The densities of the carriers are:

$$\vec{J}_p (B) = \vec{J}_p (0) + \mu_p [\vec{J}_p (B) \times \vec{B}]$$

(1.18)

$$\vec{J}_n (B) = \vec{J}_n (0) + \mu_n [\vec{J}_n (B) \times \vec{B}]$$

Here $J_p(B)$ and $J_n(B)$ are the hole and electron current densities respectively in the presence of the magnetic induction $B$. 

![Fig. 1.3 A short Hall plate. The distance between the current contacts is smaller than the distance between the insulating boundaries (ib). (The picture was taken from P.S Popović “ Hall Effect Devices (Sensor Series) Swiss Federal Institute of Technology Lausanne (E.P.F.L) 2nd Edition IoP 2004) (306x381 to 522x543)](image-url)
\[
\begin{align*}
\vec{J}_p(\vec{B}) &= q\mu_p p\vec{E}_p = \mu_p p\vec{F}_p = qp\vec{v}_p \\
\vec{J}_n(\vec{B}) &= q\mu_n n\vec{E}_n = \mu_n n\vec{F}_n = -qm\vec{v}_n
\end{align*}
\]

And \(J_p(0)\) and \(J_n(0)\) are the drift current densities due to the electric field \(E\) when \(B=0\):

\[
\begin{align*}
\vec{J}_p(0) &= q\mu_p p\vec{E} \\
\vec{J}_n(0) &= q\mu_n n\vec{E}
\end{align*}
\]

Equations (1.17),(1.18) and (1.20) are represented in figure 1.4. In a short sample, a magnetic field deflects the current from its usual way along the electrical field. Moreover using the diagrams in figure 1.4, we find

\[
\begin{align*}
tan\Theta_{lp} &= \mu_p B_{\perp}, \quad tan\Theta_{ln} = -\mu_n B_{\perp}
\end{align*}
\]

![Image](image1.png)

Fig. 1.4 The graphical representation of the vector equations (1.17),(1.18) and (1.20) that hold in a short Hall plate: (p) of a p-type semiconductor ; (n) of a n-type semiconductor. (The picture was taken from P.S Popović “ Hall Effect Devices (Sensor Series) Swiss Federal Institute of Technology Lausanne (E.P.F.L.) 2nd Edition IoP 2004)

2. The magnetoresistance effect

By inspecting the diagrams in figure 1.4, we notice that the current density vector \(\vec{J}(\vec{B})\) are smaller than \(\vec{J}(0)\). This means that in short Hall Plates a magnetic field also has additional effect: it causes a reduction in current density. This current deflection is illustrated in figure 1.5. As shown in figure 1.5(a), current density in presence of a magnetic field is smaller than that with no magnetic field. The attenuation in the current density is a consequence of the current deflection effect: the deflected current lines between the sample contacts are longer (see figure 1.5(a)). The longer current path means also greater effective resistance of the sample.

To find how much the current density decrease due to a magnetic induction, we have to solve the equation (1.18) with respect to \(\vec{J}(\vec{B})\). We can solve this equation and for perpendicular magnetic field, such that \(\vec{E} \cdot \vec{B} = 0\), the solutions of the equations (1.18) are

\[
\begin{align*}
\vec{J}_p(\vec{B}) &= \sigma_{pB} \vec{E} + \sigma_{pB} \mu_p [\vec{E} \times \vec{B}] \\
\vec{J}_n(\vec{B}) &= \sigma_{nB} \vec{E} - \sigma_{nB} \mu_n [\vec{E} \times \vec{B}]
\end{align*}
\]

where

\[
\begin{align*}
\sigma_{pB} &= \frac{\sigma_{p0}}{[1 + (\mu_p B)^2]} \quad \text{and} \quad \sigma_{nB} = \frac{\sigma_{n0}}{[1 + (\mu_n B)^2]}
\end{align*}
\]

are the effective conductivities of p-type and n-type materials in the presence of the magnetic induction \(B\) and, \(\sigma_{pB}\)and \(\sigma_{nB}\) are the conductivities at \(B=0\). Note that, at a given electric field, the coefficients \(\sigma_{pB}\) determine the longitudinal current density in an infinitely short sample. Since a materialization of the infinitely short sample is Corbino disk, we shall call the conductivity the coefficients \(\sigma_{nB}\) the Corbino conductivity.

The first term on the right-hand side of equation (1.22) represent the lateral current densities. The integral of this current density over the contact surface is the device current. From equations (1.22) and (1.23) we deduce that the device current decreases as the magnetic field, increases. Therefore, if a sample exposed in magnetic field, everything happens as if the conductivity of the sample material were decrese. The corresponding relative increase in the effective material resistivity is given by

\[
\begin{align*}
\frac{\rho_{pB} - \rho_{p0}}{\rho_{p0}} &= (\mu_p B)^2 \\
\frac{\rho_{nB} - \rho_{n0}}{\rho_{n0}} &= (\mu_n B)^2
\end{align*}
\]

The increase in a material resistivity under the influence of a magnetic field is called the magnetoresistance effect. In particular, the just described increase in resistivity due to the current deflection effect in a short samples is called the geometrical magnetoresistance effect. The attribute “geometrical “ reflects the fact that the effect is related to the geometry of the current lines, as illustrated in figure 1.5.
2.1 Galvanomagnetic current components

Up to now, in order to simplify our analysis, we have always assumed that our sample is plate-like, and that the magnetic field was perpendicular to the plate. Let us drop now this assumption and consider a general case. We only assume that the vectors of electric field and magnetic induction have an angle and we seek the current density vector.

The basic equations relating the resulting current density \( \mathbf{J}(\mathbf{E}, \mathbf{B}) \) with the electric field \( \mathbf{E} \) and the magnetic induction \( \mathbf{B} \) are given by

\[
\mathbf{J}(\mathbf{E}, \mathbf{B}) = \sigma \mathbf{E} + \mu [\mathbf{J}(\mathbf{E}) \times \mathbf{B}]
\]

(1.25)

where \( \mathbf{J}(\mathbf{E}, \mathbf{B}) \equiv \mathbf{J}(\mathbf{B}) \) and \( \sigma \) is the material conductivity at \( B=0 \) and \( \mu \) is the mobility of the carriers. The total galvanomagnetic current density (figure 1.6) generally consists of three components:

\[
\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{J}_3
\]

(1.25a)

and specially we can write

\[
\mathbf{J}_1 = \left( \frac{\sigma}{1 + \mu^2 B^2} \right) \mathbf{E}
\]

\[
\mathbf{J}_2 = \left( \frac{\sigma}{1 + \mu^2 B^2} \right) \mu [\mathbf{E} \times \mathbf{B}]
\]

\[
\mathbf{J}_3 = \left( \frac{\sigma}{1 + \mu^2 B^2} \right) \mu^2 \mathbf{B} [\mathbf{E} \cdot \mathbf{B}]
\]

(1.26)

And finally the total galvanomagnetic current density is:

\[
\mathbf{J} = \sigma \mathbf{E}(1 - \mu^2 B^2) + \mu \sigma [\mathbf{E} \times \mathbf{B}] + \mu^2 \sigma \mathbf{B}(\mathbf{E} \cdot \mathbf{B})
\]

(1.27)

3. Geometrical factors

Let us have a look at the distribution of the current density and the electrical potential in a “realistic” galvanomagnetic sample. Figure 1.7, shows such a two terminal rectangular plate, which is somewhere between “very long” and “very short”. By inspecting the current and the equipotential lines, we notice the following. Near the two isolating boundaries, current is forced to flow along the boundaries and the equipotential lines are strongly inclined: a Hall electric field is generated.

The device such us assumed have intermediate geometry. Then a factor to related from geometry can be calculated. It is really that the Hall voltage of the Hall plate with an arbitrary shape can de expressed as

\[
G_H = \frac{V_H}{V_{H\infty}}
\]

(1.28)

Where \( G_H \) is a parameter called the geometrical correction factor of a Hall voltage, and \( V_{H\infty} \) denotes the Hall voltage in a corresponding long strip.

The Hall geometrical factor is a number limited by \( 0 < G_H < 1 \). For a long Hall device \( G_H=1 \); for the very short Hall device \( G_H=0 \). Similarly, for the resistance of the device exposed to a magnetic field we may write:

\[
G_R = \frac{R(B)}{R_s(B)}
\]

(1.29)

Where \( G_R \) is a parameter called the geometrical factor of magneto-resistance, \( R(B) \) is the resistance of a considered magneto-resistor at an induction \( B \), and \( R_s(B) \) denotes the resistance of a corresponding part of an infinitely long strip.

3.1 Common shapes of a Hall plates and magneto-resistors

Generally, any piece of electrically conductive material, fitted with four electrical contacts, could be used as a Hall device. However, to be reasonably efficient and convenient for a practical applications, a Hall device should preferably be made in the form of a plate. The contacts should be positioned at the boundary of the plate, far away from each other. The two contacts for sensing the Hall voltage should be placed so as to attain approximately equal potentials at a zero magnetic field. Then the potential difference at the sense contacts equals to Hall Voltage. The common potential at the sensing contacts should roughly equal the mid-potential of the current contacts. Then the largest Hall voltage can me measured. To simplify the design and fabrication of a Hall plate a highly symmetrical shape and a uniform material and thickness of the plate are usual chosen. Some commonly used shapes of Hall plates are shown in figure 1.7.

3.2. Reducing offset voltage and 1/f noise

To removing ohmic offset we use, an elegant method that exploits a property of the Hall-effect transducer to reduce system offset. A four-way symmetric Hall-effect transducer...
can be viewed as a Wheatstone bridge. Ohmic offsets can be represented as a small $\Delta R$, as shown in figure 1.8a. When bias current is applied to the drive terminals, the output voltage appearing at the sense terminals is $V_H + V_E$, where $V_H$ is the Hall voltage and $V_E$ is the offset error voltage.

Now consider what happens if we take the transducer and reconnect the bias and sense terminals, as shown in figure 1.8b. All of the terminal functions have been rotated clockwise by 90° ("Spinning Current technique"). The sense terminals are now connected to bias voltage, and the former drive terminals are now used as outputs. Because the transducer is symmetric with rotation, we should expect to see, and do see, the same Hall output voltage.

Imagine that the $1/f$ noise (flicker) at the output terminals of a Hall device as a fluctuating offset voltage. Then we can model $1/f$ noise source by a fluctuating asymmetry resistance $\Delta R$ in figure 1.8. If the biasing current of a Hall device spins fast enough so that the fluctuating voltage does not change essentially during one circle, then the system shall not “see” the difference between the static and the fluctuating offset and shall cancel both of them. In order to completely eliminate the $1/f$ noise of a Hall device, we have to set the spinning current frequency well above the $1/f$ noise corner frequency. For a very small device, this may be a very high frequency.

4. Our suggestion for a novel shape Hall device : The 3D-WHS

The lasts few months we design and we are ready to developed a novel Hall sensor device which uses elaborate spinning current technique. The novel 3D Hall device that we call “3D Wheel Hall Sensor” is presented in figures 1.9a and 1.9b. The current enters the device, as presented in the aforementioned figures, in two phases namely the even phase (PHASE-P) and the odd phase (PHASE-N).
We simulate in MatLab environment the electric field and the equipotential lines inclination on the 3D-WHS in a presence of magnetic induction to see the behavior of the device in a real Hall effect conditions and the results represented in figures 1.10a and 1.10b.

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