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Synchronization Phenomena in Coupled Non-identical Chaotic Circuits

Ch. K. Volos[1,*], A. Giakoumis[1], I. M. Kyprianidis[1], I. N. Stouboulos[1], H. E. Nistazakis[2] and G. S. Tombras[2]

[1]*Physics Department, Aristotle University of Thessaloniki, Thessaloniki, GR-54124, Greece.*
[2]*Department of Electronics, Computers, Telecommunications and Control, Faculty of Physics, National and Kapodistrian University of Athens.*

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Abstract

The last decades, the synchronization between coupled chaotic circuits has attracted the interest of the research community because it is a rich and multi-disciplinary phenomenon with broad range applications, such as in broadband communication systems, in secure communications and in cryptography. For this reason many coupling schemes between identical nonlinear circuits with chaotic behavior have been presented. However, the basic drawback of the majority of these schemes is the request the coupled circuits to be identical, due to the fact that in real world applications this is impossible. Motivated by the aforementioned inevitable feature of this class of circuits, which drives the systems out of synchronization, a unidirectional coupling scheme between non-identical, nonlinear circuits, is presented in this work. The circuit, which is used, realizes a four-dimensional modified Lorenz system, which is capable of producing chaotic and hyperchaotic attractors. Furthermore, the coupling scheme is designed by using Nonlinear Open Loop Controllers to target the synchronization state. The stability of synchronization is ensured by using Lyapunov function stability theory. Simulation results of the proposed coupling scheme by using SPICE are also presented to verify the feasibility of the proposed coupling scheme.

Keywords: Chaos, hidden oscillation, synchronization, unidirectional coupling, nonlinear open loop controller.

1. Introduction

In the last three decades the phenomenon of synchronization between coupled chaotic systems has attracted the interest of the scientific community because it is a rich and multi-disciplinary phenomenon with broad range applications, such as in a variety of complex physical, chemical, and biological systems [1-7], as well as in secure communications [8], cryptography [9,10] and broadband communication systems [11]. In more details, synchronization of chaos is a process, where two or more chaotic systems adjust a given property of their dynamics motion to a common behavior, such as identical trajectories or phase locking, due to coupling or forcing. Because of the exponential divergence of the nearby trajectories of a chaotic system, having two chaotic systems being synchronized, might be a surprise. However, today the synchronization of coupled chaotic oscillators is a phenomenon well established experimentally and reasonably well understood theoretically.

The history of chaotic synchronization's theory began with the study of the interaction between coupled chaotic systems in the 1980's and early 1990's by Fujisaka and Yamada [12], Pikovsky [13], Pecora and Carroll [14]. Since then, a wide range of research activity based on synchronization of nonlinear systems has risen and a variety of synchronization's types, depending on the nature of the interacting systems and of the coupling schemes, has been presented.

In particular, the phenomenon of complete synchronization is the most studied type of synchronization. In this case, two coupled chaotic systems are leaded to a perfect coincidence of their chaotic trajectories i.e., $x_1(t) = x_2(t)$ as $t \to \infty$.

Recently, a great interest for dynamical systems with hidden attractors has been raised. The term "*hidden attractor*" refers to the fact that in this class of systems the attractor is not associated with an unstable equilibrium and thus often goes undiscovered because they may occur in a small region of parameter space and with a small basin of attraction in the space of initial conditions [15-20]. In 2010, for the first time, a chaotic hidden attractor was discovered in the most well-known nonlinear circuit, in Chua's circuit, which is described as a three-dimensional dynamical system [21,22].

Furthermore, systems with hidden attractors have received attention due to their practical and theoretical importance in other scientific branches, such as in mechanics (unexpected responses to perturbations in a structure like a bridge or in an airplane wing) [23,24]. So, the study of these systems is an interesting topic of a significant importance.

So, in this work a hyperchaotic four-dimensional modified Lorenz system with hidden attractors is used for studying the unidirectional coupling of two non-identical systems of this kind, by using Nonlinear Open Loop Controllers (NOLCs). The simulation results from system's numerical integration as well as the circuital implementation of the proposed system in SPICE, verify the appearance of the complete synchronization phenomenon.

The paper is organized as follows. In Section 2 the four-dimensional modified Lorenz system, which is used in this work, is presented. The unidirectional coupling scheme, by using the nonlinear open loop controllers, is discussed in

* E-mail address: volos@physics.auth.gr
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Section 3. Section 4 presents the circuital implementation of the aforementioned coupling system as well as the simulation results which are obtained by using the SPICE. Finally, the conclusive remarks are drawn in the last Section.

2. The Four-Dimensional Modified Lorenz System

In this work the simplest four-dimensional hyperchaotic Lorenz-type system, which has been proposed by Gao and Zhang [23], is used. This system is an extension of a modified Lorenz system, which was studied by Schrier and Maas as well as by Munmuangsaen and Srisuchinwong [24,25]. The proposed system, which is structurally a very simple four-dimensional dynamical system having only two independent parameters (c, d), is described by the following set of differential equations.

$$\begin{cases} \dot{x}_1 = x_2 - x_1 \\ \dot{x}_2 = -x_1 x_3 + x_4 \\ \dot{x}_3 = x_1 x_2 - c \\ \dot{x}_4 = -d x_2 \end{cases} \tag{1}$$

In this section the system's dynamic behavior is investigated numerically by employing a fourth order Runge-Kutta algorithm. For this reason, the bifurcation diagram, which is a very useful tool from nonlinear dynamics theory, is used. In Fig.1 the bifurcation diagram of the variable y versus the parameter d, for c = 2.7, reveals the richness of system's dynamical behavior.

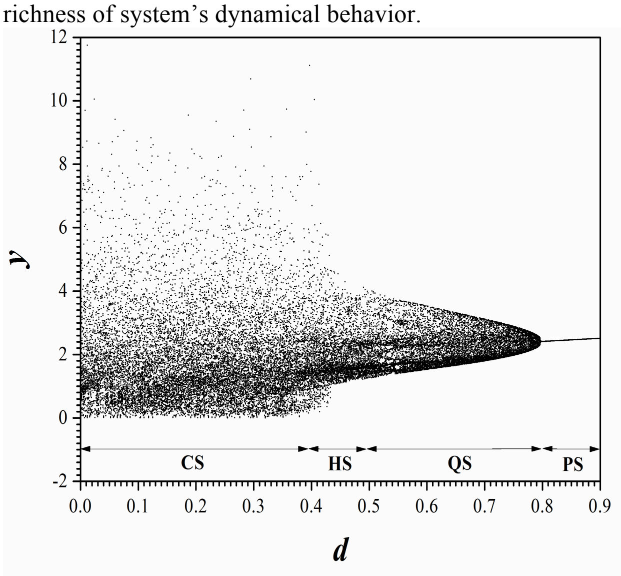

Fig. 1. Bifurcation diagram of y versus d, for c = 2.7.

System's (1) dynamics presents limit cycles, quasiperiodicity, chaos, and hyperchaos, which can make control difficult in practical applications where a particular dynamic behavior is desired. As d is decreased from d = 0.9 the system goes from a period-1 steady state (PS), through a quasi-periodic state (QS), to a chaotic state (CS). However, the very interesting feature of the specific system is the existence of hyperchaotic state (HS) for a range of $d \in [0.39, 0.49]$, where the system has two positive Lyapunov exponents, (i.e. for d = 0.4 system's Lyapunov exponents are: LE_1 = 0.12806, LE_2 = 0.01161, LE_3 = 0 and LE_4 = –1.58236), which is an indication of hyperchaos.

3. The Coupling Scheme

The unidirectional coupling scheme can be described by the following system of differential equations:

$$\begin{cases} \dot{x} = f(x) \\ \dot{y} = f(y) + U \end{cases} \tag{2}$$

where $(f(x), f(y)) \in R^n$ are the flows of the master and slave system respectively, while $U = [U_1, U_2, U_3, U_4]^T$ is the Nonlinear Open Loop Controllers [26]. The error function is defined by $\boldsymbol{e} = \beta\boldsymbol{y} - \alpha\boldsymbol{x}$, with $\boldsymbol{e} = [e_1, e_2, e_3, e_4]^T$, $\boldsymbol{x} = [x_1, x_2, x_3, x_4]^T$ and $\boldsymbol{y} = [y_1, y_2, y_3, y_4]^T$. In this work, parameters (c, d) in the slave system have been changed into (c', d'), in order the systems to be non-identical. So, the error dynamics, in the specific case of system (1), are written as:

$$\begin{cases} \dot{e}_1 = e_2 - e_1 + \beta U_1 \\ \dot{e}_2 = a x_1 x_3 - \beta y_1 y_3 + e_4 + \beta U_2 \\ \dot{e}_3 = -a x_1 x_2 + \beta y_1 y_2 - (c' - c)\beta + c(a - \beta) + \beta U_3 \\ \dot{e}_4 = -d e_2 - (d' - d)\beta y_2 + \beta U_4 \end{cases} \tag{3}$$

For stable synchronization $\boldsymbol{e} \to 0$ with $t \to \infty$. By substituting the conditions in Eqs. (3) and taking the time derivative of Lyapunov function

$$\begin{aligned} \dot{V}(e) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = \\ & e_1\left(e_2 - e_1 + \beta U_1\right) + e_2\left(a x_1 x_3 - \beta y_1 y_3 + e_4 + \beta U_2\right) + \\ & + e_3\left[-a x_1 x_2 + \beta y_1 y_2 - (c' - c)\beta + c\left(a - \beta\right) + \beta U_3\right] + \\ & + e_4\left[-d e_2 - (d' - d)\beta y_2 + \beta U_4\right] \end{aligned} \tag{4}$$

we consider the following NOLCs

$$\begin{cases} U_1 = -\dfrac{1}{\beta} e_2 \\ U_2 = -\dfrac{1}{\beta}\left(e_2 + a x_1 x_3 - \beta y_1 y_3 + e_4\right) \\ U_3 = -\dfrac{1}{\beta}\left[e_3 - a x_1 x_2 + \beta y_1 y_2 - (c' - c)\beta + c(a - \beta)\right] \\ U_4 = -\dfrac{1}{\beta}\left[e_4 - d e_2 - (d' - d)\beta y_2\right] \end{cases} \tag{5}$$

such that

$$\dot{V}(e) = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \tag{6}$$

Equation (6) ensures the asymptotic global stability of synchronization.

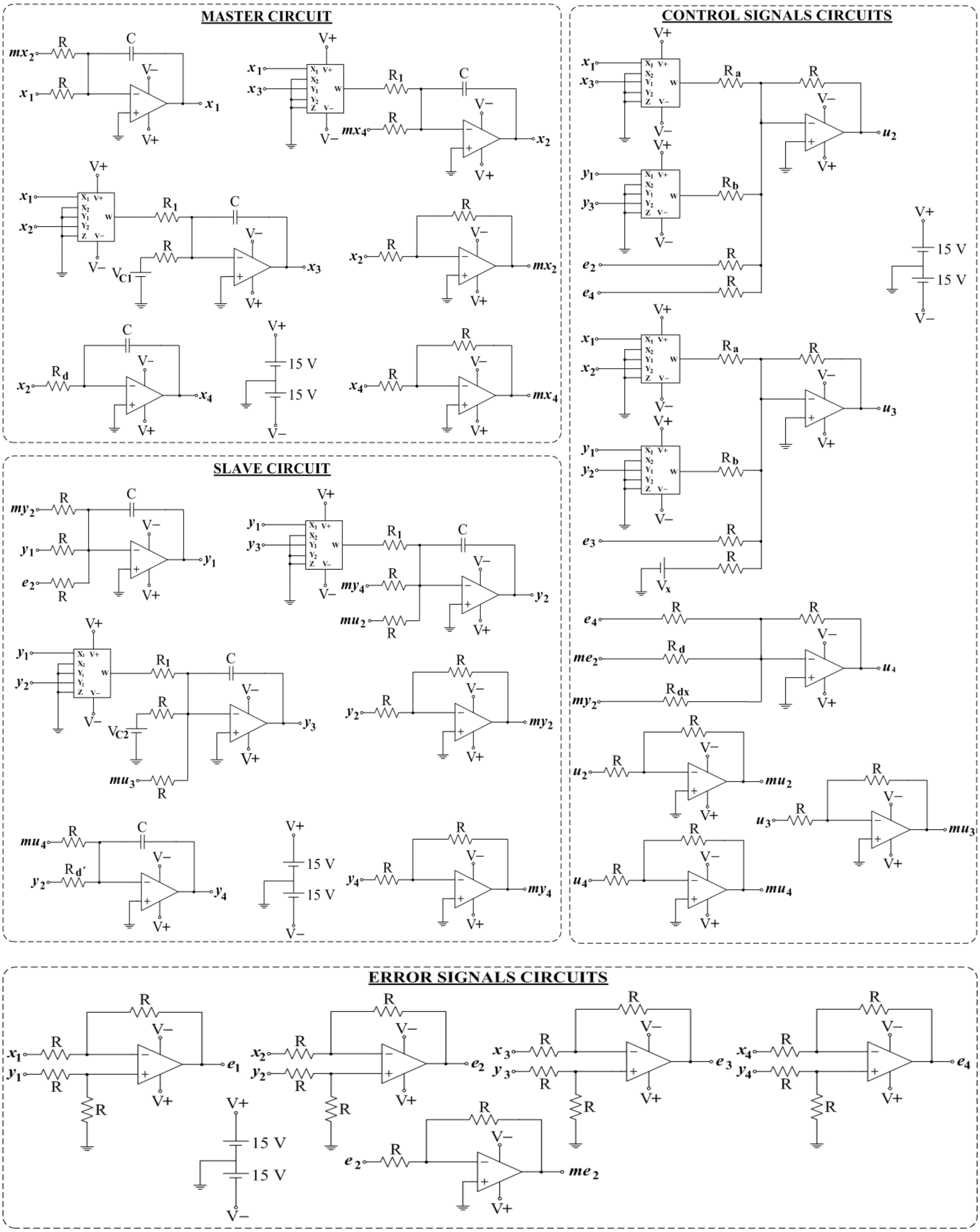

Fig. 2. Schematics of the master, slave, errors and control signals circuits, modelling the proposed coupling scheme.

4. The Circuital Realization

In Fig. 2 the schematics of the master and slave circuits, as well as the circuits for producing the error and control signals of Eqs. (3) & (5) are depicted. The circuit components have been selected as: R = 10 kΩ, $R_1 = R_a = R_b$ = 1 kΩ, R_d = 25 kΩ, $R_{d'}$ = 11.111 kΩ, R_{dx} = 20 kΩ, C = 10 nF, V_{C1} = 2.7 V, V_{C2} = 3.2 V and V_X = 0.5 V, while the power supplies of all active devices are ±15 V_{DC}. Also, the operational amplifier TL084 and the analog multiplier AD633 have been used in circuits' realization. For the chosen set of components the parameters are: $\alpha = 1$, $\beta = 1$, $c = 2.7$, $c' = 3.2$, $d = 0.4$ and $d' = 0.9$. For the chosen set of parameters, the master circuit is in hyperchaotic state, while the slave system is in periodic state, which is confirmed by using the electronic simulation package Cadense OrCAD (Fig. 3). Finally, in Fig. 4 the system's complete synchronization is verified.

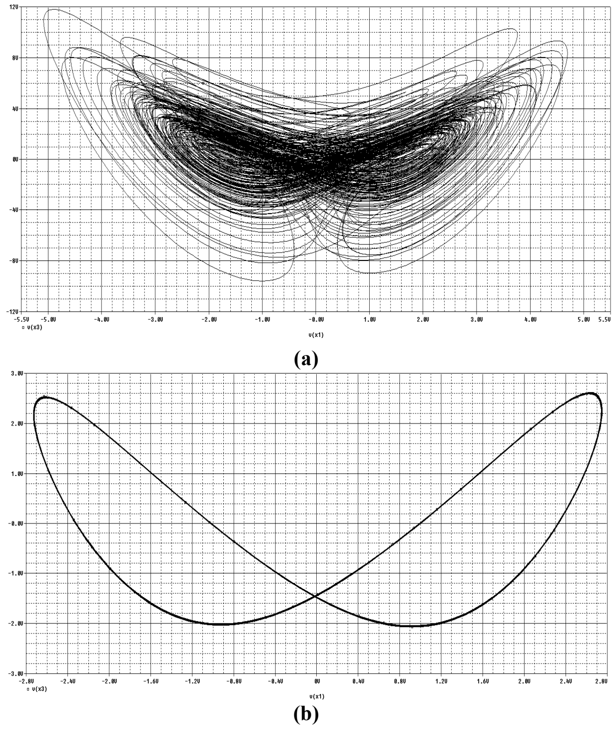

Fig. 3. (a) The hyperchaotic and (b) the periodic attractor in (x_3, x_1)-plane, for the chosen set of parameters, designed with SPICE.

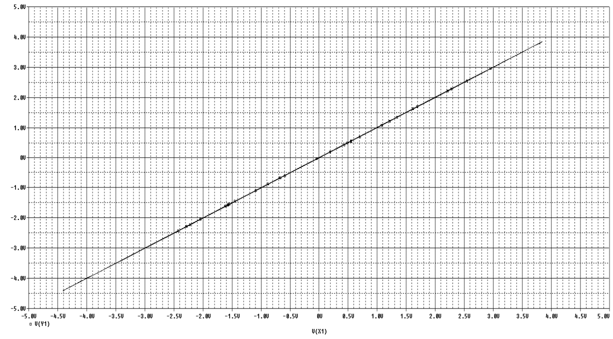

Fig. 4. Phase portrait of y_1 versus x_1. (Synchronization is verified)

5. Conclusion

In this work the synchronization of a unidirectional coupling scheme of two non-identical, nonlinear circuits, by using Nonlinear Open Loop Controllers, was presented. The simulation results from SPICE verified the feasibility of the proposed scheme even if the coupled circuits are in different states due to their different parameter values. So, as a conclusion, this work proves that the proposed scheme should be used in the design of a secure communication system, by using chaotic or hyperchaotic nonlinear circuits in a master-slave configuration.

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