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Enhanced Velocity Tracking Control using Higher-Order Model-Free Adaptive Approach for Permanent Magnet Synchronous Motor

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Abstract

The velocity of permanent magnet synchronous motors (PMSMs) must be exactly controlled to promote the development of high-performance drive systems. This work proposed an enhanced adaptive controller to improve the tracking performance of PMSM based on a new higher-order adaptive control mechanism. Firstly, the controller adopted a novel higher-order weighted one-step-ahead criterion function to generate the control law for an equivalent partial form linearization system. This model-free design depended on a pseudo partial derivative (PPD) that was derived online from the input and output (I/O) information of the controlled plant. This approach is especially useful for nonlinear systems with vague dynamics. Secondly, the design guaranteed the stability of the bounded input and output and ensured tracking error monotonic convergence under a restricted set of parameters. Thirdly, the design was simulated and applied on an actual PMSM system to demonstrate the effects of different control parameters. Results show that the approach is fit for motor control and yields satisfactory velocity tracking precision and fault tolerance along with the increased order. Moreover, the approach involves lesser calculation efforts for parameter estimation and simplifies the controller design. This study can meet the demand of velocity tracking and demonstrates the effective applications of this approach for real nonlinear motor systems that are typically difficult to model and control.

Keywords: PMSM, Higher-order, Model-free, Adaptive control, Velocity tracking

1. Introduction

Permanent magnet synchronous motors (PMSMs) are widely used in high-performance servo applications owing to the high efficiency, superior power density, and large torque to inertia ratio [1]. However, PMSMs are nonlinear multivariable dynamic systems and it is difficult to control their velocity with high precision due to the parameter perturbations and the non-modeled dynamics. Adaptive control has been widely used for such uncertain systems [2], but this approach is typically assumed that the mathematical model of the system is known and the parameters are unknown or slow time-varying [3]. For practical PMSM systems, the models are often complex to build and the parameters are hard to identify, which make the adaptive control questionable. This motivates us to study data-driven control approaches.

Data-driven control approaches mainly concentrate on the importance of input and output (I/O) information in studying systems behavior and design controller merely using I/O data of a plant. Since these approaches do not require any explicit model or the structural information of the plant, the modeling process and the non-modeled dynamics all disappear. Now, several data-driven control approaches can be found, such as simultaneous perturbation stochastic approximation control, multi-level recursive control, model free adaptive control (MFAC), unfalsified control, iterative feedback tuning, virtual reference feedback tuning and lazy learning [4], [5], [6]. Compared with other approaches, the MFAC offers low computational burden, easy implementation and strong robustness, which make it suitable for many practical applications. But its main problems that need to be solved are the utilization amount and the utilization ratio of the historical I/O information. Therefore, to solve the mentioned problems, this paper focuses on an enhanced adaptive control approach.

2. State of the art

As a novel data-driven approach, MFAC is based on 'pseudo partial derivative (PPD)', a new concept of partial derivative derived from I/O information [7], [8]. This approach differs from proportional-integral-derivative (PID) control, fuzzy control, neural network control and expert system control [9], and does not require support from the model, rule or prior knowledge.

The convergence analysis, stability analysis and general procedures of controller design are directed by fairly complete guidelines [10]. The recent developments in this field have focused on improving the designs and applications of adaptive controllers for nonlinear dynamic systems. For instance, [11] proposed a second-order universal model adaptive controller which parameters were optimized by a gradient descent algorithm, whilst [12] designed a higher-order model free adaptive controller for controlling a class of single-input single-output (SISO) nonlinear systems that could obtain promising results using only input information. However, these designs do not fully utilize the historical I/O data of the controlled plant, and

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using incomplete or missing data results in the poor robustness, oscillation and instability of these systems. Model-free predictive control [13], [14] and model-free iterative learning control [15], [16], [17] have been developed integrating MFAC with advanced control strategies to improve their performance. The former offers strong robustness and economic requirements in an optimization criterion, but its performance greatly depends on the prediction accuracy of the model. However, incorrect model parameters can lead to inaccurate predictions. The latter finds a control input that generates the desired output over a finite time interval through trial repetition. However, this process only limits the motor control.

MFAC has been widely applied to address the theoretical and actual problems in control engineering, power grids and systems, intelligent transportation systems, electrical drives and process industry [18]. This approach can also be used to control nonlinear PMSM systems.

Exploiting a larger amount of historical I/O information can improve design accuracy. However, the appropriate amount and the utilization ratio of the I/O information remain unclear. This paper proposed a higher-order model-free adaptive control (HMFAC) approach to improve the tracking performance of a PMSM system. To design the adaptive control law, this approach adopted a novel weighted one-step-ahead input criterion function with an online-derived PPD. The approach exploited a larger amount of historical I/O information in a sliding time window and improved the robustness and stability of the controlled system. Theoretical analysis and simulations were performed to validate the effectiveness of this approach.

The rest of this paper is organized as follows. Section 3 describes the HMFAC design that employs the linearization method for a SISO nonlinear system and presents the convergence and stability analyses. Section 4 presents a simulation to illustrate the effectiveness and superior performance of HMFAC. Section 5 concludes the paper.

3. Methodology

3.1 Problem formulation and dynamic linearization method

The controlled system is described by the following SISO nonlinear time-varying equation:

$$\begin{aligned} y(t+1) = f(&y(t), y(t-1), \dots, y(t-n_y), \\ &u(t), u(t-1), \dots, u(t-n_u)) \end{aligned} \tag{1}$$

where $y(t)$ and $u(t)$ are the output and input at time t respectively, n_y and n_u are the unknown orders and $f(\dots)$ is an unknown nonlinear function.

To guide our discussion, we make the following assumptions:

Assumption 1: The input and output of system (1) are observable and controllable, that is, for the desired bounded output signal $y^*(t+1)$, there exists a bounded feasible input signal that makes the practical output equal to the desired output.

Assumption 2: The partial derivatives of $f(\dots)$ with respect to control input $u(t)$ are continuous.

Assumption 3: System (1) presents the generalized Lipschitz condition, that is, $|\Delta y(t+1)| \le b|\Delta u(t)|$ for any t, where $\Delta u(t) = u(t) - u(t-1)$, $\Delta y(t+1) = y(t+1) - y(t)$, $|\Delta u(t)| \ne 0$ and b is a positive constant.

Theorem 1: For the nonlinear system (1) that satisfies assumptions 1, 2, and 3, there must exist PPD vectors $\phi(t)$ and $|\phi(t)| \le b$ when $|\Delta u(t)| \ne 0$, such that system (1) can be transformed into the following partial form dynamic linearization description:

$$\Delta y(t+1) = \phi(t)\Delta u(t) \tag{2}$$

Equation (2) is the universal model of system (1) that converts a complex SISO nonlinear system into a linear system with the time-varying parameter $\phi(t)$.

3.2 Adaptive control law algorithm

The adaptive control law finds a suitable control input $u(t)$ sequence to achieve the desired trajectory $y^*(t)$, that is,

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}[y^*(t) - y(t)] = 0 \tag{3}$$

where $e(t)$ is the tracking error of the output.

Unlike the control law in [5], the novel weighted one-step-ahead control criterion function of system (2) is defined as follows:

$$\begin{aligned} J(u(t), a_i, b_j) = &\left|\sum_{i=1}^{L_y} a_i(y^*(t-i+2) - y(t-i+2))\right|^2 \\ &+\lambda\left|\sum_{j=1}^{L_u} b_j(u(t-j+1) - u(t-j))\right|^2 \end{aligned} \tag{4}$$

where λ is a positive weight factor that restricts the variation of control inputs. The first and second items in equation (4) denote the weighted output and input errors of the previous i or j sampling instances, respectively, which are defined at sampling instant t. a_i and b_j denote the weight factors that directly determine the region and degree of previous information. $a_i = (a_1, a_2, \dots, a_{L_y})^T$ with $\sum_{i=1}^{L_y} a_i = 1$, while $b_j = (b_1, b_2, \dots, b_{L_u})^T$ with $\sum_{j=1}^{L_u} b_j = 1$.

Substituting equation (2) into equation (4) defines $J(u(t), a_i, b_j)$ as follows:

$$\begin{aligned} J(u(t), a_i, b_j) = &\left|\sum_{i=1}^{L_y} a_i(y^*(t-i+2) - y(t-i+1)\right. \\ &\left. -\phi(t-i+1)(u(t-i+1) - u(t-i)))\right|^2 \\ &+\lambda\left|\sum_{j=1}^{L_u} b_j(u(t-j+1) - u(t-j))\right|^2 \end{aligned} \tag{5}$$

Using the optimal condition $\frac{1}{2}\partial J(u(\mathrm{t}),a_i,b_j)/\partial u(\mathrm{t})=0$ yields the following:

$$
\begin{aligned}
u(\mathrm{t}) = u(\mathrm{t}-1) &+ \frac{a_1\phi(\mathrm{t})}{\lambda b_1^2 + a_1^2\phi(\mathrm{t})^2}(a_1(y^*(\mathrm{t}+1)-y(\mathrm{t})) \\
&+\sum_{i=2}^{L_y} a_i(y^*(\mathrm{t}-i+2)-y(\mathrm{t}-i+2))) \\
&-\frac{\lambda b_1}{\lambda b_1^2 + a_1^2\phi(\mathrm{t})^2}\sum_{j=2}^{L_u} b_j\Delta u(\mathrm{t}-j+1)
\end{aligned} \tag{6}
$$

Remark: The novel control criterion function (5) differs from the general function in [5]. This function contains not only the control inputs in a j sliding time window but also the output error in an i sliding time window before the current sampling instant to achieve a highly accurate control.

3.3 PPD estimation algorithm

Let $\hat{\phi}(\mathrm{t})$ denotes the estimation of the parameter $\phi(\mathrm{t})$ as described in [5]. The PPD estimation criterion function for system (2) is defined as follows:

$$
\begin{aligned}
J(\hat{\phi}(\mathrm{t})) = &[y(\mathrm{t})-y(\mathrm{t}-1)-\hat{\phi}(\mathrm{t})\Delta u(\mathrm{t}-1)]^2 \\
&+\mu[\hat{\phi}(\mathrm{t})-\hat{\phi}(\mathrm{t}-1)]^2
\end{aligned} \tag{7}
$$

where $\mu>0$ is the penalty factor of the changes in the parameter estimation.

Using the optimal condition $\frac{1}{2}\partial J(\hat{\phi}(\mathrm{t}))/\partial\hat{\phi}(\mathrm{t})=0$ yields the following estimation algorithms:

$$
\begin{aligned}
\hat{\phi}(\mathrm{t}) = \hat{\phi}(\mathrm{t}-1) + &\frac{\eta_k\Delta u(\mathrm{t}-1)}{\mu+\Delta u(\mathrm{t}-1)^2}[y(\mathrm{t}) \\
&-y(\mathrm{t}-1)-\hat{\phi}(\mathrm{t}-1)\Delta u(\mathrm{t}-1)]
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\hat{\phi}(\mathrm{t}) = \hat{\phi}(1),\ &\text{if } |\hat{\phi}(\mathrm{t})| \le \varepsilon \text{ or } |\Delta u(\mathrm{t}-1)| \le \varepsilon \\
&\text{or } sign(\hat{\phi}(\mathrm{t})) \ne sign(\hat{\phi}(1))
\end{aligned} \tag{9}
$$

where $0<\eta_k<2$ is the step-size constant series added in equation (8) to generalize the function, ε is a small positive constant and $\hat{\phi}(1)$ is the initial estimated value of $\hat{\phi}(\mathrm{t})$. Equation (9) is the reset mechanism that confirms the condition of $|\Delta u(\mathrm{t})| \ne 0$ and the tracking ability of equation (8).

The control law algorithm (6), the parameter estimation algorithms (8) and the reset mechanism (9) yield the following higher-order adaptive control scheme:

$$
\begin{aligned}
\hat{\phi}(\mathrm{t}) = \hat{\phi}(\mathrm{t}-1) + &\frac{\eta_k\Delta u(\mathrm{t}-1)}{\mu+\Delta u(\mathrm{t}-1)^2}[y(\mathrm{t}) \\
&-y(\mathrm{t}-1)-\hat{\phi}(\mathrm{t}-1)\Delta u(\mathrm{t}-1)]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\hat{\phi}(\mathrm{t}) = \hat{\phi}(1),\ &\text{if } |\hat{\phi}(\mathrm{t})| \le \varepsilon \text{ or } |\Delta u(\mathrm{t}-1)| \le \varepsilon \\
&\text{or } sign(\hat{\phi}(\mathrm{t})) \ne sign(\hat{\phi}(1))
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
u(\mathrm{t}) = u(\mathrm{t}-1) &+ \frac{a_1\phi(\mathrm{t})}{\lambda b_1^2 + a_1^2\phi(\mathrm{t})^2}(a_1(y^*(\mathrm{t}+1)-y(\mathrm{t})) \\
&+\sum_{i=2}^{L_y} a_i(y^*(\mathrm{t}-i+2)-y(\mathrm{t}-i+2))) \\
&-\frac{\lambda b_1}{\lambda b_1^2 + a_1^2\phi(\mathrm{t})^2}\sum_{j=2}^{L_u} b_j\Delta u(\mathrm{t}-j+1)
\end{aligned} \tag{12}
$$

Figure 1 shows the system structure block diagram of the proposed control scheme. A model-free controller is designed using only the I/O information of the controlled plant to yield a practical output. This information, along with the controller output, is used by an adaptive mechanism to estimate directly the parameter $\phi(\mathrm{t})$ in real time and to force asymptotically the error to zero. When the output tracking characteristic is influenced by parameter variations, non-modeled dynamics or external disturbances, the estimator corrects the adjustable parameter $\phi(\mathrm{t})$ to prevent any differences in tracking.

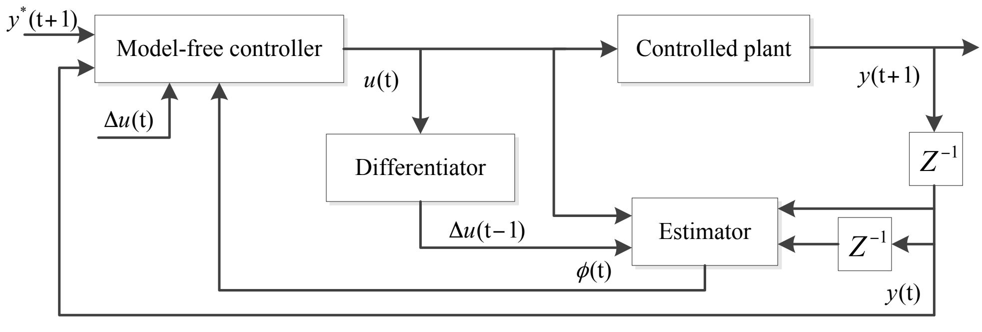

Fig. 1. System structure block diagram of the proposed approach

3.4 Convergence and stability analysis

Assumption 4: The sign of parameter $\phi(\mathrm{t})$ remains constant in any t and $\Delta u(\mathrm{t}) \ne 0$, that is, $\phi(\mathrm{t})>\varepsilon>0$ or $\phi(\mathrm{t})<-\varepsilon$. Only $\phi(\mathrm{t})>\varepsilon>0$ is considered.

Lemma [19]: Let $A = \begin{bmatrix} a_1 & a_2 & L & a_{L-1} & a_L \\ 1 & 0 & L & 0 & 0 \\ 0 & 1 & L & 0 & 0 \\ M & M & M & M & M \\ 0 & 0 & L & 1 & 0 \end{bmatrix}$. If $\sum_{i=1}^{L} |a_i| < 1$, then $s(A) < 1$, where $s(A)$ is the spectral radius of A.

Theorem 2: Following assumptions 1 to 4, when $y^*(t+1) = y^*(t) = \text{const}$ and the appropriate λ is selected, the HMFAC algorithms (10) to (12) that are applied to system (2) guarantee the following:

(1) $\lim_{t\to\infty} |y^* - y(t+1)| = 0$ and

(2) $\{y(t)\}$ and $\{u(t)\}$ are bounded sequences.

The proof is given in the following sections.

3.4.1 Convergence analysis

$\Delta\phi(t) = \hat{\phi}(t) - \phi(t)$ denotes the estimation error of PPD. Subtracting $\phi(t)$ in equation (10) yields the following:

$$\begin{aligned} \Delta\phi(t) &= [1 - \frac{\eta_k \Delta u(t-1)^2}{\mu + \Delta u(t-1)^2}][\Delta\phi(t-1) + \phi(t-1)] + \\ &\quad \frac{\eta_k \Delta u(t-1)}{\mu + \Delta u(t-1)^2} \Delta y(t) - \phi(t) \\ &= [1 - \frac{\eta_k \Delta u(t-1)^2}{\mu + \Delta u(t-1)^2}]\Delta\phi(t-1) + \phi(t-1) - \phi(t) \end{aligned} \tag{13}$$

Taking the absolute value of equation (13) yields the following:

$$|\Delta\phi(t)| = \left|1 - \frac{\eta_k \Delta u(t-1)^2}{\mu + \Delta u(t-1)^2}\right| |\Delta\phi(t-1)| + |\phi(t-1) - \phi(t)| \tag{14}$$

$\eta_k \Delta u(t-1)^2 / (\mu + \Delta u(t-1)^2)$ in equation (14) increases monotonously by $\Delta u(t-1)^2$ with a minimum value of $\eta_k \varepsilon^2 / (\mu + \varepsilon^2)$. Therefore, when $0 < \eta_k < 2$, $\mu > 0$ and $|\phi(t)| \le b$, we obtain the following:

$$0 \le \left|1 - \frac{\eta_k \Delta u(t-1)^2}{\mu + \Delta u(t-1)^2}\right| \le \left|1 - \frac{\eta_k \varepsilon^2}{\mu + \varepsilon^2}\right| \overset{\Delta}{=} d_1 < 1 \text{ and} \tag{15}$$

$$\begin{aligned} |\Delta\phi(t)| &\le d_1 |\Delta\phi(t-1)| + 2b \\ &\le d_1^2 |\Delta\phi(t-2)| + 2bd_1 + 2b \le L \\ &\le d_1^{t-1} |\Delta\phi(1)| + 2b(d_1^{t-2} + d_1^{t-3} + L + 1) \\ &= d_1^{t-1} |\Delta\phi(1)| + 2b \frac{1 - d_1^{t-1}}{1 - d_1} \end{aligned} \tag{16}$$

Equation (16) implies the boundedness of $\Delta\phi(t)$. Therefore, $\hat{\phi}(t)$ is bounded with the bounded $\phi(t)$.

The system tracking error can be defined as follows:

$$\begin{aligned} e(t+1) &= y^*(t+1) - y(t+1) \\ &= y^*(t+1) - y(t) - \phi(t)\Delta u(t) \\ &= e(t) - \phi(t)\Delta u(t) \end{aligned} \tag{17}$$

where $\Delta u(t)$ can be obtained from control law (12) as follows:

$$\begin{aligned} \Delta u(t) &= \frac{a_1 \hat{\phi}(t)}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} (a_1 e(t) + \sum_{i=2}^{L_y} a_i e(t-i+2)) \\ &\quad - \frac{\lambda b_1}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} \sum_{j=2}^{L_u} b_j \Delta u(t-j+1) \end{aligned} \tag{18}$$

Let

$$A(t) = \begin{bmatrix} -\frac{\lambda b_1 b_2}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} & -\frac{\lambda b_1 b_3}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} & L & -\frac{\lambda b_1 b_L}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} & 0 \\ 1 & 0 & L & 0 & 0 \\ 0 & 1 & L & 0 & 0 \\ M & M & M & M & M \\ 0 & 0 & L & 1 & 0 \end{bmatrix},$$

$\Delta U(t) = [\Delta u(t), \Delta u(t-1), L, \Delta u(t-L_u+1)]^T$,

$g(t) = \frac{a_1 \hat{\phi}(t)}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} (a_1 e(t) + \sum_{i=2}^{L_y} a_i e(t-i+2))$ and

$C = [1, 0, L, 0]^T \in R^L$.

Equation (18) can be rewritten as follows:

$$\Delta U(t) = A(t)\Delta U(t-1) + Cg(t) \tag{19}$$

If $\phi(t)$ and $\hat{\phi}(t)$ are bounded, then $\lambda > 0$. Therefore, the following inequality holds when $b_1 \ge 0.5$:

$$\begin{aligned} \left(\sum_{i=2}^{L_u} \left| \frac{\lambda b_1 b_i}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} \right| \right)^{\frac{1}{L_u - 1}} &= \left(\frac{\lambda b_1}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} \sum_{i=2}^{L_u} b_i \right)^{\frac{1}{L_u - 1}} \\ &= \left(\frac{\lambda b_1 (1 - b_1)}{\lambda b_1^2 + a_1^2 \hat{\phi}(t)^2} \right)^{\frac{1}{L_u - 1}} \overset{\Delta}{=} M_1 < 1 \end{aligned} \tag{20}$$

From equation (20) and the Lemma, there exists $\varepsilon_1 > 0$ such that

$$\|A(t)\|_v \le s(A(t)) + \varepsilon_1 \le M_1 + \varepsilon_1 \overset{\Delta}{=} d_2 < 1 \tag{21}$$

where $\|A(t)\|_v$ is the consistent matrix norm of $A(t)$.

Using $\|\Delta U(0)\|_v = 0$, the norm on both sides of equation (19) yields the following:

$$\begin{aligned} \|\Delta U(t)\|_v &= \|A(t)\|_v \|\Delta U(t-1)\|_v + |g(t)| \\ &< d_2 \|\Delta U(t-1)\|_v + |g(t)| \\ &< d_2^2 \|\Delta U(t-2)\|_v + d_2 |g(t-1)| + |g(t)| \\ &< L < \sum_{i=1}^{t} d_2^{i-1} |g(t-i+1)| \end{aligned} \tag{22}$$

Substituting equation (19) into equation (17) yields the following:

$$\begin{aligned} e(\mathrm{t}+1) &= \mathrm{e(t)} - \phi(\mathrm{t})\mathrm{C}^T \Delta U(\mathrm{t}) \\ &= \mathrm{e(t)} - \phi(\mathrm{t})\mathrm{C}^T (A(\mathrm{t})\Delta U(\mathrm{t}-1) + \mathrm{Cg(t)}) \\ &= e(\mathrm{t}) - \frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})}{\lambda b_1^2 + a_1^2\hat{\phi}(\mathrm{t})^2}(a_1\,\mathrm{e(t)} + \\ &\sum_{t=2}^{L_y} a_i\,\mathrm{e(t-i+2)}) - \phi(\mathrm{t})\mathrm{C}^T A(\mathrm{t})\Delta U(\mathrm{t}-1) \end{aligned} \tag{23}$$

Let

$$B(\mathrm{t}) = \begin{bmatrix} 1-\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})(a_1+a_2)}{\lambda b_1^2+a_1^2\hat{\phi}(\mathrm{t})^2} & -\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})a_3}{\lambda b_1^2+a_1^2\hat{\phi}(\mathrm{t})^2} & \mathrm{L} & -\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})a_{L_y}}{\lambda b_1^2+a_1^2\hat{\phi}(\mathrm{t})^2} & 0 \\ 1 & 0 & \mathrm{L} & 0 & 0 \\ 0 & 1 & \mathrm{L} & 0 & 0 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ 0 & 0 & \mathrm{L} & 1 & 0 \end{bmatrix} \text{and}$$

$E(\mathrm{t}+1) = \begin{bmatrix} e(\mathrm{t}+1) & e(\mathrm{t}) & \mathrm{L} & e(\mathrm{t}-L_y+2) \end{bmatrix}^T$.

Equation (23) can be rewritten as follows:

$$E(\mathrm{t}+1) = B(\mathrm{t})E(\mathrm{t}) - \phi(\mathrm{t})C^T A(\mathrm{t})\Delta U(\mathrm{t}-1) \tag{24}$$

If $\lambda_{\min} = \frac{b^2}{4b_1^2}$, then $\lambda > \lambda_{\min}$. Therefore, the following inequalities hold when $a_1 + a_2 \geq 0.5$:

$$\begin{aligned} \left|\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})}{\lambda b_1^2 + a_1^2\hat{\phi}(\mathrm{t})^2}\right| &\leq \left|\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})}{2\sqrt{\lambda}b_1 a_1\hat{\phi}(\mathrm{t})}\right| \\ &\leq \left|\frac{b}{2\sqrt{\lambda}b_1}\right| < \left|\frac{b}{2\sqrt{\lambda_{\min}}b_1}\right| = 1 \end{aligned} \tag{25}$$

$$\begin{aligned} &\left(\left|1-\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})}{\lambda b_1^2+a_1^2\hat{\phi}(\mathrm{t})^2}(a_1+a_2)\right| + \left|\sum_{i=3}^{L_y}\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})}{\lambda b_1^2+a_1^2\hat{\phi}(\mathrm{t})^2}a_i\right|\right)^{\frac{1}{L_y-1}} \\ &= \left(1+\frac{a_1\phi(\mathrm{t})\hat{\phi}(\mathrm{t})}{\lambda b_1^2+a_1^2\hat{\phi}(\mathrm{t})^2}(1-2(a_1+a_2))\right)^{\frac{1}{L_y-1}} \\ &< \left(1+\frac{b}{2\sqrt{\lambda_{\min}}b_1}(1-2(a_1+a_2))\right)^{\frac{1}{L_y-1}} \stackrel{\Delta}{=} M_2 \leq 1 \end{aligned} \tag{26}$$

From equation (26) and the Lemma, there exists $\varepsilon_1 > 0$ such that

$$\|B(\mathrm{t})\|_v \leq s(B(\mathrm{t})) + \varepsilon_1 \leq M_2 + \varepsilon_1 \stackrel{\Delta}{=} d_3 < 1 \tag{27}$$

where $\|B(\mathrm{t})\|_v$ is the consistent matrix norm of $B(\mathrm{t})$.

The norm on both sides of (24) yields the following:

$$\begin{aligned} \|E(\mathrm{t}+1)\|_v &\leq \|B(\mathrm{t})\|_v\|E(\mathrm{t})\|_v + \phi(\mathrm{t})\|A(\mathrm{t})\|_v\|\Delta U(\mathrm{t}-1)\|_v \\ &< d_3\|E(\mathrm{t})\|_v + \phi(\mathrm{t})d_2\|\Delta U(\mathrm{t}-1)\|_v < \mathrm{L} \\ &< d_3^t\|E(1)\|_v + d_2(\sum_{i=1}^{t} d_3^{t-i}\phi(i)\|\Delta U(i-1)\|_v) \end{aligned} \tag{28}$$

Let $h(\mathrm{t}+1) = d_3^t\|E(1)\|_v + d_2(\sum_{i=1}^{t} d_3^{t-i}\phi(i)\|\Delta U(i-1)\|_v)$.

Therefore,

$$\begin{aligned} h(\mathrm{t}+2) &= d_3^{t+1}\|E(1)\|_v + d_2(\sum_{i=1}^{t+1} d_3^{t+1-i}\phi(i)\|\Delta U(i-1)\|_v) \\ &= d_3 h(\mathrm{t}+1) + d_2\phi(\mathrm{t}+1)\|\Delta U(t)\|_v \\ &< d_3 h(\mathrm{t}+1) + d_2\phi(\mathrm{t}+1)(d_2\|\Delta U(t-1)\|_v + |g(\mathrm{t})|) \\ &< d_3 h(\mathrm{t}+1) + d_2\phi(\mathrm{t}+1)(d_2\|\Delta U(t-1)\|_v + \\ &\frac{1}{\phi(\mathrm{t})}\|\mathrm{C}^T E(\mathrm{t}) - \mathrm{C}^T B(\mathrm{t})E(\mathrm{t})\|_v) \\ &< d_3 h(\mathrm{t}+1) + d_2\phi(\mathrm{t}+1)(d_2\|\Delta U(t-1)\|_v + \frac{\|E(\mathrm{t})\|}{\phi(\mathrm{t})}) \\ &< d_3 h(\mathrm{t}+1) + d_2\phi(\mathrm{t}+1)(d_2\|\Delta U(t-1)\|_v + \frac{h(\mathrm{t})}{\phi(\mathrm{t})}) \\ &< (d_3 + \frac{d_2\phi(\mathrm{t}+1)}{d_3\phi(\mathrm{t})})h(\mathrm{t}+1) \end{aligned} \tag{29}$$

If $0 < \phi(\mathrm{t}) \leq b$, $a_1 + a_2 \geq 0.5$ and $b_1 \geq 0.5$, then $\lambda > \lambda_{\min} = b^2/4b_1^2$. Therefore, the following inequality holds when $L_u = L_y = L$:

$$\begin{aligned} &d_3 + \frac{d_2\phi(\mathrm{t}+1)}{d_3\phi(\mathrm{t})} \\ &= M_2 + \frac{M_1\phi(\mathrm{t}+1)}{M_2\phi(\mathrm{t})} + \varepsilon_1 \\ &< \left(1+\frac{b(1-2(a_1+a_2))}{2\sqrt{\lambda}b_1}\right)^{\frac{1}{L-1}} + \frac{\left(\frac{\sqrt{\lambda}(1-b_1)}{2a_1\hat{\phi}(\mathrm{t})}\right)^{\frac{1}{L-1}}\phi(\mathrm{t}+1)}{\phi(\mathrm{t})\left(1+\frac{b(1-2(a_1+a_2))}{2\sqrt{\lambda}b_1}\right)^{\frac{1}{L-1}}} + \varepsilon_1 \\ &< 1 - \left(\frac{\lambda}{4a_1\hat{\phi}(\mathrm{t})(b-\sqrt{\lambda})}\right)^{\frac{1}{L-1}}\frac{\phi(\mathrm{t}+1)}{\phi(\mathrm{t})} + \varepsilon_1 \\ &< 1 - \left(\frac{\frac{b^2}{4b_1^2}\phi(\mathrm{t}+1)^{L-1}}{4a_1 b(b-\sqrt{\frac{b^2}{4b_1^2}})\phi(\mathrm{t})^{L-1}}\right)^{\frac{1}{L-1}} + \varepsilon_1 \stackrel{\Delta}{=} d_4 < 1 \end{aligned} \tag{30}$$

In this case,

$$\begin{aligned} \lim_{t\to\infty} h(\mathrm{t}+2) &< \lim_{t\to\infty} d_4 h(\mathrm{t}+1) < \mathrm{L} < \lim_{t\to\infty} d_4^t h(2) \\ &= \lim_{t\to\infty} d_4^t d_3\|E(1)\|_v = 0 \end{aligned} \tag{31}$$

Equations (28) and (31) validate the conclusion (1) of Theorem 2.

3.4.2 Stability analysis

Given that $y^*(\mathrm{t}) = \mathrm{const}$, the convergence of $e(\mathrm{t})$ implies the boundedness of $y(\mathrm{t})$.

Subject to control input $u(\mathrm{t})$,

$$
\begin{aligned}
|u(\mathrm{t})| &\le |u(\mathrm{t})-\mathrm{u}(\mathrm{t}-1)| + |u(\mathrm{t}-1)| \\
&\le |u(\mathrm{t})-\mathrm{u}(\mathrm{t}-1)| + |u(\mathrm{t}-1)-\mathrm{u}(\mathrm{t}-2)| + |u(\mathrm{t}-2)| \\
&\le \cdots \le |\Delta u(\mathrm{t})| + |\Delta u(\mathrm{t}-1)| + \cdots + |\Delta u(2)| + |\Delta u(1)| \\
&\le \sum_{i=1}^{t} \|\Delta u(i)\|_v < \sum_{i=1}^{t}\sum_{j=1}^{i} d_2^{i-j}|g(j)| \\
&< \frac{1}{1-d_2}\sum_{i=1}^{t}|g(i)| < \frac{1}{1-d_2}\sum_{i=1}^{t}\frac{\|E(i)\|}{\phi(i)} \\
&< \frac{1}{(1-d_2)\min\limits_{i=1,\ldots,t}\phi(i)}\sum_{i=1}^{t}\|E(i)\| \\
&< \frac{1}{(1-d_2)\min\limits_{i=1,\ldots,t}\phi(i)}\sum_{i=1}^{t}h(i) \\
&< \frac{h(2)}{d_4(1-d_4)(1-d_2)\min\limits_{i=1,\ldots,t}\phi(i)}
\end{aligned}
\tag{32}
$$

which implies the boundedness of control input $u(\mathrm{t})$ and validates conclusion (2) of Theorem 2.

4. Result analysis and discussion

The PMSM simulation results illustrate the asymptotic convergence and tracking performance of the proposed higher-order adaptive control approach.

4.1 Data generator of a practical PMSM system

A practical PMSM served as an I/O data generator to implement the proposed approach. No explicit model and structural information of the PMSM were included in the controller design.

The nonlinear model of PMSM is described as follows [20]:

$$
\begin{cases}
\dot{\theta}_m = \omega_m \\
mr^2\dot{\omega}_m = T_e - T_L - T_{friction} - T_{ripple}
\end{cases}
\tag{33}
$$

where w_m and θ_m denote the mechanical angular velocity and mechanical angle, respectively. T_e , T_L , $T_{friction}$ and T_{ripple} denote the electromagnetic torque, load torque, friction torque and ripple torque, respectively. m denotes the slide weight and load, and r denotes the outer diameter of the rotor.

The assumed friction and ripple torque were modelled as follows:

$$
\begin{cases}
T_{friction} = (T_c + (T_s - T_c)e^{-(\omega_m/\omega_{md})^{\delta}} + T_v\omega_m)\mathrm{sgn}(\omega_m) \\
T_{ripple} = F\sin(\omega_0\theta_m)
\end{cases}
\tag{34}
$$

where T_c is the minimum coulomb friction torque, T_s is the static friction torque, T_v is the viscous friction torque, ω_{md} is the desired angular velocity, ω_0 is the angular velocity of the ripple torque, F is the swing of the ripple torque and δ is an additional empirical parameter.

Using a practical PMSM system (which parameters are listed in Table 1) and discretizing equation (33) yield the following:

$$
\begin{cases}
y(\mathrm{t}+1) = y(\mathrm{t}) + z(\mathrm{t}) \\
\mathrm{z}(\mathrm{t}+1) = z(\mathrm{t}) + \dfrac{1}{1.152}(u(\mathrm{t}) - 8 - (1.6 + 1.6e^{-(\mathrm{z(t)}/z_d(\mathrm{t}))^2} \\
\qquad + 1.6z(\mathrm{t}))\mathrm{sgn}(\mathrm{z(t)}) - 1.6\sin(900\,\mathrm{y(t)}))
\end{cases}
\tag{35}
$$

where $y(\mathrm{t})$ and $z(\mathrm{t})$ are the system outputs denoting θ_m and w_m, respectively, and $u(\mathrm{t})$ is the system control input T_e .

The output desired trajectory in the simulations was set as follows:

$$
z^*(\mathrm{t}+1) = \begin{cases}
100, & 0 \le \mathrm{t} < 100 \\
600, & 100 \le \mathrm{t} < 300 \\
450, & \mathrm{t} \ge 300
\end{cases}
\tag{36}
$$

Table 1. PMSM system parameters

Parameters	Symbols	Values	Units
Rated torque	T_N	400	*Nm*
Rated velocity	n	6000	*rpm*
Slide weight	m	45	*kg*
Outer diameter of rotor	r	161.9	*mm*
Minimum coulomb friction torque	T_c	1.6	*Nm*
Static friction torque	T_s	3.2	*Nm*
Viscous friction torque	T_v	1.6	*Nm*
Additional parameter	δ	2	–
Swing of the ripple torque	F	1.6	*Nm*
Angular velocity of the ripple torque	ω_0	900	*rad / s*

4.2 Simulation analysis

Based on algorithms (10) to (12) and discretization model (35), a simulation was performed under the following conditions.

4.2.1 Influence of weight factor λ

The first order of the control law was employed for simplicity. Figure 2 shows the results, whilst Table 2 lists the parameters.

Table 2. Simulation parameters

λ	System initial values	Controller parameters
$\lambda = 0.1$ or $\lambda = 2$ or $\lambda = 10$	$y(1) = -1$, $y(2) = 1$, $u(1) = 0$, $\Delta u(1) = 0$, $\hat{\phi}(1) = 2$	$\eta_k = 0.6$, $\mu = 1$, $\varepsilon = 10^{-5}$, $L_u = L_y = 1$, $a_i = b_j = (1, 0, \ldots, 0)^T$

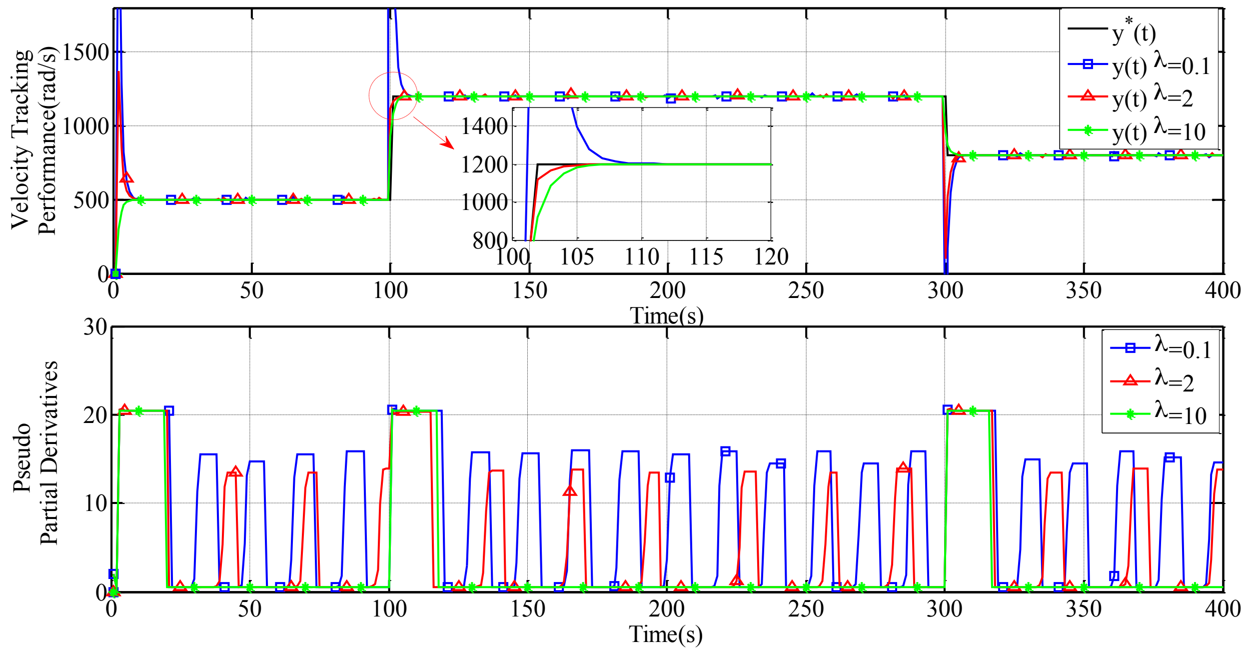

Fig. 2. Velocity responses of HMFAC with different λ values Times

The values of weight factor λ strongly influenced the system dynamic properties. The velocity overshoot decreased with increasing λ, which indicated an improved relative stability and a reduced rapidity. Stability and rapidity must be balanced when selecting the value of λ in practical applications. Figure 2 illustrates PPD as a slow time-varying bounded parameter relating to the system action point or system dynamics.

4.2.2 Influence of the input and output orders L_u and L_y

Figure 3 shows that exploiting additional historical information can enhance the accuracy of the adaptive control. Table 3 lists the parameters.

The simulation results in Figure 3 show that the system response is highly precise, has a small overshoot, and is highly stable when the introduction of additional historical input and output data increases the amount of orders. However, using excessive historical information will generate oscillations at the mutation instant of the desired output.

Table 3. Simulation parameters

Orders	System initial values	Controller parameters
L_u=L_y=1	$y(1)=-1$, $y(2)=1$, $u(1)=0$, $\Delta u(1)=0$, $\hat{\phi}(1)=2$	$\eta_k=0.6$, $\mu=1$, $\varepsilon=10^{-5}$, $\lambda=2$, $a_i=b_j=(1,0,\ldots,0)^T$
L_u=L_y=3	$y(1)=y(2)=\ldots=y(4)=1$, $u(1)=u(2)=u(3)=0$, $\Delta u(1)=\Delta u(2)=\Delta u(3)=0$, $\hat{\phi}(1)=\hat{\phi}(2)=\hat{\phi}(3)=2$	$\eta_k=0.6$, $\mu=1$, $\varepsilon=10^{-5}$ $\lambda=2$, $a_i=(0.6,0.2,0.2,0,\ldots,0)^T$ $b_j=(0.6,0.2,0.2,0,\ldots,0)^T$
L_u=L_y=5	$y(1)=y(2)=\ldots=y(6)=1$, $u(1)=u(2)=\ldots=u(5)=0$, $\Delta u(1)=\Delta u(2)=\ldots=\Delta u(5)=0$ $\hat{\phi}(1)=\hat{\phi}(2)=\ldots=\hat{\phi}(5)=2$	$\eta_k=0.6$, $\mu=1$, $\varepsilon=10^{-5}$, $\lambda=2$, $a_i=(0.4,0.3,0.1,0.1,0.1,0,\ldots,0)^T$ $b_j=(0.4,0.3,0.1,0.1,0.1,0,\ldots,0)^T$

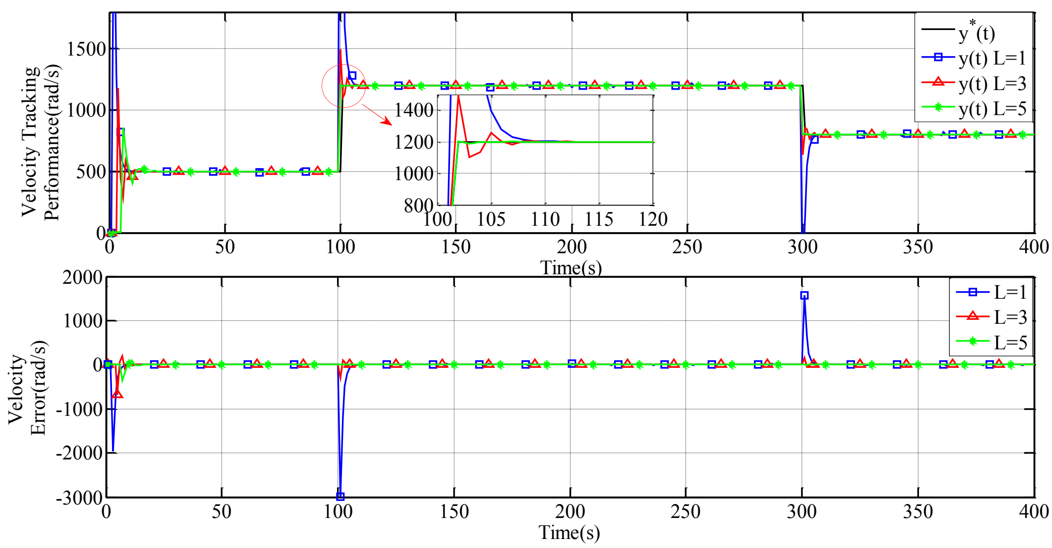

Fig. 3. Velocity responses with different L_u and L_y values

4.2.3 Comparison with traditional PID

A second-order control law was used to compare the proposed control algorithm with the traditional PID algorithm. Table 4 lists the parameters of this law. The PID parameters were fine-tuned off line to balance stability and rapidity. Figure 4 illustrates the superiority of the proposed method.

Table 4. Simulation parameters

Control scheme	System initial values and parameters
HMFAC	$y(1)=-1$, $y(2)=y(3)=1$, $u(1)=u(2)=0$, $\Delta u(1)=\Delta u(2)=0$, $\hat{\phi}(1)=\hat{\phi}(2)=2$, $\eta_k=0.6$, $\mu=1$, $\varepsilon=10^{-5}$, $\lambda=10$, $L_u=L_y=2$, $a_i=b_j=(0.6,0.4,0,\ldots,0)^T$
PID	$K_p=0.01$, $T_i=0.5$, $T_d=1$

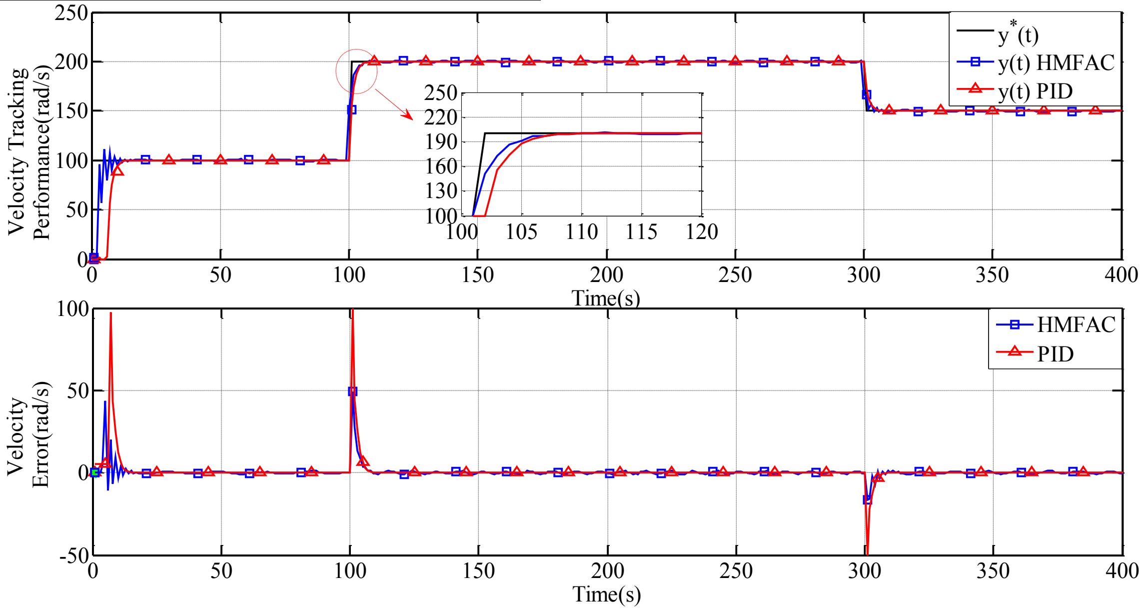

Fig. 4. Comparison between HMFAC and PID

Simulations above indicate the following:

(1) The nonlinear PMSM system demonstrates satisfactory adaptability and stability under the proposed control scheme.

(2) The stability and rapidity of the closed loop PMSM system can be balanced by selecting the appropriate weight factors and orders. Higher values of λ, L_u and L_y indicate higher stability for the controlled plant, whilst lower values indicate favorable rapidity.

(3) PPD is a slow time-varying bounded parameter that relates to the system action point, the control inputs or the dynamics of the system. This parameter can reflect all possible complex behaviors of a nonlinear system to a certain extent.

5. Conclusions

Using the dynamic linearization technique, a new higher-order model-free adaptive control approach was proposed to promote the use of historical input and output information for the PMSM control system. The proposed controller efficiently controlled the velocity and achieved zero-speed tracking. The order increased along with accuracy, especially when the desired output mutation was reached. The following conclusions were obtained:

(1) Favorable asymptotic convergence and improved tracking performance could be achieved through proper parameter coordination. The design did not use an explicit model or the structural information of the plant, which would have simplified the controller design as demonstrated in the simulation.

(2) The weight factor in the controller strongly influenced the dynamic properties. A higher weight would lower the velocity overshoot and negatively affect the rapidity of the controlled system. Stability and rapidity must be balanced when implementing the proposed approach according to different control targets.

(3) The proposed approach demonstrated higher precision, smaller overshoot, better stability and strong fault tolerance along with increasing orders. However, oscillations were observed at the mutation instant of the desired output when the amount of orders exceeded a certain threshold.

(4) The proposed method only had a scalar parameter PPD and demonstrated similar or better performance, involved lesser calculation efforts and could be implemented much easier than the traditional PID.

The proposed approach can meet the demands of many nonlinear motor systems that are difficult to model and control. However, this paper did not consider the accurate measurement approaches of the I/O data, thereby presenting an opportunity for future research.

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