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Research Article

Analysis, Adaptive Control and Adaptive Synchronization of a Nine-Term Novel 3-D Chaotic System with Four Quadratic Nonlinearities and its Circuit Simulation

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Abstract

This research work describes a nine-term novel 3-D chaotic system with four quadratic nonlinearities and details its qualitative properties. The phase portraits of the 3-D novel chaotic system simulated using MATLAB, depict the strange chaotic attractor of the system. For the parameter values chosen in this work, the Lyapunov exponents of the novel chaotic system are obtained as $L_1 = 6.8548$, $L_2 = 0$ and $L_3 = -32.8779$. Also, the Kaplan-Yorke dimension of the novel chaotic system is obtained as $D_{KY} = 2.2085$. Next, an adaptive controller is design to achieve global stabilization of the 3-D novel chaotic system with unknown system parameters. Moreover, an adaptive controller is designed to achieve global chaos synchronization of two identical novel chaotic systems with unknown system parameters. Finally, an electronic circuit realization of the novel chaotic system is presented using SPICE to confirm the feasibility of the theoretical model.

Keywords: Chaos, chaotic systems, Lyapunov exponents, Kaplan-Yorke dimension, circuit simulation.

1. Introduction

The discovery of chaos in nature and physical systems is an active area of research [1]. The applications and importance of chaos theory are well-known in the literature. Poincaré was the first to notice the possibility of chaos according to which a deterministic system exhibits aperiodic behavior that depends on the initial conditions, thereby rendering long-term prediction impossible [2].

Chaotic systems are nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and also with dense periodic orbits [3]. The sensitivity to initial conditions of a chaotic system is indicated by a positive Lyapunov exponent. A dissipative chaotic system is characterized by the condition that the sum of the Lyapunov exponents of the chaotic system is negative.

Since the discovery of a chaotic system by Lorenz [4] while he was modelling weather patterns with a 3-D model, there is great interest in the literature in the modelling of new chaotic systems. Many paradigms of 3-D chaotic systems have been discovered such as Rössler system [5], Rabinovich system [6], ACT system [7], Sprott systems [8], Chen system [9], Lü system [10], Shaw system [11], Feeny system [12], Shimizu system [13], Liu-Chen system [14], Cai system [15], Tigan system [16], Colpitt's oscillator [17], Zhou system [18], etc.

Recently, many 3-D chaotic systems have been discovered such as Li system [19], Pan system [20], Sundarapandian system [21], Yu-Wang system [22], Sundarapandian-Pehlivan system [23], Zhu system [24], Vaidyanathan systems [25-31], Vaidyanathan-Madhavan system [32], Pehlivan-Moroz-Vaidyanathan system [33], Jafari system [34], Pham system [35], etc.

The study of chaos theory in the last few decades had a big impact on the foundations of Science and Engineering and has found several engineering applications.

Some important applications of chaos theory can be cited as oscillators [36-38], lasers [39, 40], robotics [41-45], chemical reactors [46,47], biology [48,49], ecology [50,51], neural networks [52-54], secure communications [55-58], cryptosystems [59-62], economics [63-65], etc.

Furthermore, control and synchronization of chaotic systems are important research problems in the chaos literature.

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The study of control of a chaotic system investigates methods for finding feedback control laws that globally asymptotically stabilize or regulate the outputs of a chaotic system. Some important methodologies used for this study are active control [66-69], adaptive control [70-76], sliding mode control [77,78], backstepping control [79-81], etc.

A pioneering research work on the global chaos synchronization of two chaotic systems was published by Pecora and Carroll [82]. After this seminal work, many different methodologies have been developed for synchronization of chaotic systems such as active control [83-93], time-delayed feedback control [94,95], adaptive control [96-108], sampled-data feedback control [109-112], backstepping control [113-119], sliding mode control [120-124], etc.

In this research paper, we detail the properties of our recently discovered a nine-term 3-D novel chaotic system with four quadratic nonlinearities [125]. First, we detail the basic qualitative properties of the 3-D novel chaotic system. We show that the novel chaotic system is dissipative and we derive the Lyapunov exponents and Kaplan-Yorke dimension of the novel chaotic system.

Next, we derive an adaptive backstepping control law that stabilizes the novel chaotic system when the system parameters are unknown.

Furthermore, we also derive an adaptive backstepping control law that achieves global chaos synchronization of two identical 3-D novel chaotic systems with unknown parameters.

All the main results in this research paper have been established using adaptive control theory and Lyapunov stability theory. MATLAB simulations are shown to illustrate the phase portraits of the novel chaotic system, dynamics of the Lyapunov exponents, adaptive stabilization and adaptive chaos synchronization for the 3-D novel chaotic system described in this research paper.

Finally, an electronic circuit realization of the 3-D novel jerk chaotic system using SPICE simulations is presented to confirm the feasibility of the theoretical model.

2. A 3-D Novel Chaotic System

In a recent work [125], we described a nine-term 3-D novel chaotic system modelled by

$$\begin{cases} \dfrac{dx_1}{dt} = a(x_2 - x_1) - x_2 x_3 \\ \dfrac{dx_2}{dt} = b x_1 - x_2 - x_1 x_3 \\ \dfrac{dx_3}{dt} = -c x_3 + p(x_2^2 - x_1^2) \end{cases} \quad (1)$$

In Eq. (1), a, b, c and p are assumed to be positive constant parameters.

In [125l, it was shown that the system (1) is chaotic when the parameters take the following values:

$$a = 22, \; b = 600, \; c = 3, p = 11 \quad (2)$$

For the numerical simulations of the novel chaotic system (1), we have taken the parameter values as in the chaotic case (2) and the initial conditions of the system (1) as $x_1(0) = 0.5$, $x_2(0) = 2$ and $x_3(0) = 30$.

Figure 1 depicts the chaotic attractor of the novel system (1) in 3-D view, while in Figs. 2-4, the 2-D projections of the strange chaotic attractor of the novel chaotic system (1) on (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes, are shown, respectively.

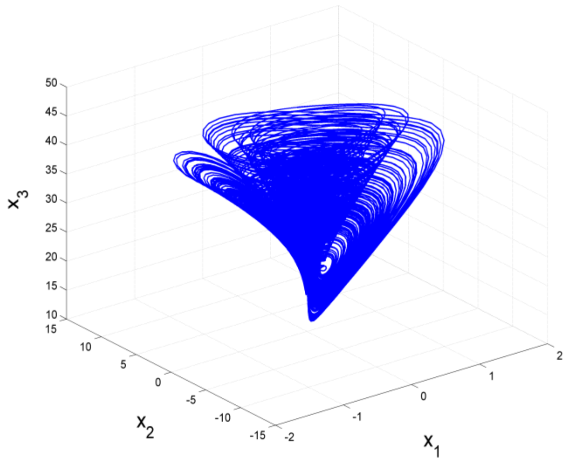

Fig. 1. The strange attractor of the novel chaotic system.

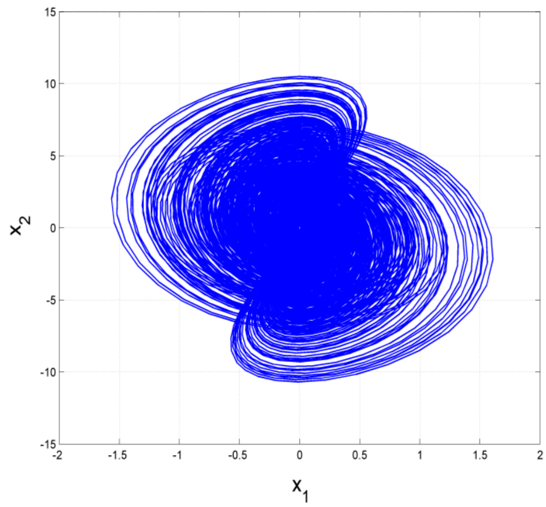

Fig. 2. 2-D projection of the novel chaotic system on (x_1, x_2)-plane.

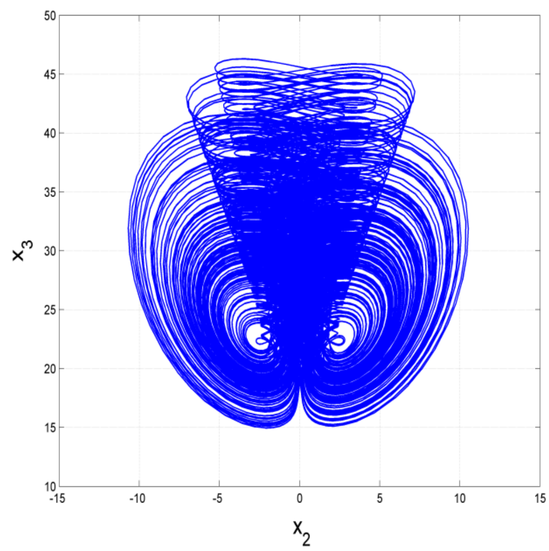

Fig. 3. 2-D projection of the novel chaotic system on (x_2, x_3)-plane.

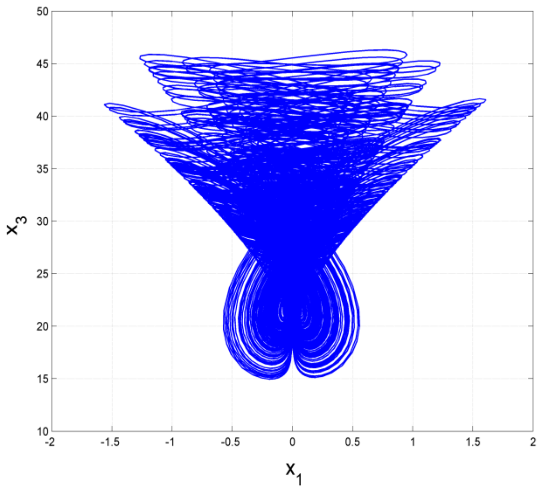

Fig. 4. 2-D projection of the novel chaotic system on (x_1, x_3)-plane.

3. Properties of the 3-D Novel Chaotic System

In this section, we analyse 3-D novel chaotic system (1) and detail its fundamental properties like dissipativity, symmetry and invariance, equilibria, Lyapunov exponents and Kaplan-Yorke dimension.

3.1. Dissipativity

In vector notation, we may express the system (1) as:

$$\frac{dx}{dt} = f(\boldsymbol{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \quad (3)$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) &= a(x_2 - x_1) - x_2 x_3 \\ f_2(x_1, x_2, x_3) &= b x_1 - x_2 - x_1 x_3 \\ f_3(x_1, x_2, x_3) &= -c x_3 + p(x_2^2 - x_1^2) \end{cases} \quad (4)$$

Let Ω be any region in $\boldsymbol{R}^3$ with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\frac{dV}{dt} = \int_{\Omega(t)} (\nabla \cdot f) dx_1 dx_2 dx_3 \quad (5)$$

The divergence of the novel chaotic system (1) is easily found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a - 1 - c = -\mu < 0 \quad (6)$$

where

$$\mu = a + 1 + c > 0 \quad (7)$$

as a and c are positive parameters.

Substituting (6) into (5), we obtain the first order ODE

$$\frac{dV}{dt} = -\mu\, V(t) \quad (8)$$

Integrating (8), we obtain the unique solution as:

$$V(t) = \exp(-\mu\, t)\, V(0) \quad (9)$$

Since $\mu > 0$, is evident from Eq. (9) that $V(t) \to 0$ exponentially as $t \to \infty$.

This shows that the novel chaotic system (1) is dissipative.

Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2. Symmetry and Invariance

The novel chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \quad (10)$$

The transformation (10) persists for all values of the system parameters. Thus, the novel chaotic system (1) has rotation symmetry about the x_3-axis. As a consequence, any non-trivial trajectory of the system (1) must have a twin trajectory.

It is easy to check that the x_3-axis is invariant for the flow of the novel chaotic system (1). Hence, all orbits of the system (1) starting from the x_3-axis stay in the x_3-axis for all values of time.

3.3. Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the following system of equations

$$\begin{cases} a(x_2 - x_1) - x_2 x_3 &= 0 \\ b x_1 - x_2 - x_1 x_3 &= 0 \\ -c x_3 + p(x_2^2 - x_1^2) &= 0 \end{cases} \quad (11)$$

We take the parameter values as in the chaotic case (2).

A simple calculation yields three equilibrium points of the chaotic system (1), viz.

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ E_1 = \begin{bmatrix} 0.0042 \\ 2.4474 \\ 21.9619 \end{bmatrix}, E_2 = \begin{bmatrix} -0.0042 \\ -2.4474 \\ 21.9619 \end{bmatrix} \quad (12)$$

The Jacobian matrix of the system (1) at x is given by

$$J(x) = \begin{bmatrix} -22 & 22 - x_3 & -x_2 \\ 600 - x_3 & -1 & -x_1 \\ -22x_1 & 22x_2 & -3 \end{bmatrix} \quad (13)$$

The Jacobian matrix at E_0 is obtained as:

$$J_0 = J(E_0) = \begin{bmatrix} -22 & 22 & 0 \\ 600 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} \tag{14}$$

Using MATLAB, we find the eigenvalues of J_0 as:

$$\lambda_1 = -126.8701, \lambda_2 = -3, \lambda_3 = 103.8701 \tag{15}$$

Thus, the equilibrium E_0 is a *saddle-point*, which is unstable.

The Jacobian matrix at E_1 is obtained as:

$$J_1 = J(E_1) = \begin{bmatrix} -22.0000 & 0.0381 & -2.4474 \\ 578.0381 & -1 & -0.0042 \\ -0.0924 & 53.8428 & -3 \end{bmatrix} \tag{16}$$

Using MATLAB, we find the eigenvalues of J_1 as:

$$\lambda_1 = -52.4131, \lambda_{2,3} = 13.2065 \pm 35.7625i \tag{17}$$

Thus, the equilibrium E_1 is a *saddle-focus,* which is unstable.

The Jacobian matrix at E_2 is obtained as:

$$J_2 = J(E_2) = \begin{bmatrix} -22.0000 & 0.0381 & 2.4474 \\ 578.0381 & -1 & 0.0042 \\ 0.0924 & -53.8428 & -3 \end{bmatrix} \tag{16}$$

Using MATLAB, we find the eigenvalues of J_2 as:

$$\lambda_1 = -52.4131, \lambda_{2,3} = 13.2065 \pm 35.7625i \tag{17}$$

Thus, the equilibrium E_2 is a *saddle-focus,* which is unstable.

Hence, all the three equilibrium points of the system (1) are unstable.

3.4. Lyapunov Exponents and Kaplan-Yorke Dimension

For the chosen parameter values (2), the Lyapunov exponents of the novel chaotic system (1) are obtained using MATLAB as:

$$L_1 = 6.8548, L_2 = 0, L_3 = -32.8779 \tag{18}$$

Since the spectrum of Lyapunov exponents (18) has a positive term L_1, it follows that the 3-D novel system (1) is chaotic.

The maximal Lyapunov exponent (MLE) of the novel chaotic system (1) is $L_1 = 6.8548$. Since this is a large value when compared to the Lyapunov exponents of Lorenz system, Rössler system, Chen's system, etc., we conclude that our 3-D novel chaotic system (1) is a highly chaotic system.

Since the sum of the Lyapunov exponents in (18) is negative, it follows that the novel chaotic system (1) is dissipative.

Also, the Kaplan-Yorke dimension of the novel chaotic system (1) is calculated as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.2085 \tag{19}$$

4. Adaptive Control of the 3-D Novel Chaotic System

In this section, we derive an adaptive feedback control law for globally stabilizing the 3-D novel chaotic system with unknown system parameters. Lyapunov stability theory is applied to establish the main result of this section.

We consider the 3-D novel chaotic system given by

$$\begin{cases} \frac{dx_1}{dt} &= a(x_2 - x_1) - x_2x_3 + u_1 \\ \frac{dx_2}{dt} &= bx_1 - x_2 - x_1x_3 + u_2 \\ \frac{dx_3}{dt} &= -cx_3 + p(x_2^2 - x_1^2) + u_3 \end{cases} \tag{20}$$

In (20), a, b, c and p are unknown constant parameters, and u is an adaptive control law to be determined using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ of the unknown parameters a, b, c, p, respectively.

We take the adaptive control law defined by

$$\begin{cases} u_1 &= -\hat{a}(t)(x_2 - x_1) + x_2x_3 - k_1x_1 \\ u_2 &= -\hat{b}(t)x_1 + x_2 + x_1x_3 - k_2x_2 \\ u_3 &= \hat{c}(t)x_3 - \hat{p}(t)(x_2^2 - x_1^2) - k_3x_3 \end{cases} \tag{21}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (21) into (20), we obtain the closed-loop control system as:

$$\begin{cases} \frac{dx_1}{dt} &= (a - \hat{a}(t))(x_2 - x_1) - k_1x_1 \\ \frac{dx_2}{dt} &= \left(b - \hat{b}(t)\right)x_1 - k_2x_2 \\ \frac{dx_3}{dt} &= -\left(c - \hat{c}(t)\right)x_3 + \left(p - \hat{p}(t)\right)(x_2^2 - x_1^2) - k_3x_3 \end{cases} \tag{22}$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \\ e_c(t) &= c - \hat{c}(t) \\ e_p(t) &= p - \hat{p}(t) \end{cases} \tag{23}$$

Differentiating (23) with respect to t, we obtain

$$\begin{cases} \frac{de_a}{dt} &= -\frac{d\hat{a}}{dt} \\ \frac{de_b}{dt} &= -\frac{d\hat{b}}{dt} \\ \frac{de_c}{dt} &= -\frac{d\hat{c}}{dt} \\ \frac{de_p}{dt} &= -\frac{d\hat{p}}{dt} \end{cases} \tag{24}$$

By using (23), we rewrite the closed-loop system (22) as:

$$\begin{cases} \frac{dx_1}{dt} &= e_a(x_2 - x_1) - k_1x_1 \\ \frac{dx_2}{dt} &= e_bx_1 - k_2x_2 \\ \frac{dx_3}{dt} &= -e_cx_3 + e_p(x_2^2 - x_1^2) - k_3x_3 \end{cases} \tag{25}$$

To find an update law for the parameter estimates, we shall use Lyapunov stability theory.

We consider the quadratic Lyapunov function given by

$$V = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right) \tag{26}$$

Clearly, V is a positive definite function on R^7. Differentiating V along the trajectories of the systems (25) and (24), we obtain the following.

$$\frac{dV}{dt} = \left\{ \begin{array}{l} -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \\ +e_a\left[x_1(x_2 - x_1) - \frac{d\hat{a}}{dt}\right] + e_b\left[x_1x_2 - \frac{d\hat{b}}{dt}\right] \\ +e_c\left[-x_3^2 - \frac{d\hat{c}}{dt}\right] + e_p\left[x_3(x_2^2 - x_1^2) - \frac{d\hat{p}}{dt}\right] \end{array} \right\} \tag{27}$$

In view of (27), we take the parameter update law as follows.

$$\begin{cases} \frac{d\hat{a}}{dt} & = & x_1(x_2 - x_1) \\ \frac{d\hat{b}}{dt} & = & x_1x_2 \\ \frac{d\hat{c}}{dt} & = & -x_3^2 \\ \frac{d\hat{p}}{dt} & = & x_3(x_2^2 - x_1^2) \end{cases} \tag{28}$$

Next, we establish the main result of this section.

Theorem 1. *The 3-D novel chaotic system (20) with unknown parameters is globally and exponentially stabilized by the adaptive feedback control law (21) and the parameter update law (28), where k_1, k_2, k_3 are positive constants.*

Proof. We prove this result via Lyapunov stability theory.

We consider the quadratic Lyapunov function V defined by (26), which is positive definite on R^7.

Next, by substituting the parameter update law (28) into (27), we obtain the time derivative of V as

$$\frac{dV}{dt} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \tag{29}$$

Thus, it is clear that $\frac{dV}{dt}$ is a negative semi-definite function on R^7.

From (29), it follows that the state vector $\boldsymbol{x}(t) = (x_1(t), x_2(t), x_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t))$ are globally bounded.

We define $k = \min(k_1, k_2, k_3)$.

Then it follows from (29) that

$$\frac{dV}{dt} \leq -kx_1^2 - kx_2^2 - kx_3^2 = -k\|\boldsymbol{x}\|^2 \tag{30}$$

That is,

$$\|\boldsymbol{x}\|^2 \leq -\frac{dV}{dt} \tag{31}$$

Integrating the inequality (31) from 0 to t, we get

$$\int_0^t \|\boldsymbol{x}(\tau)\|^2 d\tau \leq V(0) - V(t) \tag{32}$$

From (32), it follows that $x(t) \in \boldsymbol{L}_2$, while from (25), it can be deduced that $\frac{dx}{dt} \in \boldsymbol{L}_\infty$.

Thus, using Barbalat's lemma [126], we can conclude that $\boldsymbol{x}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\boldsymbol{x}(0) \in \boldsymbol{R}^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (20) and (28), when the adaptive control law (21) is applied.

The parameter values of the novel 3-D chaotic system (20) are chosen as in the chaotic case (2). The positive gain constants are taken as $k_i = 6$, for $i = 1, 2, 3$.

Moreover, as initial conditions of the novel chaotic system (20), we have chosen $x_1(t) = 6.4$, $x_2(t) = -4.7$ and $x_3(t) = 2.5$.

Furthermore, as initial conditions of the parameter estimates of the unknown parameters, we have chosen $\hat{a}(0) = 5.4, \hat{b}(0) = 12.3, \hat{c}(0) = 7.4$ and $\hat{p}(0) = 8.6$.

In Fig. 5, the exponential convergence of the controlled states $x_1(t)$, $x_2(t)$, $x_3(t)$ is depicted, when the adaptive control law (21) and parameter update law (28) are implemented.

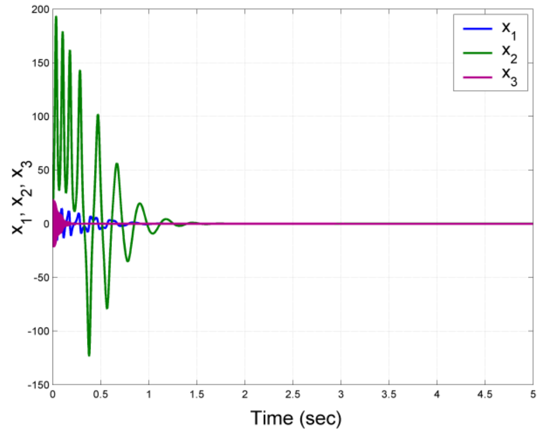

Fig. 5. Time-History of the controlled states $x_1(t)$, $x_2(t)$, $x_3(t)$.

5. Adaptive Synchronization of the Identical 3-D Novel Chaotic Systems

In this section, we derive an adaptive control law for globally and exponentially synchronizing the identical 3-D novel chaotic systems with unknown system parameters.

Thus, the master system is given by the novel chaotic system dynamics

$$\begin{cases} \frac{dx_1}{dt} & = & a(x_2 - x_1) - x_2x_3 \\ \frac{dx_2}{dt} & = & bx_1 - x_2 - x_1x_3 \\ \frac{dx_3}{dt} & = & -cx_3 + p(x_2^2 - x_1^2) \end{cases} \tag{33}$$

Also, the slave system is given by the novel chaotic system dynamics

$$\begin{cases} \frac{dy_1}{dt} & = & a(y_2 - y_1) - y_2y_3 + u_1 \\ \frac{dy_2}{dt} & = & by_1 - y_2 - y_1y_3 + u_2 \\ \frac{dy_3}{dt} & = & -cy_3 + p(y_2^2 - y_1^2) + u_3 \end{cases} \tag{34}$$

In (33) and (34), the system parameters a, b, c, p are unknown and the design goal is to find an adaptive feedback control u that uses estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ for the parameters a, b, c, p respectively so as to render the states of the systems (33) and (34) fully synchronized asymptotically.

The synchronization error between the novel chaotic systems (33) and (34) is defined as:

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{35}$$

Thus, the synchronization error dynamics is obtained as:

$$\begin{cases} \frac{de_1}{dt} = a(e_2 - e_1) - y_2 y_3 + x_2 x_3 + u_1 \\ \frac{de_2}{dt} = b e_1 - e_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \frac{de_3}{dt} = -c e_3 + p(y_2^2 - x_2^2 - y_1^2 + x_1^2) + u_3 \end{cases} \tag{36}$$

We take the adaptive control law defined by

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) + y_2 y_3 - x_2 x_3 - k_1 e_1 \\ u_2 = -\hat{b}(t) e_1 + e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 = \hat{c}(t) e_3 - \hat{p}(t)(y_2^2 - x_2^2 - y_1^2 + x_1^2) - k_3 e_3 \end{cases} \tag{37}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (37) into (36), we obtain the closed-loop error dynamics as:

$$\begin{cases} \frac{de_1}{dt} = (a - \hat{a}(t))(e_2 - e_1) - k_1 e_1 \\ \frac{de_2}{dt} = \left(b - \hat{b}(t)\right) e_1 - k_2 e_2 \\ \frac{de_3}{dt} = -\left(c - \hat{c}(t)\right) e_3 \\ \qquad + \left(p - \hat{p}(t)\right)(y_2^2 - x_2^2 - y_1^2 + x_1^2) - k_3 e_3 \end{cases} \tag{38}$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \end{cases} \tag{39}$$

Differentiating (39) with respect to t, we obtain

$$\begin{cases} \frac{de_a}{dt} = -\frac{d\hat{a}}{dt} \\ \frac{de_b}{dt} = -\frac{d\hat{b}}{dt} \\ \frac{de_c}{dt} = -\frac{d\hat{c}}{dt} \\ \frac{de_p}{dt} = -\frac{d\hat{p}}{dt} \end{cases} \tag{40}$$

By using (39), we rewrite the closed-loop system (38) as:

$$\begin{cases} \frac{de_1}{dt} = e_a(e_2 - e_1) - k_1 e_1 \\ \frac{de_2}{dt} = e_b e_1 - k_2 e_2 \\ \frac{de_3}{dt} = -e_c e_3 + e_p(y_2^2 - x_2^2 - y_1^2 + x_1^2) - k_3 e_3 \end{cases} \tag{41}$$

We consider the quadratic Lyapunov function given by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right) \tag{42}$$

Differentiating V along the trajectories of the systems (41) and (40), we obtain the following.

$$\frac{dV}{dt} = \left\{ \begin{array}{l} -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ +e_a \left[e_1(e_2 - e_1) - \frac{d\hat{a}}{dt}\right] + e_b \left[e_1 e_2 - \frac{d\hat{b}}{dt}\right] \\ +e_c \left[-e_3^2 - \frac{d\hat{c}}{dt}\right] \\ +e_p \left[e_3(y_2^2 - x_2^2 - y_1^2 + x_1^2) - \frac{d\hat{p}}{dt}\right] \end{array} \right\} \tag{43}$$

In view of (43), we take the parameter update law as follows.

$$\begin{cases} \frac{d\hat{a}}{dt} = e_1(e_2 - e_1) \\ \frac{d\hat{b}}{dt} = e_1 e_2 \\ \frac{d\hat{c}}{dt} = -e_3^2 \\ \frac{d\hat{p}}{dt} = e_3(y_2^2 - x_2^2 - y_1^2 + x_1^2) \end{cases} \tag{44}$$

Next, we establish the main result of this section.

Theorem 2. *The 3-D novel chaotic systems (33) and (34) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive feedback control law (37) and the parameter update law (44), were k_1, k_2, k_3 are positive constants.*

Proof. We prove this result via Lyapunov stability theory.

We consider the quadratic Lyapunov function V defined by (42), which is positive definite on R^7.

Next, by substituting the parameter update law (44) into (43), we obtain the time derivative of V as:

$$\frac{dV}{dt} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{45}$$

Thus, it is clear that $\frac{dV}{dt}$ is a negative semi-definite function on R^7.

From (45), it follows that the synchronization error vector $\boldsymbol{e}(t) = (e_1(t), e_2(t), e_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t), e_p(t))$ are globally bounded.

We define $k = \min(k_1, k_2, k_3)$.

Then it follows from (45) that

$$\frac{dV}{dt} \le -k e_1^2 - k e_2^2 - k e_3^2 = -k\|\boldsymbol{e}\|^2 \tag{46}$$

That is,

$$\|\boldsymbol{e}\|^2 \le -\frac{dV}{dt} \tag{47}$$

Integrating the inequality (47) from 0 to t, we get

$$\int_0^t \|\boldsymbol{e}(\tau)\|^2 d\tau \le V(0) - V(t) \tag{48}$$

From (48), it follows that $\boldsymbol{e}(t) \in \boldsymbol{L}_2$, while from (41), it can be deduced that $\frac{de}{dt} \in \boldsymbol{L}_\infty$.

Thus, using Barbalat's lemma [126], we can conclude that $\boldsymbol{e}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\boldsymbol{e}(t) \in \boldsymbol{R}^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (33), (34) and (44), when the adaptive control law (37) is applied.

The parameter values of the novel 3-D chaotic systems (33) and (34) are taken as in the chaotic case (2). The gain constants are taken as $k_i = 6$, for $i = 1, 2, 3$.

Furthermore, as initial conditions of the master system (33), we take $x_1(t) = 6.2$, $x_2(t) = -5.4$ and $x_3(t) = 8.3$. As initial conditions of the slave system (34), we take $y_1(0) = -7.3$, $y_2(0) = 4.7$ and $y_3(0) = 3.9$.

Also, as initial conditions of the parameter estimates, we take $\hat{a}(0) = 3.4, \hat{b}(0) = 18.3, \hat{c}(0) = 12.4$ and $\hat{p}(0) = 5.3$.

In Figs. 6-8, the synchronization of the states of the master system (33) and slave system (34) is depicted, when the adaptive control law (37) and parameter update law (44) are implemented. In Fig. 9, the time-history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$ is depicted.

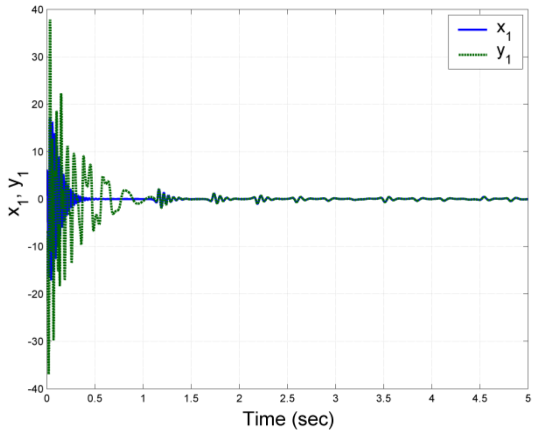

Fig. 6. Synchronization of the states $x_1(t)$ and $y_1(t)$.

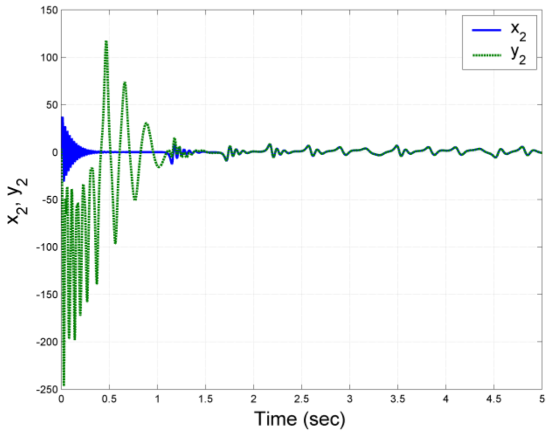

Fig. 7. Synchronization of the states $x_2(t)$ and $y_2(t)$.

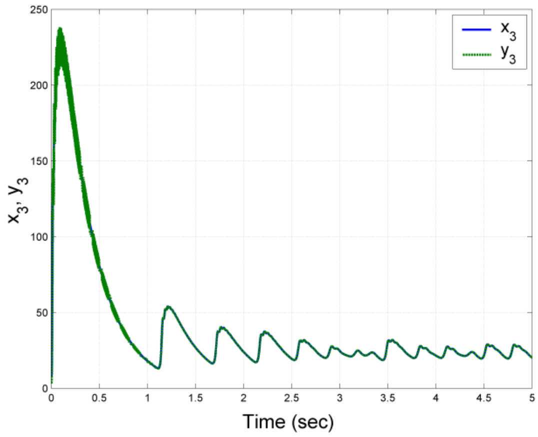

Fig. 8. Synchronization of the states $x_3(t)$ and $y_3(t)$.

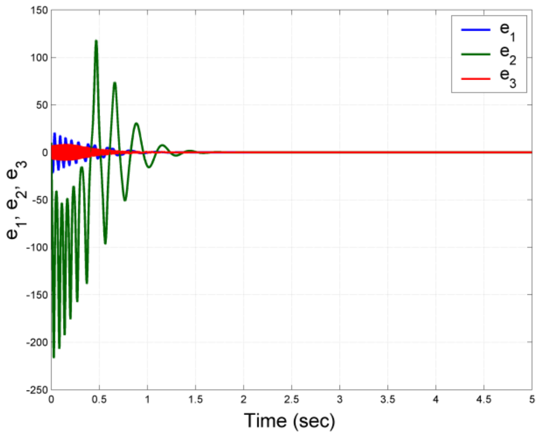

Fig. 9. Time-History of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$.

6. Circuit Realization of the Novel Chaotic System

In this section, we present an electronic circuit modelling the new 3-D system (1) in order to show its feasibility. Because the circuit is designed following an approach based on operational amplifiers [30,33,35], the state variables of system (1) are scaled down to obtain chaotic attractors in the dynamical range of operational amplifiers. As a result, 3-D system (1) can be written as:

$$\begin{cases} \frac{dX_1}{dt} = 2aX_2 - aX_1 - 20X_2X_3 \\ \frac{dX_2}{dt} = \frac{b}{2}X_1 - X_2 - 5X_1X_2 \\ \frac{dX_3}{dt} = -cX_3 + \frac{2d}{5}X_2^2 - \frac{d}{10}X_1^2 \end{cases} \tag{49}$$

in which $X_1 = x_1$, $X_2 = \frac{x_2}{2}$ and $X_3 = \frac{x_3}{10}$. The electronic circuit has been designed using common off-the-shelf components and its schematic is represented in Fig. 10.

By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\begin{cases} \frac{dv_{C_1}}{dt} & = & \frac{1}{R_1C_1}v_{C_2} - \frac{1}{R_2C_1}v_{C_1} - \frac{1}{10R_3C_1}v_{C_2}v_{C_3} \\ \frac{dv_{C_2}}{dt} & = & \frac{1}{R_4C_2}v_{C_1} - \frac{1}{R_5C_2}v_{C_2} - \frac{1}{10R_6C_2}v_{C_1}v_{C_3} \\ \frac{dv_{C_3}}{dt} & = & -\frac{1}{R_7C_3}v_{C_3} + \frac{1}{10R_8C_3}v_{C_2}^2 - \frac{1}{10R_9C_3}v_{C_1}^2 \end{cases} \quad (50)$$

where v_{C_1}, v_{C_2}, v_{C_3} are the voltages across the capacitors C_1, C_2 and C_3, respectively.

It is noting that the state variables X_1, X_2, X_3 of system (49) are also the voltages v_{C_1}, v_{C_2}, v_{C_3}, respectively.

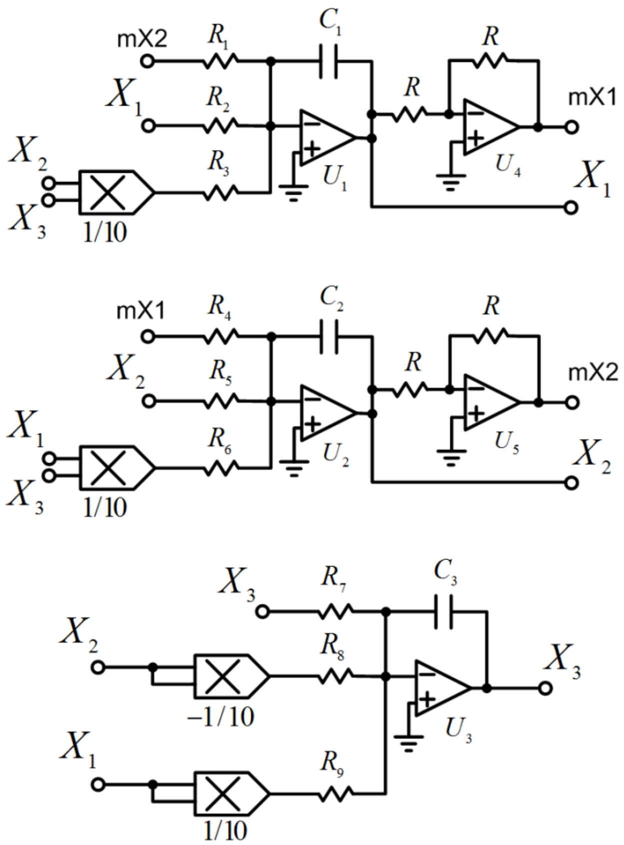

Fig. 10. Circuital diagram for implementing novel nine-term 3-D chaotic system (49).

The values of the electronic components in Fig. 10 are selected to match known parameters of system (1): $R_1 = R_8 = 9.09$ kΩ, $R_2 = 18.18$ kΩ, $R_3 = 2$ kΩ, $R_4 = 1.33$ kΩ, $R_5 = R = 400$ kΩ, $R_6 = 8$ kΩ, $R_7 = 133.33$ kΩ, $R_9 = 36.36$ kΩ, and $C_1 = C_2 = C_3 = 1$ nF. The power supplies of all active devices are ±15 Volts.

The proposed circuit is implemented by using the electronic simulation package Cadence OrCAD. The obtained phase portraits in (v_{C_1}, v_{C_2})-plane, (v_{C_2}, v_{C_3})-plane, and (v_{C_1}, v_{C_3})-plane are shown in Figs. 11-13, respectively. Obviously, the circuital results agree well with numerical simulation results (see Figs. 2-4).

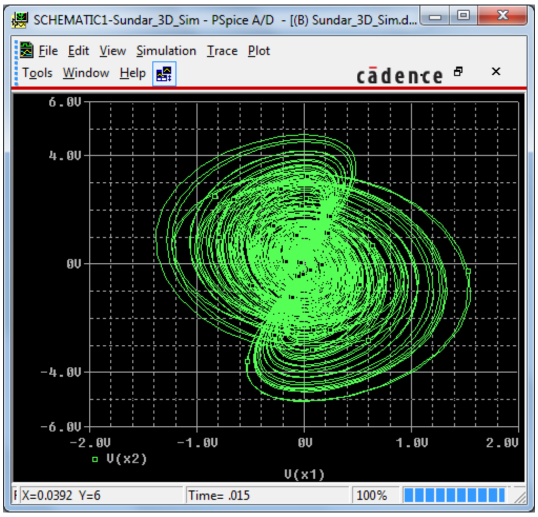

Fig. 11. Chaotic attractor obtained from the circuit in Fig. 10 in (v_{C_1}, v_{C_2})-plane.

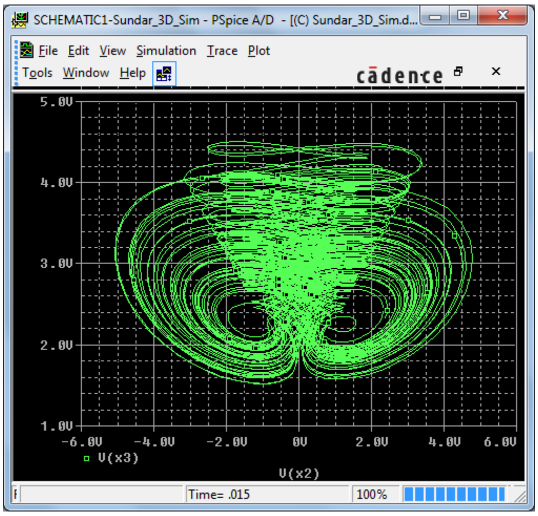

Fig. 12. Chaotic attractor obtained from the circuit in Fig. 10 in (v_{C_2}, v_{C_3})-plane.

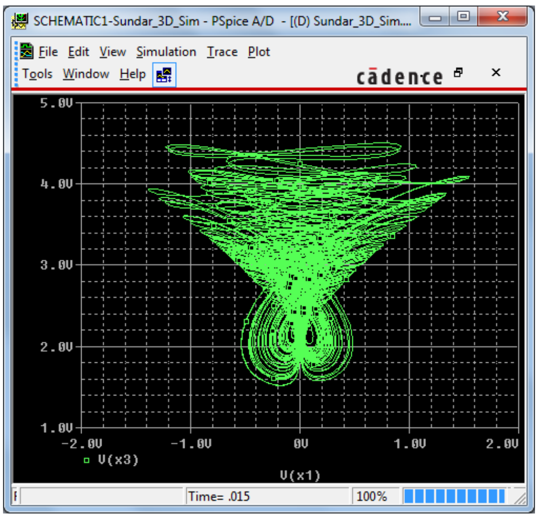

Fig. 13. Chaotic attractor obtained from the circuit in Fig. 10 in (v_{C_1}, v_{C_3})-plane.

7. Conclusion

A new chaotic system with nine-term has been studied in this paper. The dynamical features of the novel system have been illustrated by discussing phase portraits, Lyapunov exponent, and Kaplan-Yorke dimension. Moreover, adaptive control schemes have been proposed to stabilize and synchronize two such new chaotic systems. Furthermore circuital results obtained form an electronic circuit hve validated the theoretical results. Due to its feasibility and its chaotic behaviour, the proposed system can be applied in various engineering chaos-based applications such as crypto-systems, random number generators, or path-planning for autonomous mobile robots.

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