Journal of Engineering Science and Technology Review 8 (2) (2015) 130 - 141
Special Issue on Synchronization and Control of Chaos: Theory, Methods and Applications

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

Research Article

Analysis, Adaptive Control and Synchronization of a Seven-Term Novel 3-D Chaotic System with Three Quadratic Nonlinearities and its Digital Implementation in LabVIEW

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Received 8 September 2014; Revised 13 October 2014; Accepted 25 October 2014

Abstract

This research work proposes a seven-term novel 3-D chaotic system with three quadratic nonlinearities and analyses the fundamental properties of the system such as dissipativity, symmetry, equilibria, Lyapunov exponents and Kaplan-Yorke dimension. The phase portraits of the novel chaotic system simulated using MATLAB depict the strange chaotic attractor of the novel system. For the parameter values and initial conditions chosen in this work, the Lyapunov exponents of the novel chaotic system are obtained as $L_1 = 2.71916, L_2 = 0$ and $L_3 = -13.72776$. Also, the Kaplan-Yorke dimension of the novel chaotic system is obtained as $D_{KY} = 2.19808$. Next, an adaptive controller is designed to stabilize the novel chaotic system with unknown system parameters. Also, an adaptive controller is designed to achieve global chaos synchronization of two identical novel chaotic systems with unknown system parameters. Finally, an electronic circuit realization of the novel chaotic system is depicted using LabVIEW to confirm the feasibility of the theoretical chaotic model.

Keywords: Chaos, chaotic systems, synchronization, Lyapunov exponents, Kaplan-Yorke dimension, circuit simulation.

1. Introduction

Chaotic systems are commonly defined as nonlinear dynamical systems which are very sensitive to initial conditions, topologically mixing and also with dense periodic orbits [1]. There is great interest in the chaos literature in the discovery of chaotic behavior in nature and physical systems.

Historically, Poincaré was the first to notice the possibility of chaos when he was working on a three-body problem. A significant development also occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Since then, many paradigms of 3-D chaotic systems have been found in the chaos literature such as Rössler system [3], Rabinovich system [4], ACT system [5], Sprott systems [6], Chen system [7], Lü system [8], Shaw system [9], Feeny system [10], Shimizu system [11], Liu-Chen system [12], Cai system [13], Tigan system [14], Colpitt's oscillator [15], WINDMI system [16], Zhou system [17], etc.

Recently, many 3-D chaotic systems have been discovered such as Li system [18], Elhadj system [19], Pan system [20], Sundarapandian system [21], Yu-Wang system [22], Sundarapandian-Pehlivan system [23], Zhu system [24], Vaidyanathan systems [25-31], Vaidyanathan-Madhavan system [32], Pehlivan-Moroz-Vaidyanathan system [33], Jafari system [34], Pham system [35], etc.

The study of chaos theory has many important applications in science and engineering such as oscillators [36-37], lasers [38,39], robotics [40-43], chemical reactors [44,45], biology [46,47], ecology [48,49], cardiology [50], memristors [51-53], neural networks [54-56], secure communications [57-60], cryptosystems [61-64], economics [65-67], etc.

Chaos control and chaos synchronization are important research problems in the chaos theory. In the last three decades, many mathematical methods have been developed successfully to address these research problems.

The study of control of a chaotic system investigates methods for designing feedback control laws that globally or locally asymptotically stabilize or regulate the outputs of a chaotic system.

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ISSN: 1791-2377

Many methods have been developed for the control and tracking of chaotic systems such as active control [68-72], adaptive control [73-79], backstepping control [80-82], sliding mode control [83, 84], etc.

Chaos synchronization problem deals with the synchronization of a couple of systems called the *master* or *drive* system and the *slave* or *response* system. To solve this problem, control laws are designed so that the output of the slave system tracks the output of the master system asymptotically with time. Because of the butterfly effect, this is a challenging problem even when the initial conditions of the master and slave systems are nearly identical because of the exponential divergence of the outputs of the two systems in the absence of any control. The synchronization of chaotic systems has applications in secure communications [85-87], cryptosystems [88, 89], encryption [90, 91], etc.

In the chaos literature, many different methodologies have been also proposed for the synchronization and anti-synchronization of chaotic systems such as PC method [92], active control [93-103], time-delayed feedback control [104,105], adaptive control [106-119], sampled-data feedback control [120-121], backstepping control [122-128], sliding mode control [129-134], etc.

In this research paper, we propose a seven-term novel chaotic system with three quadratic nonlinearities. The paper gives a detailed analysis of the fundamental properties of the chaotic system such as dissipativity, symmetry, equilibria, Lyapunov exponents, etc. The Lyapunov exponents of the novel chaotic system are found as $L_1 = 2.71916$, $L_2 = 0$ and $L_3 = -13.72226$. The Kaplan-Yorke dimension of the novel chaotic system is found as $D_{KY} = 2.19808$.

Next, we derive an adaptive control law that stabilizes the 3-D novel chaotic system when the system parameters are unknown. The main control result is proved using Lyapunov stability theory and MATLAB simulations are shown to illustrate the stabilization of the chaotic system.

Furthermore, we derive an adaptive control law that asymptotically synchronizes the identical 3-D novel chaotic systems when the system parameters are unknown. The main synchronization result is proved using Lyapunov stability theory and MATLAB simulations are shown to illustrate the chaos synchronization of the identical novel chaotic systems.

Finally, an electronic circuit realization of the 3-D novel chaotic system using LabVIEW is presented to confirm the feasibility of the theoretical chaos model and control design.

2. A Seven-Term 3-D Novel Chaotic System

The dynamics of the seven-term novel 3-D chaotic system with three quadratic nonlinearities is described by

$$\begin{cases} \dfrac{dx_1}{dt} = a(x_2 - x_1) + cx_2x_3 \\ \dfrac{dx_2}{dt} = bx_1 - x_1x_3 \\ \dfrac{dx_3}{dt} = x_1x_2 - x_3 \end{cases} \quad (1)$$

where x_1, x_2, x_3 are the states and a, b, c are positive parameters.

The nonlinear system (1) depicts a strange chaotic attractor when the parameter values are taken as:

$$a = 10, \; b = 15, \; c = 12 \quad (2)$$

We take the initial conditions as:

$$x_1(0) = 0.6, \; x_2(0) = 1.8, \; x_3(0) = 1.2 \quad (3)$$

The 3-D portrait of the strange chaotic attractor (1) for the parameter values (2) and the initial conditions (3) is depicted in Fig. 1, and the 2-D portraits (projections on the three coordinate planes) are depicted in Figs. 2-4.

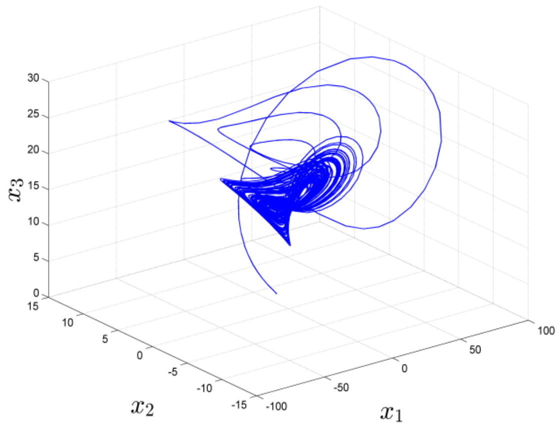

Fig. 1. The strange chaotic attractor of the novel chaotic system in R^3.

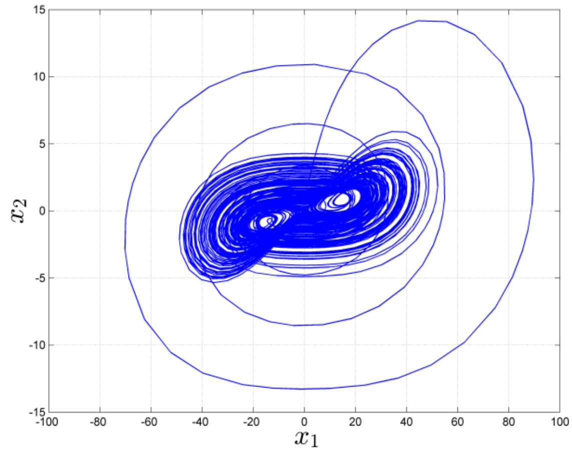

Fig. 2. The 2-D projection of the attractor on the (x_1, x_2) plane.

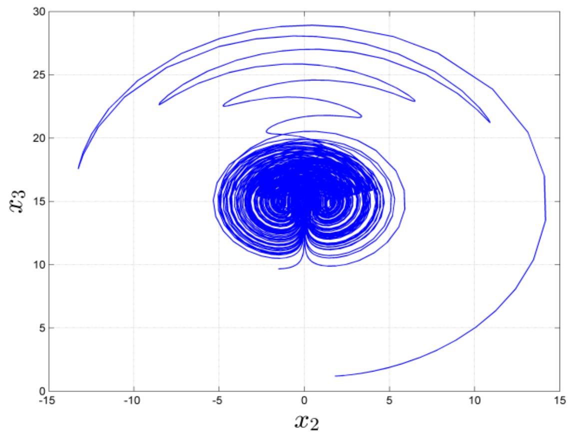

Fig. 3. The 2-D projection of the attractor on the (x_2, x_3) plane.

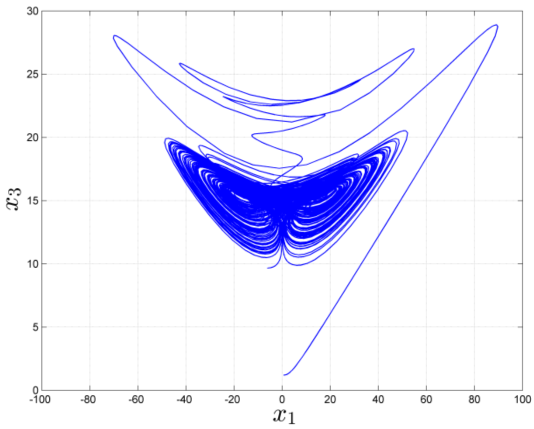

Fig. 4. The 2-D projection of the attractor on the (x_1, x_3) plane.

3. Analysis of the 3-D Novel Chaotic System

In this section, qualitative properties of the 3-D novel chaotic system are detailed.

3.1. Dissipativity

In vector notation, we may express the system (1) as:

$$\frac{dx}{dt} = f(\boldsymbol{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \quad (4)$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) &= a\,(x_2 - x_1) + c x_2 x_3 \\ f_2(x_1, x_2, x_3) &= b x_1 - x_1 x_3 \\ f_3(x_1, x_2, x_3) &= x_1 x_2 - x_3 \end{cases} \quad (5)$$

We take the parameter values as in the chaotic case, *viz.* $a = 10$, $b = 15$ and $c = 12$.

Let Ω be any region in $\boldsymbol{R}^3$ with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\frac{dV}{dt} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1 dx_2 dx_3 \quad (6)$$

The divergence of the novel chaotic system (1) is easily found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a - 1 = -11 \quad (7)$$

Substituting (7) into (6), we obtain the first order ODE.

$$\frac{dV}{dt} = -11\, V(t) \quad (8)$$

Integrating (8), we obtain the unique solution as:

$$V(t) = \exp(-11\, t)\, V(0) \quad (9)$$

It is evident from Eq.(9) that $V(t) \to 0$ exponentially as $t \to \infty$.

This shows that the novel chaotic system (1) is dissipative.

Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2. Symmetry and Invariance

It is easy to see that the system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \to (-x_1, -x_2, x_3). \quad (10)$$

Thus, the system (1) has rotation symmetry about the x_3-axis and any non-trivial trajectory of the system (1) must have a twin trajectory. It is also easy to see that the x_3-axis is invariant under the flow of the system (1).

3.3. Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the following system of equations (with $a = 10$, $b = 15$ and $c = 12$).

$$\begin{cases} a(x_2 - x_1) + c x_2 x_3 &= 0 \\ b x_1 - x_1 x_3 &= 0 \\ x_1 x_2 - c x_3 &= 0 \end{cases} \quad (11)$$

The equilibria of (11) are calculated as

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, E_1 = \begin{bmatrix} 16.88194 \\ 0.88852 \\ 15.00000 \end{bmatrix}, E_2 = \begin{bmatrix} -16.88194 \\ -0.88852 \\ 15.00000 \end{bmatrix} \quad (12)$$

The Jacobian matrix of the system (1) at $\boldsymbol{x}$ is given by

$$J(\boldsymbol{x}) = \begin{bmatrix} -a & a + c x_3 & c x_2 \\ b - x_3 & 0 & -x_1 \\ x_2 & x_1 & -1 \end{bmatrix} \quad (13)$$

The Jacobian matrix at E_0 is obtained as:

$$J_0 = J(E_0) = \begin{bmatrix} -10 & 10 & 0 \\ 15 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (14)$$

Using MATLAB, we find the eigenvalues of J_0 as:

$$\lambda_1 = -18.2288,\ \lambda_2 = -1,\ \ \lambda_3 = 8.2288 \quad (15)$$

Thus, the equilibrium E_0 is a *saddle-point,* which is unstable.

The Jacobian matrix at E_1 is obtained as:

$$J_1 = J(E_1) = \begin{bmatrix} -10 & 190 & 10.6622 \\ 0 & 0 & -16.8819 \\ 0.8885 & 16.8819 & -1 \end{bmatrix} \quad (16)$$

Using MATLAB, we find the eigenvalues of J_1 as:

$$\lambda_1 = -15.7860,\ \lambda_{2,3} = 2.3930 \pm 18.8508i \tag{17}$$

Thus, the equilibrium E_1 is a *saddle-focus,* which is unstable.

The Jacobian matrix at E_2 is obtained as:

$$J_2 = J(E_2) = \begin{bmatrix} -10 & 190 & -10.6622 \\ 0 & 0 & 16.8819 \\ -0.8885 & -16.8819 & -1 \end{bmatrix} \tag{18}$$

Using MATLAB, we find the eigenvalues of J_2 as:

$$\lambda_1 = -15.7860,\ \lambda_{2,3} = 2.3930 \pm 18.8508i \tag{19}$$

Thus, the equilibrium E_2 is a *saddle-focus,* which is unstable. Hence, all the three equilibria of the system (1) are unstable equilibria.

3.4. Lyapunov Exponents and Kaplan-Yorke Dimension

For the chosen parameter values (2), the Lyapunov exponents of the novel chaotic system (1) are obtained using MATLAB as:

$$L_1 = 2.71916, L_2 = 0, L_3 = -13.72776 \tag{20}$$

Since the spectrum of Lyapunov exponents (20) has a positive term L_1, it follows that the 3-D novel system (1) is chaotic.

The maximal Lyapunov exponent (MLE) of the novel chaotic system (1) is $L_1 = 2.71916$.

The sum of the Lyapunov exponents is obtained as:

$$L_1 + L_2 + L_3 = -11.086 < 0. \tag{21}$$

Thus, it follows that the novel chaotic system (1) is dissipative.

Also, the Kaplan-Yorke dimension of the novel chaotic system (1) is calculated as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.19808 \tag{22}$$

Figure 5 depicts the dynamics of the Lyapunov exponents of the novel chaotic system (1).

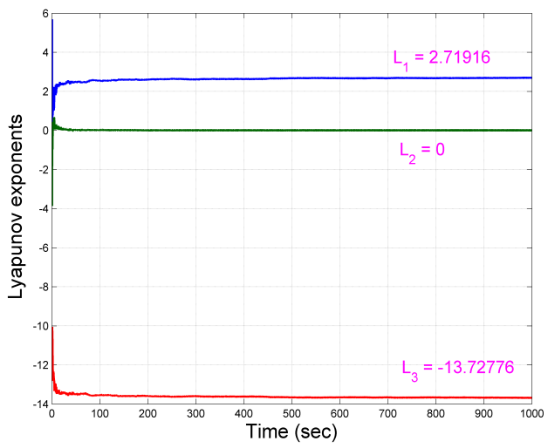

Fig. 5. Dynamics of the Lyapunov Exponents of the Novel System

4. Adaptive Control of the Novel Chaotic System

In this section, new results are derived for an adaptive controller to stabilize the novel chaotic system with unknown system parameters.

We consider the controlled novel 3-D chaotic system

$$\begin{cases} \frac{dx_1}{dt} &= a(x_2 - x_1) + cx_2x_3 + u_1 \\ \frac{dx_2}{dt} &= bx_1 - x_1x_3 + u_2 \\ \frac{dx_3}{dt} &= x_1x_2 - x_3 + u_3 \end{cases} \tag{23}$$

In (23), the parameters a, b, c are unknown and u_1, u_2, u_3 are adaptive controls to be determined using estimates $A(t)$, $B(t)$, $C(t)$ for the unknown parameters a, b, c, respectively.

We consider the adaptive feedback control law

$$\begin{aligned} u_1 &= -A(t)(x_2 - x_1) - C(t)x_2x_3 - k_1x_1 \\ u_2 &= -B(t)x_1 + x_1x_3 - k_2x_2 \\ u_3 &= -x_1x_2 + x_3 - k_3x_3 \end{aligned} \tag{24}$$

In (24), k_1, k_2, k_3 are positive gain constants. Also, $A(t)$, $B(t)$ and $C(t)$ are estimates for the unknown system parameters a, b, c, respectively.

Substituting (24) into (23), we get

$$\begin{cases} \frac{dx_1}{dt} &= (a - A(t))(x_2 - x_1) \\ &\quad + (c - C(t))x_2x_3 - k_1x_1 \\ \frac{dx_2}{dt} &= (b - B(t))x_1 - k_2x_2 \\ \frac{dx_3}{dt} &= -k_3x_3 \end{cases} \tag{25}$$

The parameter estimation errors are defined by

$$\begin{cases} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \end{cases} \tag{26}$$

Using (26), we can simplify the dynamics (25) as

$$\begin{cases} \frac{dx_1}{dt} &= e_a(x_2 - x_1) + e_cx_2x_3 - k_1x_1 \\ \frac{dx_2}{dt} &= e_bx_1 - k_2x_2 \\ \frac{dx_3}{dt} &= -k_3x_3 \end{cases} \tag{27}$$

Differentiating (26) with respect to t, we get

$$\begin{cases} \frac{de_a}{dt} &= -\frac{dA}{dt} \\ \frac{de_b}{dt} &= -\frac{dB}{dt} \\ \frac{de_c}{dt} &= -\frac{dC}{dt} \end{cases} \tag{28}$$

To derive an update law for the parameter estimates, we use adaptive control theory.

We consider the candidate Lyapunov function defined by

$$V = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2) \tag{29}$$

Clearly, V is a quadratic, positive definite function defined on R^6.

Differentiating V along the trajectories of (28) and (29), we obtain

$$\begin{aligned}\frac{dV}{dt} &= -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \\ &+e_a\left[x_1(x_2-x_1)-\frac{dA}{dt}\right] \\ &+e_b\left[x_1x_2-\frac{dB}{dt}\right]+e_c\left[x_1x_2x_3-\frac{dC}{dt}\right]\end{aligned} \tag{30}$$

In view of (30), we take the parameter update law as:

$$\begin{cases}\frac{dA}{dt} &= x_1(x_2-x_1) \\ \frac{dB}{dt} &= x_1x_2 \\ \frac{dC}{dt} &= x_1x_2x_3\end{cases} \tag{31}$$

Theorem 1.*The novel chaotic system (23) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (24) and the parameter update law (31), where k_1, k_2, k_3 are positive constants.*

Proof. We prove this result using Lyapunov stability theory. For this purpose, we consider the quadratic Lyapunov function V defined by (29), which is positive definite on R^6.

Substituting the parameter update law (31) into (30), we obtain the time derivative of V as:

$$\frac{dV}{dt} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2, \tag{32}$$

which is a negative semi-definite function on R^6.

Thus, we can conclude that the state vector $x(t)$ and the parameter estimation error are globally bounded, i.e. $[x_1(t),\ x_2(t),\ x_3(t), e_a(t), e_b(t), e_c(t)]^T \in L_\infty$.

We define $k = \min\{k_1, k_2, k_3\}$.

Then it follows from (32) that

$$\frac{dV}{dt} \le -k\|x\|^2 \text{ or } k\|x\|^2 \le -\frac{dV}{dt} \tag{33}$$

Integrating the inequality (33) from 0 to t , we get

$$k\int_0^t \|x(\tau)\|^2 d\tau \le V(0) - V(t) \tag{34}$$

From (34), it follows that $x(t) \in L_2$.

Using (27), we can conclude that $\dot{x} \in L_\infty$.

Thus, using Barbalat's lemma [135], we conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof. ■

For numerical simulations, the parameter values of the novel chaotic system (23) are taken as in the chaotic case, *viz*. $a = 10$, $b = 15$ and $c = 12$. We take the gain constants as $k_i = 5$ for $i = 1, 2, 3$.

The initial values of the chaotic system (23) are taken as $x_1(0) = 7.5$, $x_2(0) = 5.3$ and $x_3(0) = -6.2$.

The initial values of the parameter estimates are taken as $A(0) = 10.4$, $B(0) = 6.3$ and $C(0) = 2.5$.

Fig. 6 depicts the time-history of the controlled novel chaotic system. It is clear that the controlled system (23) is globally exponentially stable when the adaptive control law (24) and the parameter update law (31) are implemented.

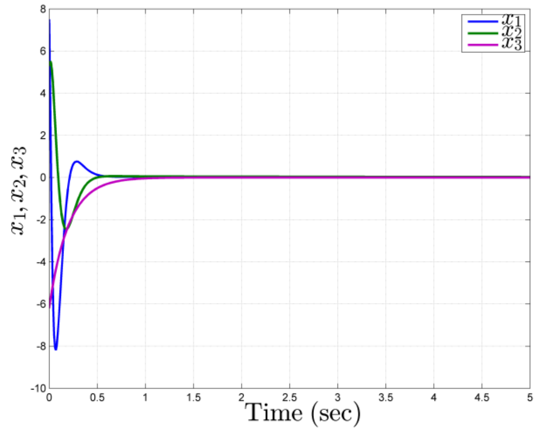

Fig. 6. Time-history of the controlled novel chaotic system.

5. Adaptive Synchronization of the Identical Novel Chaotic Systems

In this section, we derive new results for the adaptive synchronization of identical novel chaotic systems with unknown system parameters.

As the master system, we take the novel chaotic system

$$\begin{cases}\frac{dx_1}{dt} &= a(x_2-x_1)+cx_2x_3 \\ \frac{dx_2}{dt} &= bx_1 - x_1x_3 \\ \frac{dx_3}{dt} &= x_1x_2 - x_3\end{cases} \tag{35}$$

where x_1, x_2, x_3 are state variables and a, b, c are unknown, constant, parameters of the system.

As the slave system, we take the novel chaotic system

$$\begin{cases}\frac{dy_1}{dt} &= a(y_2-y_1)+cy_2y_3+u_1 \\ \frac{dy_2}{dt} &= by_1 - y_1y_3+u_2 \\ \frac{dy_3}{dt} &= y_1y_2 - y_3+u_3\end{cases} \tag{36}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are adaptive controllers to be designed.

The synchronization error between the identical chaotic systems is defined as:

$$\begin{cases}e_1(t) &= y_1(t) - x_1(t) \\ e_2(t) &= y_2(t) - x_2(t) \\ e_3(t) &= y_3(t) - x_3(t)\end{cases} \tag{37}$$

The error dynamics is calculated as:

$$\begin{cases}\frac{de_1}{dt} &= a(e_2-e_1)+c(y_2y_3-x_2x_3)+u_1 \\ \frac{de_2}{dt} &= be_1 - y_1y_3 + x_1x_3 + u_2 \\ \frac{de_3}{dt} &= -e_3 + y_1y_2 - x_1x_2 + u_3\end{cases} \tag{38}$$

We consider the adaptive control law

$$\begin{cases} u_1 &= -A(t)(e_2 - e_1) - C(t)\big(y_2 y_3 - x_2 x_3\big) \\ & \quad -k_1 e_1 \\ u_2 &= -B(t)e_1 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 &= e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{cases} \tag{39}$$

where k_1, k_2, k_3 are positive gains and $A(t)$, $B(t)$, $C(t)$ are estimates of the unknown parameters a, b, c, respectively.

The parameter estimation errors are defined by

$$\begin{cases} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \end{cases} \tag{40}$$

Substituting (39) into the error dynamics (38), we get

$$\begin{cases} \frac{de_1}{dt} &= \big(a - A(t)\big)(e_2 - e_1) \\ & \quad +\big(c - C(t)\big)\big(y_2 y_3 - x_2 x_3\big) - k_1 e_1 \\ \frac{de_2}{dt} &= (b - B(t))e_1 - k_2 e_2 \\ \frac{de_3}{dt} &= -k_3 e_3 \end{cases} \tag{41}$$

Using (40), we can simplify the error dynamics (40) as:

$$\begin{cases} \frac{de_1}{dt} &= e_a(e_2 - e_1) \\ & \quad +e_c\big(y_2 y_3 - x_2 x_3\big) - k_1 e_1 \\ \frac{de_2}{dt} &= e_b e_1 - k_2 e_2 \\ \frac{de_3}{dt} &= -k_3 e_3 \end{cases} \tag{42}$$

Differentiating (40) with respect to t, we get

$$\begin{cases} \frac{de_a}{dt} &= -\frac{dA}{dt} \\ \frac{de_b}{dt} &= -\frac{dB}{dt} \\ \frac{de_c}{dt} &= -\frac{dC}{dt} \end{cases} \tag{43}$$

Next, we use Lyapunov stability theory for finding an update law for the parameter estimates.

Consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right), \tag{44}$$

which is positive definite on R^6.

Differentiating V along the trajectories of (42) and (43), we get

$$\begin{aligned} \frac{dV}{dt} &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ & \quad +e_a\left[e_1(e_2 - e_1) - \frac{dA}{dt}\right] + e_b\left[e_1 e_2 - \frac{dB}{dt}\right] \\ & \quad +e_c\left[e_1(y_2 y_3 - x_2 x_3) - \frac{dC}{dt}\right] \end{aligned} \tag{45}$$

In view of (45), we take the parameter update law as:

$$\begin{cases} \frac{dA}{dt} &= e_1(e_2 - e_1) \\ \frac{dB}{dt} &= e_1 e_2 \\ \frac{dC}{dt} &= e_1(y_2 y_3 - x_2 x_3) \end{cases} \tag{46}$$

Theorem 2.*The identical novel chaotic systems (35) and (36) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (39) and the parameter update law (46), where k_1, k_2, k_3 are positive constants.*

Proof. We prove this result using Lyapunov stability theory.

For this purpose, we consider the quadratic Lyapunov function V defined by (44), which is positive definite on R^6.

Substituting the parameter update law (46) into (45), we obtain the time derivative of V as:

$$\frac{dV}{dt} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \tag{47}$$

which is a negative semi-definite function on R^6.

Thus, we can conclude that the synchronization error $e(t)$ and the parameter estimation error are globally bounded.

We define $k = \min\{k_1, k_2, \underline{k_3}\}$. Then we get

$$\frac{dV}{dt} \le -k\|e\|^2 \text{ or } k\|e\|^2 \le -\frac{dV}{dt} \tag{48}$$

Integrating the inequality (48) from 0 to t, we get

$$k\int_0^t \|e(\tau)\|^2 d\tau \le V(0) - V(t) \tag{49}$$

From (49), it follows that $e(t) \in L_2$. Using (42), we can conclude that $\dot{x} \in L_\infty$.

Thus, using Barbalat's lemma [135], we conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.This completes the proof. ■

For numerical simulations, the parameter values of the novel chaotic systems (35) and (36) are taken as in the chaotic case, *viz.* $a = 10$, $b = 15$ and $c = 12$. We take the gain constants as $k_i = 5$ for $i = 1, 2, 3$.

The initial conditions of the master system (35) are taken as $x_1(0) = 5.2$, $x_2(0) = 8.3$ and $x_3(0) = -2.7$.

The initial conditions of the slave system (36) are taken as $y_1(0) = -3.5$, $y_2(0) = 4.6$ and $y_3(0) = 5.4$.

The initial conditions of the parameter estimates are taken as $A(0) = 2.1$, $B(0) = 9.4$ and $C(0) = 3.8$.

Figures 7-9 describe the complete synchronization of the novel chaotic systems (35) and (36), while Fig. 10 describes the time-history of the synchronization errors e_1, e_2, e_3.

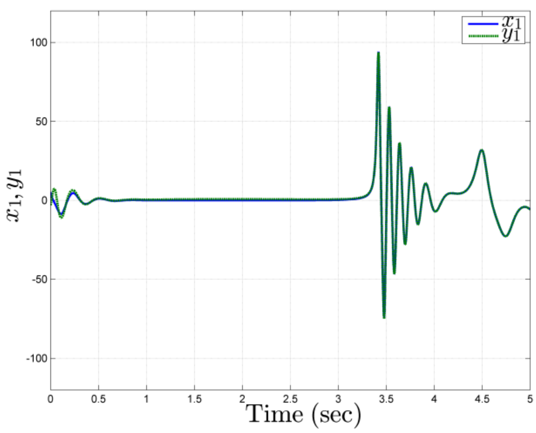

Fig. 7. Synchronization of the states x_1 and y_1 of the chaotic systems.

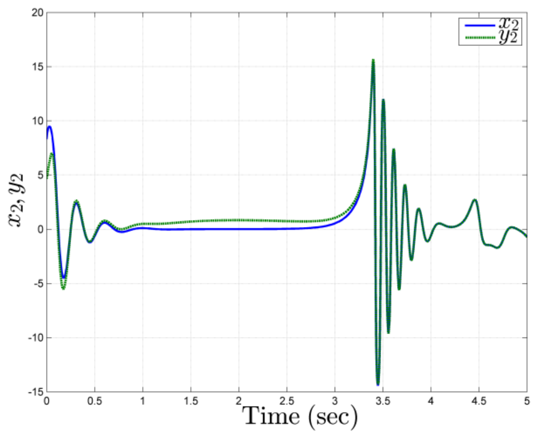

Fig. 8. Synchronization of the states x_2 and y_2 of the chaotic systems.

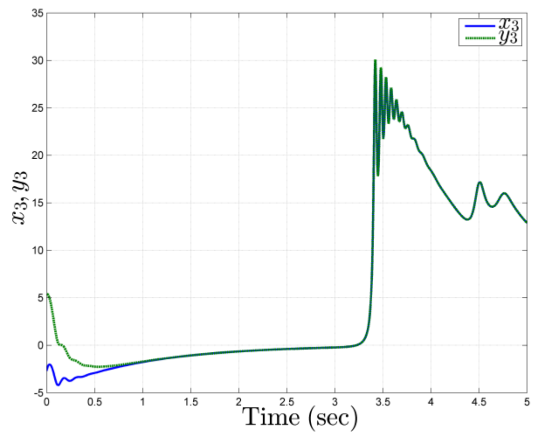

Fig. 9. Synchronization of the states x_3 and y_3 of the chaotic systems.

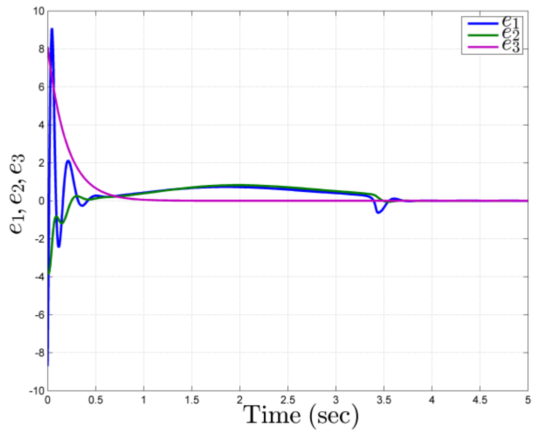

Fig. 10. Time-history of the synchronization errors.

6. LabVIEW Implementation of the 3-D Novel Chaotic system and its Adaptive Synchronization

The proposed 3-D novel chaotic system is then implemented in LabVIEW. The state equations of the new chaotic system are implemented in CSD tool book of LabVIEW. Fixed step simulation method is applied for the iteration process. The configuration parameters are altered to match the simulation environment. Simulation time waveform converters are deployed to generate time based signals for waveform plotting. Reshape array blocks are used to get 1D array elements from double precision data. The LabVIEW VI model is shown in Fig. 11. The time waveforms and phase portraits of the master system are shown in Figs. 12 & 13 respectively. The designed controller for the adaptive synchronization is shown in Fig. 14. The errors plots of the synchronized systems are shown in Fig. 15.

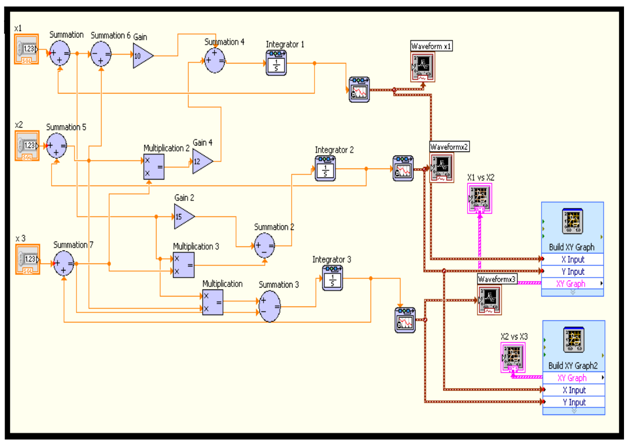

Fig. 11. VI Model for the new chaotic system.

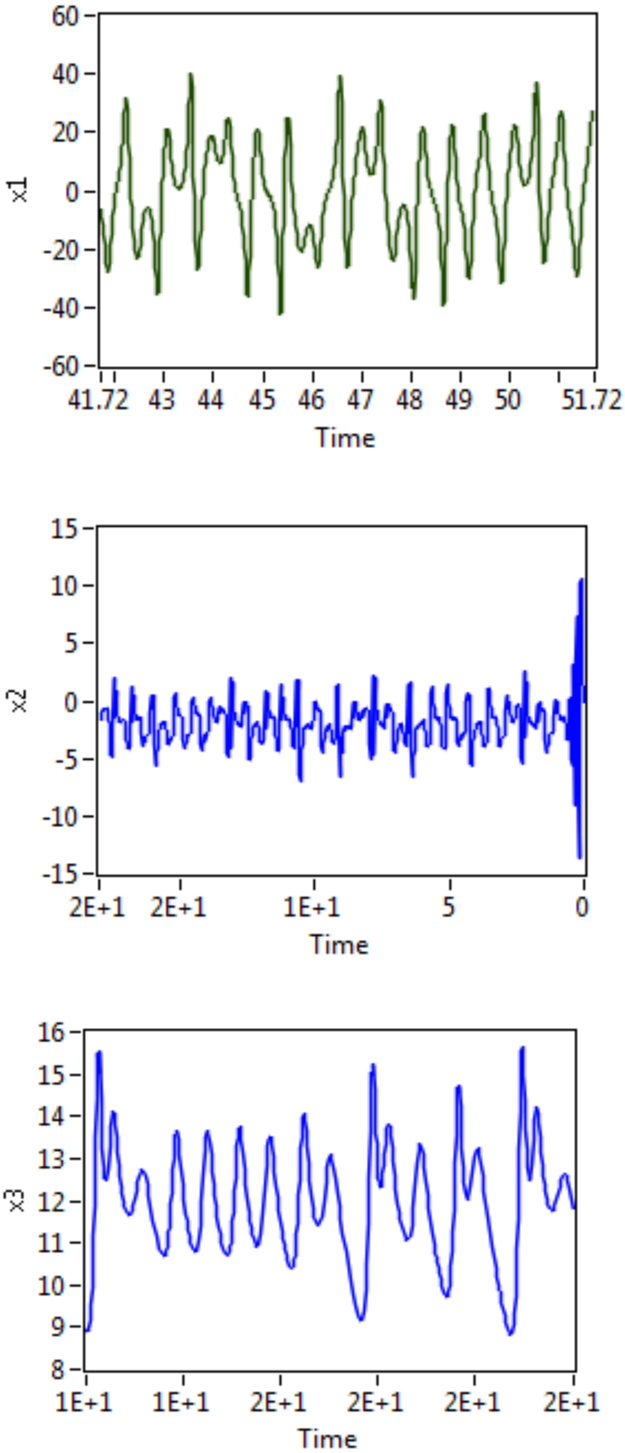

Fig. 12. Time waveforms of the state variables.

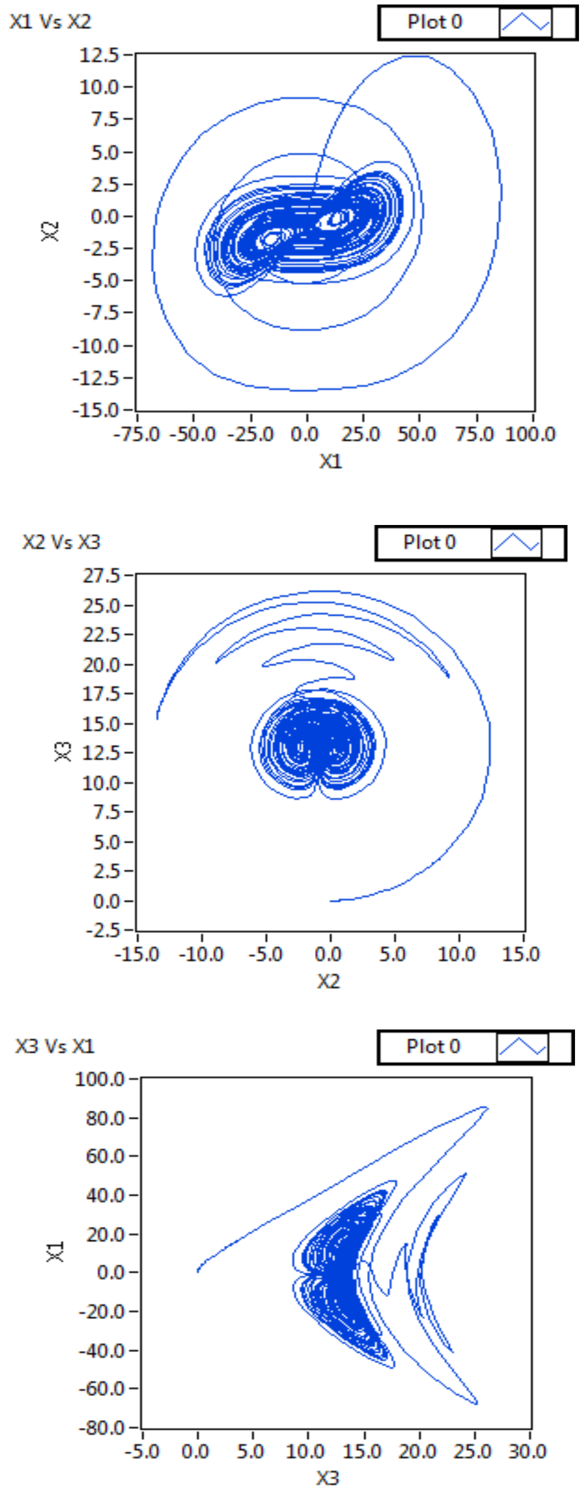

Fig. 13. 2-D Phase Portraits of the novel chaotic attractor.

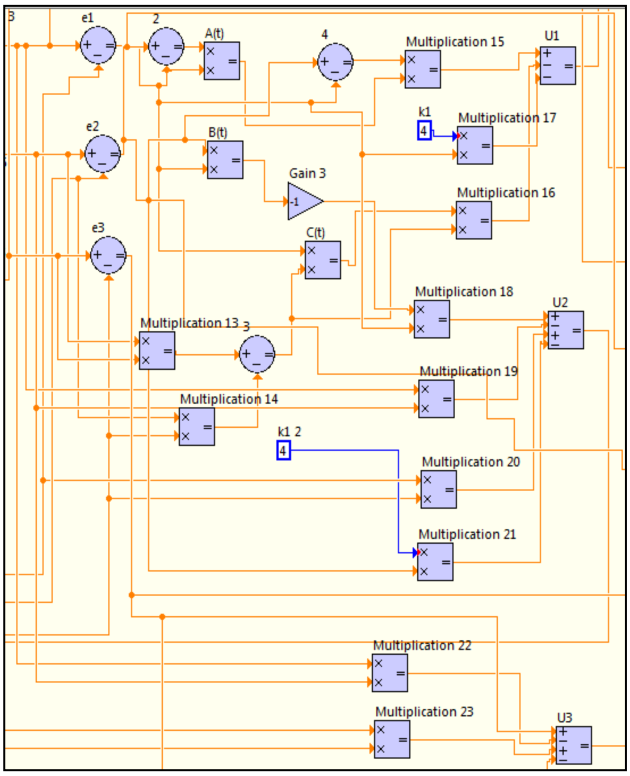

Fig. 14. The controller u implemented in LabVIEW.

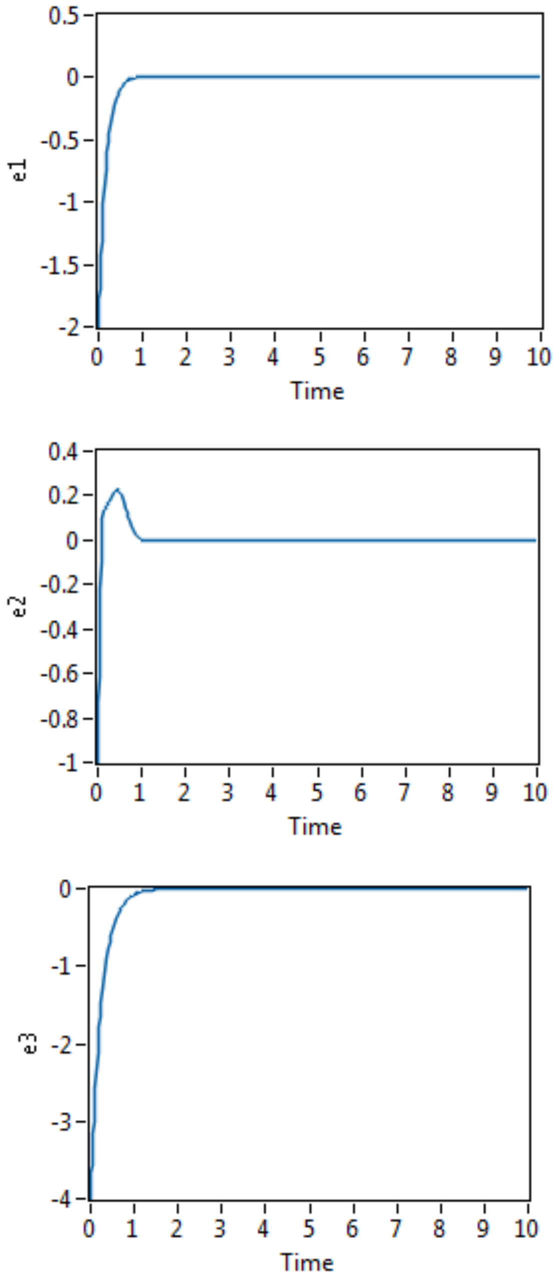

Fig. 15. Time-history of the synchronization errors e_1, e_2, e_3.

7. Conclusion

In this work a seven-term novel 3-D chaotic system with three quadratic terms was presented. The fundamental properties of the system such as dissipativity, symmetry, equilibria, Lyapunov exponents and Kaplan-Yorke dimension as well as its phase portraits were described in detail. Also, an adaptive controller for stabilizing the proposed system with unknown parameters was designed. Furthermore, the case of chaos synchronization, of two identical chaotic systems of this type, by using an adaptive controller was studied. Finally, the LabVIEW implementation of the novel chaotic system was presented for confirming the feasibility of the theoretical chaotic system.

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