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Numerical Simulation of Mixed Convection in a Inclined Thick Duct

F.Z. Bakhti[1] and M. Siameur[2,*]

[1]*Department of Mechanics, University of M'sila (Algeria)*
[2]*Department of Mechanics, University of Batna (Algeria)*

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Abstract

In this work, a numerical study of a laminar mixed convection in a inclined thick duct is considered. A uniform heat flux is applied over the entire circumference of the tube. The governing differential equations were solved by a finite volume method. The SIMPLE algorithm for pressure- velocity coupling was adopted. A parametric study is carried out to analyze the effect of the Grashof number, the angle of the duct inclination and the wall conductivity on the thermal-fluid fields. Several numbers of Grashof (4. 104, 4. 106, 4. 107) are considered for an angle of inclination equal: 0°, 30° and 60°. For material of the wall, we chose Report/ratio of conductivity (K=kp/kf) of the copper (K=19000), iron (K=3600) and aluminium (K=11500). The results are analyzed by examining the velocity, pressure and temperature fields§. The axial evolution of the Nusselt number and that of the parietal constraints are examined for various studied cases.

Keywords: Mixed convection, inclined tube, laminar flow, conjugate problem, numerical study

1. Introduction

The study of the mixed heat convection in the inclined ducts aroused an interest growing during these last decades. This interest is dictated by the role played by such configurations in many industrial applications in various fields such as the electronic components cooling, exchangers of heat, solar collectors and several other fields of the engineering.

The phenomena which appear in the case of the mixed convection in axisymmetric geometry are complex and depend on several parameters; such as Reynolds, Grashof and Prandtl numbers; as well as the angle of inclination of the tube, its thickness and thermal conductivity.

A great number of numerical, analytical and experimental studies relate to heated tubes or isotherms in mixed convection. The majority of these studies consider the two-dimensional case in vertical tubes, with negligence of the conduction of heat in material of the duct.

The studies aiming at the more complicated problems of conjugate transfer (conduction in the wall and convection inside a tube) are rather rare. Among those, it is necessary to quote that of Reynolds [1] who obtained an analytical solution for the flow developed in pure convection forced inside a duct subjected to a circumferential heat flux not uniform with the interface solid-fluid. Patankar et al. [2] carried out a numerical study of the mixed convection in a horizontal tube, for a flow developed with uniform heating with the interface solid-fluid on a half (higher or lower) and the other thermically isolated half. Bernier and Bliga [3] studied the mixed convection by taking account of conduction parietal for a flow under development in a vertical duct. They showed that the axial diffusion of heat in the wall of a tube becomes negligible for the low values of the thermal report/ratio of conductivity solid-fluid or thicknesses of the wall. Lin et al. [4] obtained a numerical solution for a unsteady laminar aiding and opposing mixed convection heat transfer in vertical flat duct. They indicated that unsteady heat transfer characteristics in the flow are principally determined by wall-to fluid heat capacity ratios. Correlation equations for the time variations of local Nusselt numbers with wall –to-fluid heat capacity ratios are proposed. Lee and Yan [5] studied the transient conjugated forced convection heat transfer in an infinitely long pipe, in which the external surface over finite length of pipe is subject to a constant wall temperature. They showed that the unsteady heat transfer depend strongly on the ratios of outside and inside radix, wall-to-fluid conductivity, and thermal diffusivity. The decrease in diffusivity ratio from 10 to 0.1 results in a longer time for the system to reach the steady state. Lee and Yan [6] presented the numerical solution for transient conjugated mixed convection in a vertical pipe maintained at either uniform heat flux or uniform wall temperature, by analyzing the effects of the ratio of Grashof number to Reynolds Gr/Re, conductivity ratio K, diffusivity ratio, and dimensionless wall thickness.

A fully implicit finite difference method is used by Yan [7] for numerically study the unsteady conjugated heat transfer in turbulent channel flows. The solutions take wall conduction and heat capacity effects into considerations. They showed that wall conduction plays a significant role in the transient conjugated heat transfer problem.

Lee and Yan [8] presented a numerical solution of transient mixed convection heat transfer in both parallel plate channels and circular pipe; experiencing a sudden change in ambient temperature the solutions take wall conduction and wall heat capacity effects into account. Their results showed that wall effects play an important role in

* E-mail address: msiameur@yahoo.fr
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unsteady heat transfer. It is found that an increase in the outside heat transfer coefficient result in a higher heat transfer exchange an shorter time period required for the system achieving the steady –state conditions.

Ouzzane and Galanis [9] have analyzed numerically the effects of the axial wall conduction and the non- uniform heat flux condition on the mixed convection in inclined circular duct. They found that the conduction through the tube material affects considerably the thermal and hydrodynamic fields especially when the uniform heat flux is applied at the top half of the tube section. Bilir and Ates [10], Faghri and Sparrow [11] have presented a numerical analysis unsteady forced convection in vertical duct, they found that both heat conduction in the wall and wall heat capacity play an important role in the case of transient conjugated heat transfer.

As regards the influence of the inclination of a tube on the transfer of heat, few studies were carried out until now. However, two tendencies, in what milked with the conclusions, seem to take shape. Certain authors like Sabbagh et al. [12] or Barozzi et al. [13] stipulate that the rate of transfer of heat decreases unceasingly between horizontal and the vertical inclination, while others like Nguyen [14] and Orfi [15] affirm that the inclination between 20° and 30° led to maximize the value of the Nusselt number.

Laouadi et al. [16] presents a numerical study, which highlights the influence of the wall of the tube on the transfer of heat. They noted that when the thermal conductivity of the tube is low compared to that of the fluid, the number of Nusselt falls if the inclination is increased. On the other hand, they fund an inclination, which optimizes the number of Nusselt when the thermal conductivity of the tube is large compared to that of the fluid.

In the present investigation, we study the laminar mixed convection of the air (Pr=0.71) in a inclined duct of length L, radius d(d=2Ro) and thickness δ tilted of an angle α compared to the horizontal plane(figure 1). The tube is uniformly heated by a constant heat flux q_p. Numerical simulations were made for number of Grachof between 5 x10 ^{4} and 5x10 ^{8} and that for various angle of inclination: 0°, 30° and 60° and of the reports/ratios of parietal conductivity equal to K=3600 (for Iron), K=11500 (for aluminum) and K = 19000 (for copper).

The flow is steady and laminar, two-dimensional and axisymmetric. The fluid is Newtonian, incompressible and obeying the approximation of Boussinesq. The thermo-physical quantities of the fluid are all assumed to be constant except for the density in the buoyancy force.

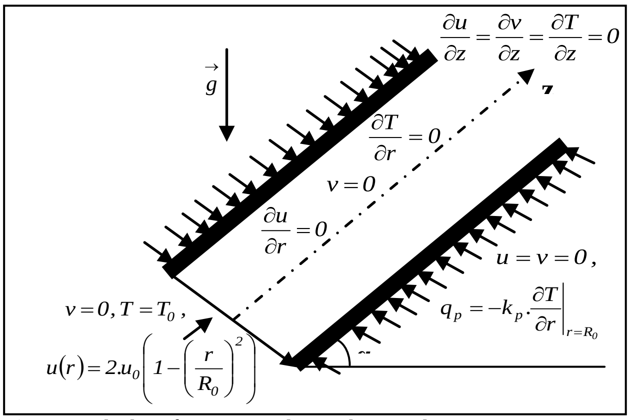


Fig. 1. Studied configuration and Boundary conditions

2. Mathematical Formulation:

With the Boussinesq approximation and assumptions, the dimensionless governing equations can be written as:

-Continuity equation

$$\frac{\partial U}{\partial Z}+\frac{\partial V}{\partial R}+\frac{V}{R}=0 \qquad (1)$$

-Axial momentum equation

$$\left[U\frac{\partial U}{\partial Z}+V\frac{\partial U}{\partial R}\right]=-\frac{\partial P}{\partial Z}+ \\ +\frac{1}{R_e}\left[\frac{\partial^2 U}{\partial Z^2}+\frac{1}{R}\frac{\partial U}{\partial R}+\frac{\partial^2 U}{\partial R^2}\right]+\frac{Gr_{qp}}{R_e{}^2}\theta.\sin\alpha \qquad (2)$$

-Radial momentum equation

$$\left[U\frac{\partial V}{\partial Z}+V\frac{\partial V}{\partial R}\right]=-\frac{\partial P}{\partial R}+\frac{1}{R_e}\left[\frac{\partial^2 V}{\partial Z^2}+\frac{1}{R}\frac{\partial V}{\partial R}+\frac{\partial^2 V}{\partial R^2}\right]- \\ -\frac{V}{(R_e.R^2)}+\frac{Gr_{qp}}{R_e{}^2}\theta.\cos\alpha \qquad (3)$$

-Energy equation in the fluid:

$$\left[U\frac{\partial \theta}{\partial Z}+V\frac{\partial \theta}{\partial R}\right]=\frac{1}{P_e}\left[\frac{\partial^2 \theta}{\partial Z^2}+\frac{1}{R}\frac{\partial \theta}{\partial R}+\frac{\partial^2 \theta}{\partial R^2}\right] \qquad (4)$$

- Energy equation in the pipe wall:

$$\frac{A}{P_e}\left[\frac{\partial^2 \theta}{\partial Z^2}+\frac{1}{R}\frac{\partial \theta}{\partial R}+\frac{\partial^2 \theta}{\partial R^2}\right]=0 \qquad (5)$$

$A=\frac{a_p}{a_f}$, wall- to- fluide diffusivity ratio

The other dimensionless variables are defined as follows:

U= $\frac{u}{u_0}$, V= $\frac{v}{u_0}$, Z= $\frac{z}{d}$,

R= $\frac{r}{d}$, $\theta=\frac{(T-T_0)}{q_p(\frac{d}{K_f})}$ P= $\frac{p^*}{\rho_0 u_0{}^2}$.,

$P^*=P+\rho_0 gz$

It is noted that the same equations are used in the area occupied by the fluid and in the solid wall. In order to make sure that velocity in the solid are negligible, the coefficients of diffusion of the quantity of the movement in the solid must tend towards the infinite one.

3. Dimensionless boundary conditions

Since the flow is axisymmetric, the flowing dimensionless boundary conditions for Eqs. (1)-(5) can be adopted

- At the symmetry (R=0, Z):

$$\frac{\partial U}{\partial R}=0, V=0, \frac{\partial \theta}{\partial R}=0$$

- At the wall

(R= R_0, Z): $U=V=0, \quad \frac{\partial \theta}{\partial R}=\frac{1}{K}=\frac{k_f}{k_p}$

- Inlet of the duct

$(Z=0,R): U(R)=2.(1-(2R)^2), V=0, \theta=0$

- Outlet of the duct (Z=L/d, R):

$$\frac{\partial U}{\partial Z}=\frac{\partial V}{\partial Z}=\frac{\partial \theta}{\partial Z}=0$$

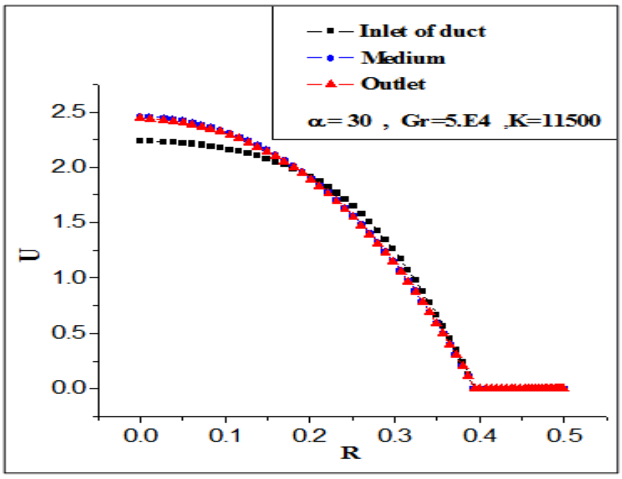
4. Resolution of the Systems of Equations:

The numerical resolution of the equations is based on the finite volumes method, the discretization of the equations is completed by using the Power Low Differencing Scheme "PLDS" for the discretization of the terms of convection. The algorithm "SIMPLE" (Semi-Implicit Method for Presses Linked Equation) was adopted to ensure the coupling pressure velocity, and the discretized equations obtained were solved by using the method of sweeping associated with the algorithm Thomas "TDMA" (Tri-Diagonal Matrix Algorithm)

Results and Discussion:

Velocity field and streamlines

The figures (2) and (3) respectively represent the profiles and the velocity field for the different values of Grashof number. We chose to represent the profile velocity at the inlet; in the medium and the outlet of duct to see the evolution axial velocity along the tube. The velocity have a traditional parabolic profile of the laminar flow, we notice that the heating of the fluid does not have any effect on the profile velocity, we can also note that the velocity increases as one advances along duct to reach a maximum value in the zone of establishment of flow.



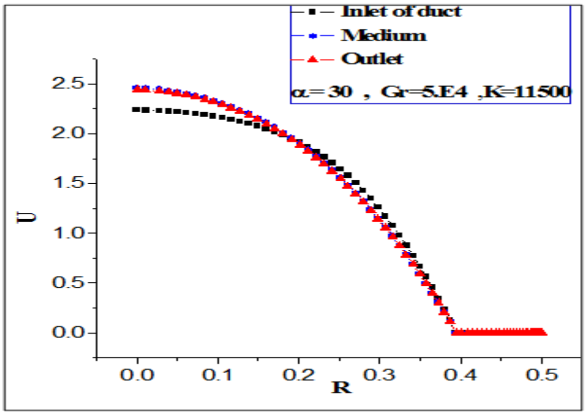


Fig. 2. Velocity profile for the different sections of duct

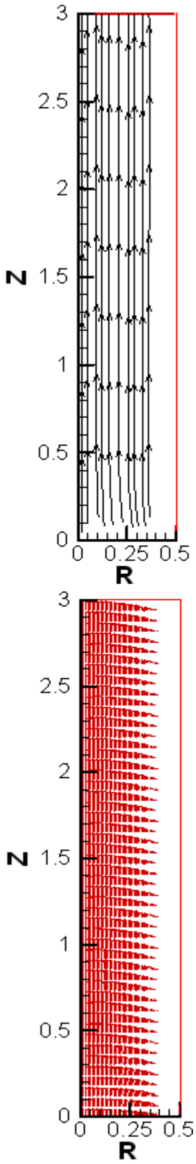
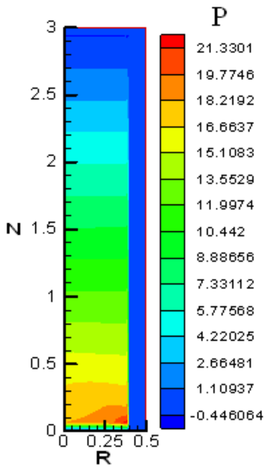


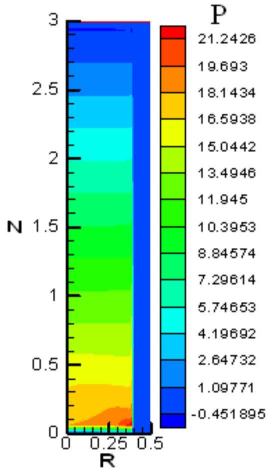
Fig. 3. Velocity fields and streamlines for Gr=5. 10^6, (α=30 and K=11500)

Pressure Field:

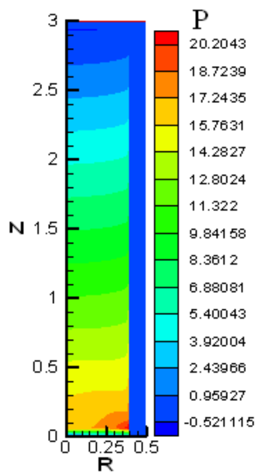
The influence of the variation of the Grashof number on the fields of pressure is presented in the figure 4. We note that the pressure decreases as one advances along duct until reaching a minimal value (P=0) (satisfaction of the condition of exit). It is also noted that the pressure is null on the level of the wall. The values of the pressure decrease when the values of a number of Grashof thus increase we note that this last influences the fields of pressure.



Gr=5. 10^5



Gr=5. 10^6



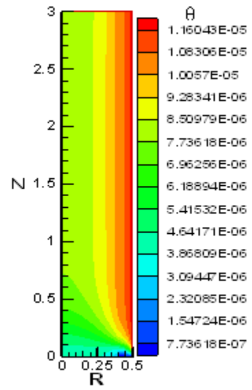
Gr=5. 10^7

Fig. 4. Pressure fields for different values of Grashof number. α=30°, K=11500

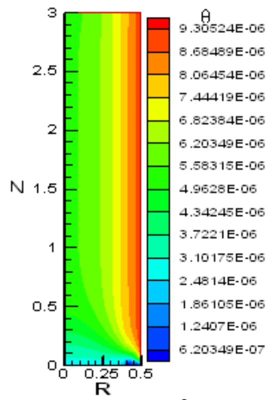
Temperature Fields:

The Figure 5 represents the temperature fields for the various values of the Grashof numbe. We note that for the three values of Gr the temperature takes its greater value meadows of the wall and it decreases until it reaches its lowest value on the level of the axis of duct.

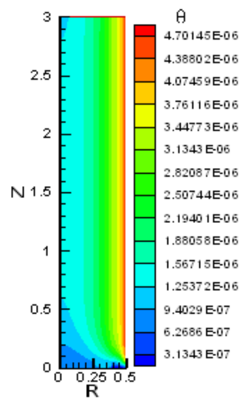
For the values of Gr (Gr< 5.10^6) the temperature tends to becoming homogeneous at the exit of duct, the transfer of heat by conduction starting from the wall of duct dominates that by convection at the entry. For the values of Gr ($Gr > 5.10^6$) we note a considerable difference between the values of temperature meadows of the wall and those meadows of the axis, in this case the transfer of heat by convection dominates that by conduction.



Gr=5. 10^5



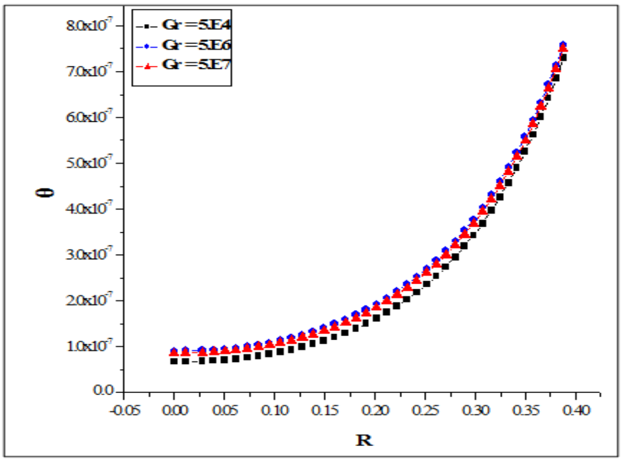
Gr=5.10^6



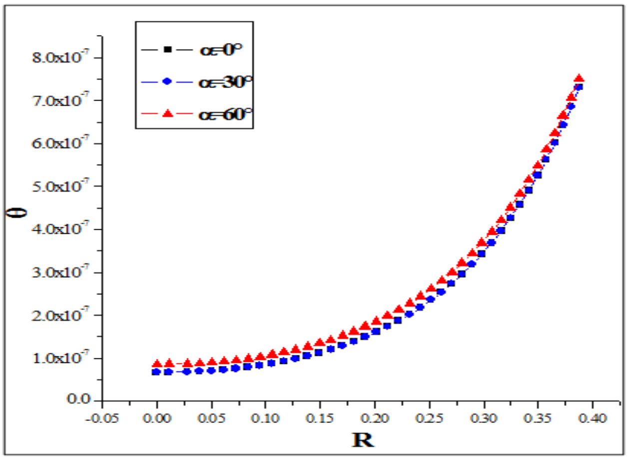
Gr=5.10^7

Fig. 5. Temperature fields for the different values of the Grashof number α=30°, K=11500

The profiles of temperature in the fluid and the solid for the various values of the Gr., α and K are represented in the figures (6), (7) and (8). According to these figures we notice that for the solid, neither the variation of the Gr. that of α influences the profile of temperature, only the report/ratio of the conductivity of the wall (K) which influences, the latter is explained by high conduction when conductivity takes large values. In addition for the fluid, we note that the variation of the three parameters (Gr., α and K) influences but in a way distinct on the profile from temperature this is explained by the variation of the variation in temperature between the various points.

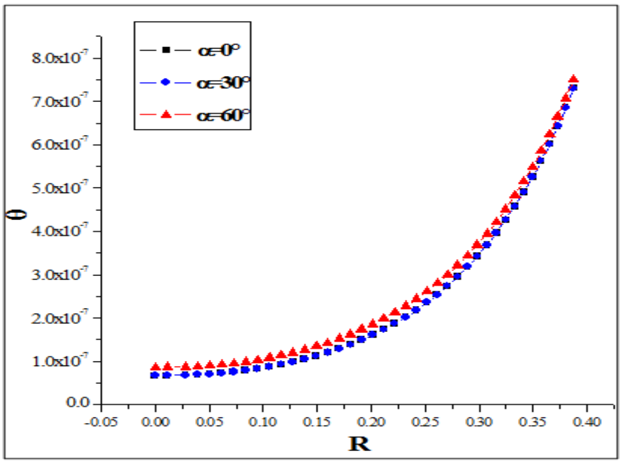


(a) For the fluid

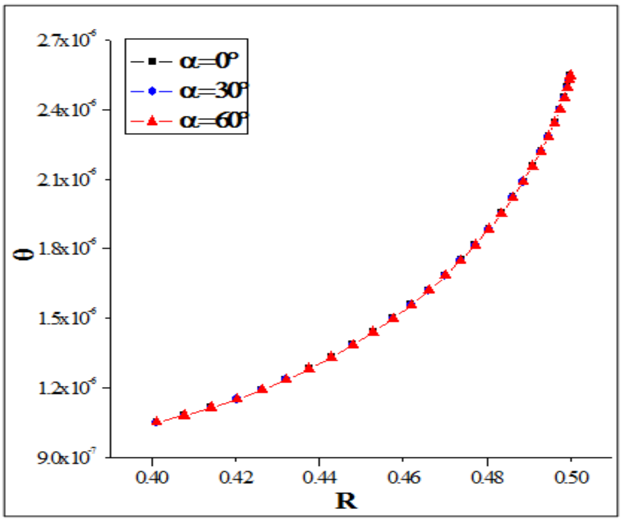


(b) For the wall

Fig. 6. Profile of temperature for different values of Grashof number (α=30°, K=11500)

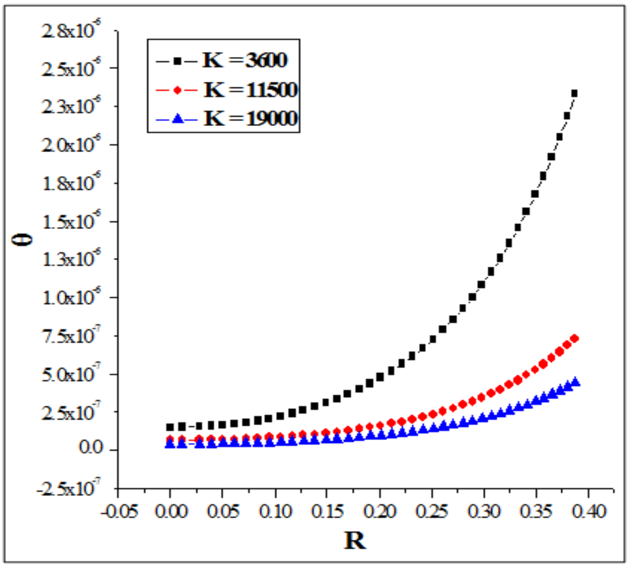


(a) For the fluid

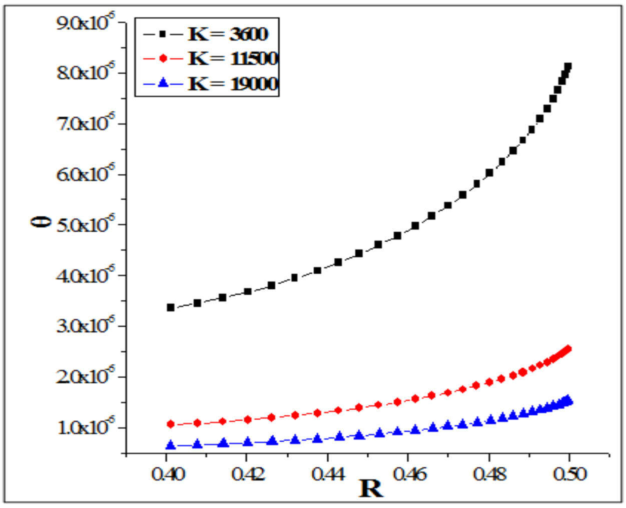


(b)For the wall

Fig. 7. Profile of temperature for different values of the angle α (Gr=5.10^6 K=11500)



(a) For the fluid



(b)For the wall

Fig. 8. Profile of temperature for different values of conductivity K (α =30° Gr=5.10^6)

Axial evolution of the Nusselt number:

Effect of variation of Grashof number on the Nusselt number

The axial evolution of the number of Nusselt for various numbers of Grashof is represented on figure 9. It is clear that the increase in the rate of heating to the external wall of the tube (increase in the number of Grashof), involves a secondary flow disturbing the principal flow. This implies a significant improvement in the transfer of heat, translated by the increase in the number of Nusselt. Near to the input area to the tube, the three curves of Nu are completely confused.

From Z=0.25, the curve corresponding to Gr = 5.10^8 is taken down to take definitely higher values. Thus it is concluded that the increase in the number of Grashof, entrained an improvement connects transfer of heat.

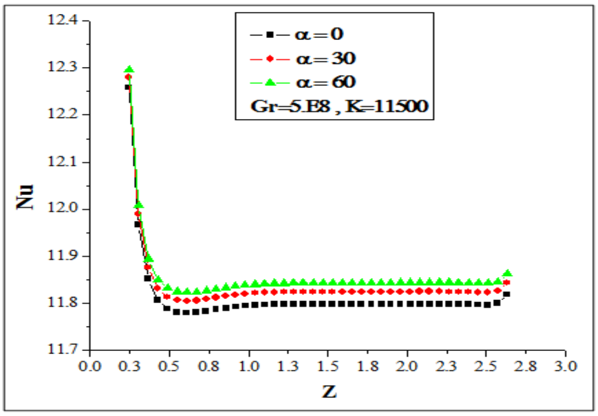


Fig. 9. Axial evolution of the Nusselt number for the different values of α

Effect of variation of the conductivity of the wall

The Figure 10 represents the axial evolution of the number of Nusselt for various materials of the wall. According to this figure we distinguished two zones from evolution: in the first zone, the three curves are confused what explains why the convection is better in this zone. Then we notice in the second zone, the detachment of the curve corresponds to the value K = 3600 (of Iron) by taking higher values from Z = 0.75. This curve is followed by that corresponds to K=11500 (of Aluminium) then by that of K=19000 (of Copper). We can conclude that the convection is better for materials with low conductivities (Iron in this case).

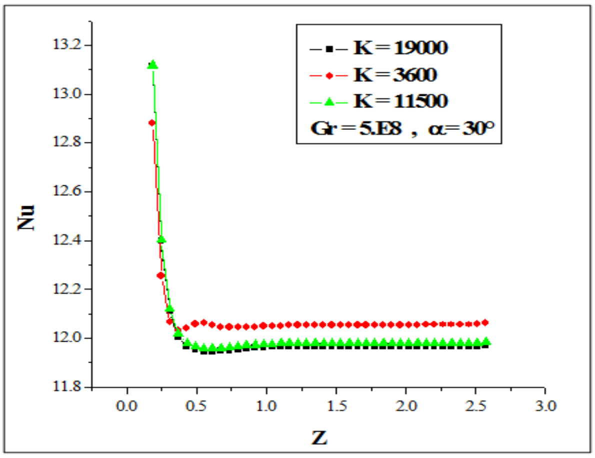


Fig. 10. Axial evolution of Nusselt number for the different values of K

Variation of the Parietal constraint:

Figures 11, 12, 13 represent the axial evolutions of the parietal constraint for various values of Grashof and the three values of the angles and parietal conductivity. It is noted that the curves represent a very fast reduction in the vicinity immediate of the entry of duct, before reaching an asymptotic value, this position corresponds to the maximum of the intensity which the secondary movements due to the natural convection can reach. The increase in the rate of heating (increase in Gr.) involves an increase in the parietal pressure. On the other hand it is noted that the effect of the angle of inclination (Fig.12) and the conductivity of material (Fig.13) is almost negligible

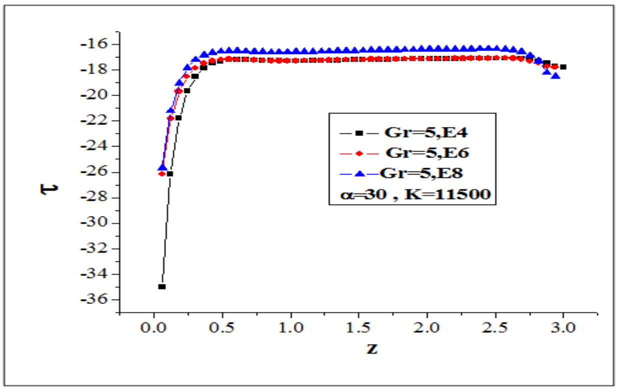


Fig. 11. Axial evolution of the parietal constraint for the various values of the number of Grashof

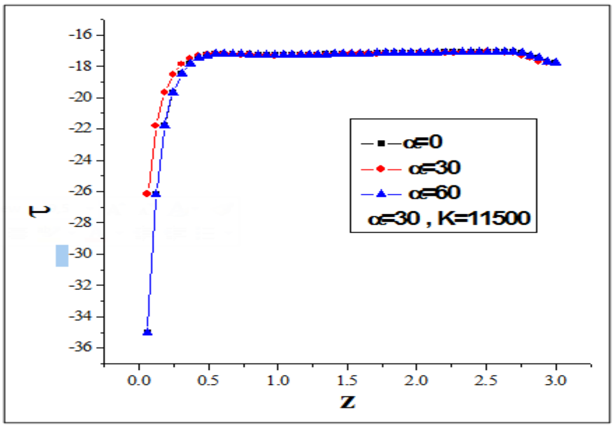


Fig. 12. Axial evolution of the parietal constraint for the various values of the angle α

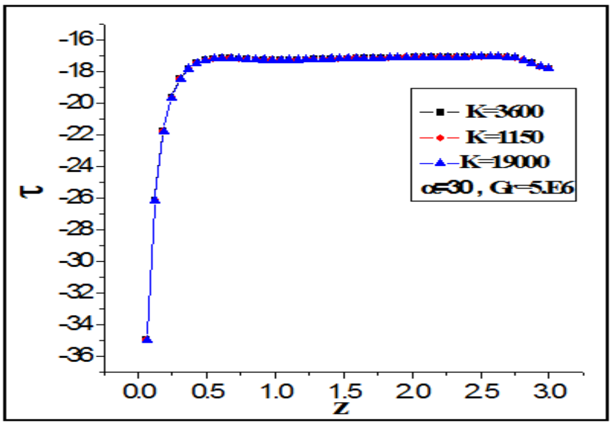


Fig. 13. Axial evolution of the parietal Constraint for the various Values of the report/ratio of conductivity K

5. Conclusion

In the present work, numerical study has been performed to investigate the steady conjugated laminar mixed convection in inclined duct, submitted to uniform and constant wall heat flux. The solution takes wall conduction and wall heat capacity in to account.

The various investigated parameters were the Reynolds and Grashof numbers, the thermal conductivity and diffusivity ratios between the tube wall and the fluid, and the dimensionless wall thickness

The discretization of the equations simulating this phenomenon (equation of mass, of energy and momentum) is made by finite volume method, the SIMPLE algorithm is adopted for assured the coupling velocity pressure and the method of sweeping associated with the algorithm with Thomas is used for the resolution of the discretized equations.

In order to see the influence of the variation of the number of Grashof, the angle of inclination and the conductivity of material of the wall on the thermal and hydrodynamic fields of the flow. We made numerical simulations for numbers of Grashof equalize with: 5.10^4, 5.10^6 and 5.10^7 and for angle of inclination: 0°, 30° and 60°. For material we chose copper (K=19000), iron (K=3600) and aluminium (K=11500).

We presented the fields velocity, pressure and temperature, the axial evolution of the number of Nusselt that of the parietal constraints for the various studied cases.

We found that the convection is better for:

1. Great values of the Grashof number
2. For the great values of α.
3. for an iron wall.

Nomenclature

A - Report/ratio of diffusivity ($A=a_p/a_f$) [-]
a - Thermal diffusivity($=k/\rho.Cp$) [m .s^{-2}]
C_p - Spécifique heat at constant pressure [J.Kg-1 K^{-1}]
D - Diameter of the tube [m]
Gr_{qp} - Grashof number based on the heat flux ($=g.\beta.q_p.d^3/\nu^2$) [-]
k - Conductivity[W.m^{-2}.K^{-1}]
K - Report/ratio of conductivity ($K=k_p/k_f$) [-]
L - Length of the duct [m]
Nu - Nusselt number ($=h.d/k_f$) [-]
P - Pressure [N.m^{-2}]
P - Dimensionless pressure [-]
p^* - Motrice pressure [N.m^{-2}]
Pe - Peclet number (=Re.Pr) [-]
Pr - Prandtl number ($=\nu/a$) [-]
r - Radial coordinates [m]
R - Dimensionless radial Coordinates [-]
R_0 - The ray of the duct [m]
Ra - Rayleigh number (=Gr.Pr) [-]
Re - Reynolds number ($=u_0.d/\nu$) [-]
T - Temperature [K]
u, v - Components velocity axial and radial [m .S^{-1}]
U, V - Dimensionless Components axial and radial velocity [-]
z - Coordinates axial [m]
Z - Dimensionless Axial coordinates [-]

Greek letters

α - Angle of inclination[degree]
β- - Isobaric coefficient of thermal expansion fluid[-]
ΔT - Variation in temperature enters the wall and the ambient conditions [K]
θ - Dimensionless temperature
ν - Kinematic viscosity [m². S^{-1}]
ρ - Density [Kg.m^{-3}]
ρ_0 - Density à T_0 [Kg.m^{-3}]
τ - Dimensionless parietal constraint [-]

Subscripts

0 -Inlet
F - Fluid
p - Wall

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