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Research Article

Bidirectional Coupling Scheme of Chaotic Systems and its Application in Secure Communication System

Aceng Sambas[*, 1,2], Mada Sanjaya WS[2,3] and Mustafa Mamat[1]

[1]*Department of Information Technology, Universiti Sultan Zainal Abidin, Kuala Terengganu, Malaysia.*
[2]*Bolabot Techno Robotic School, Jl. A.H. Nasution No. 105, Bandung, West Java, Indonesi*a.
[3]*Department of Physics, Universiti Islam Negeri Sunan Gunung Djati, Bandung, West Java, Indonesia.*

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Abstract

In this paper, in order to show some interesting phenomena of three dimensional autonomous ordinary differential equations, the chaotic behavior as a function of a variable control parameter, has been studied. The initial study in this paper is to analyze the phase portraits, the Lyapunov exponents, the Poincaré maps and the bifurcation diagrams. Moreover, some appropriate comparisons are made to contrast some of the existing results. Finally, the effectiveness of the bidirectional coupling scheme between two identical Jerk circuits in a secure communication system is presented in details. Finally, the simulation results are shown to demonstrate that the proposed method is correct and feasible

Keywords: Jerk circuit, nonlinear circuit, Poincaré map, bifurcation diagram, bidirectional coupling.

1. Introduction

Chaotic phenomena are fascinating to many researchers in various field such as biology [1], robotics [2,3], bits generators [4], psychology [5], ecology [6,7] and economy [8,9]. Deterministic chaotic systems have the property of being sensitive to initial conditions. Trajectories of a chaotic system starting from very near initial conditions in phase space tend to diverge exponentially. Nevertheless it was demonstrated that certain chaotic systems can be connected such that their chaotic movements are synchronized [10-13].

The first who study the topic of chaotic synchronization were Yamada and Fujisaka in 1983 [14] and Afraimovich et al. in 1986 [15]. However, it was not until 1990, when Pecora and Carroll (PC) introduced their method of chaotic synchronization [16] and suggested application to secure communications, that the topic started to arouse major interest [17-19]. Many researchers demonstrated, using simulation, that chaos can be synchronized and applied to secure communication schemes, such us, in secure fiber-optical communication scheme using chaos [20], in secure communication based on chaotic cipher [21], in secure communication with chaotic lasers [22] and wireless communication with chaos [23].

The paper is organized as follows. In section 2, the details of the proposed autonomous Jerk circuit's simulation using MATLAB 2010 and MultiSIM 10.0, are presented. In Section 3, the bidirectional coupling method is applied in order to synchronize two identical autonomous Jerk circuits. The chaotic masking communication scheme by using the above mentioned synchronization technique is presented in Section 4. Finally, in Section 5, the concluding remarks are given.

2. Jerk Circuit

Sprott found the functional form of three-dimensional dynamical systems which exhibit chaos. Jerk equation has a simple nonlinear function, which can be implemented with an autonomous electronic circuit [24]. Furthermore, Alpana Pandey modifies the system of Jerk equations into a system of simple quadratic equations. In this work, the Jerk circuit, which was firstly presented by Alpana Pandey in 2013 [25,26], is used. This is a three-dimensional autonomous nonlinear system that is described by the following system of ordinary differential equations:

$$\left.\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -x - y - az - bx^2 \end{aligned}\right\} \quad (1)$$

The new system has one quadratic term and two positive real constants a and b. The parameters and initial conditions of the Jerk system (1) are chosen as: a = 0.5, b = 0.125 and

* E-mail address: acenx.bts@gmail.com
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$(x_0, y_0, z_0) = (0.001, 0.010, 0.100)$, so that the system shows the expected chaotic behavior.

2.1 Numerical Simulations

In this section, we present the numerical simulation to illustrate the dynamical behavior of Jerk circuit of system (1). For numerical simulation of chaotic system defined by a set of differential equation such as Jerk circuit, different integration techniques can be used. In the MATLAB 2010 numerical simulation, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used. Figures 1(a)-(c) show the projections of the phase space orbit on to the *x*–*y* plane, the *y*–*z* plane and the *x*–*z* plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Jerk system presents chaotic attractors of Rössler type. Also, it is known from the nonlinear theory, that the spectrum of Lyapunov exponents provides additional useful information about system's behavior. In a three dimensional system, like this, there has been three Lyapunov exponents (λ_1, λ_2, λ_3). In more details, for a 3D continuous dissipative system the values of the Lyapunov exponents are useful for distinguishing among the various types of orbits. So, the possible spectra of attractors, of this class of dynamical systems, can be classified in four groups, based on Lyapunov exponents [27-30].

- For a fixed point: $\lambda_1, \lambda_2, \lambda_3 < 0$.
- For a limit point: $\lambda_1 = 0, \lambda_2, \lambda_3 < 0$.
- For a two-torus: $\lambda_1, \lambda_2 = 0, \lambda_3 < 0$.
- For a strange attractor: $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 < 0$.

So, in Figs.2(a) & 2(b) the dynamics of the proposed system's Lyapunov exponents for the variation of the parameter $a \in [0.49, 0.65]$ is shown. For $0.49 \le a \le 0.57$ a strange attractor is displayed as the system has one positive Lyapunov exponent, while for values of $0.57 < a \le 0.65$ is a transition to limit point behavior as the system has two negative Lyapunov exponents.

Also, the word bifurcation denotes a situation in which the solutions of a nonlinear system of differential equations alter their character with a change of a parameter on which the solutions depend. Bifurcation theory studies these changes (e.g. appearance and disappearance of the stationary points, dependence of their stability on the parameter etc). A MATLAB program was written to obtain the bifurcation diagrams for Jerk circuit of Figs.3(a) & 3(b). So, in this diagram a possible bifurcation diagram for system (1), in the range of $0.4 \le a \le 0.65$, is shown. For the chosen value of $0.49 \le a < 0.554$ the system displays the expected chaotic behavior. Also, for $0.554 \le a < 0.65$, a reverse period doubling route is presented.

A Poincaré section is often used to reduce a three-dimensional continuous system to a lower-dimensional discrete map. The strength behinds this tool is that these sections have the same topological properties as their continuous counterparts [31]. In the chaotic state the phase portrait is very dense, in the sense that the trajectories of the motion are very close to each other. It can be only indicative of the minima and maxima of the motion. Any other characterization of the motion is difficult to be interpreted. So, one way to capture the qualitative features of the strange attractor is to obtain the Poincaré map [32,33]. Figures 4(a)-(c) shows the Poincaré section map by using MATLAB, for $a = 0.5$ and $b = 1.25$.

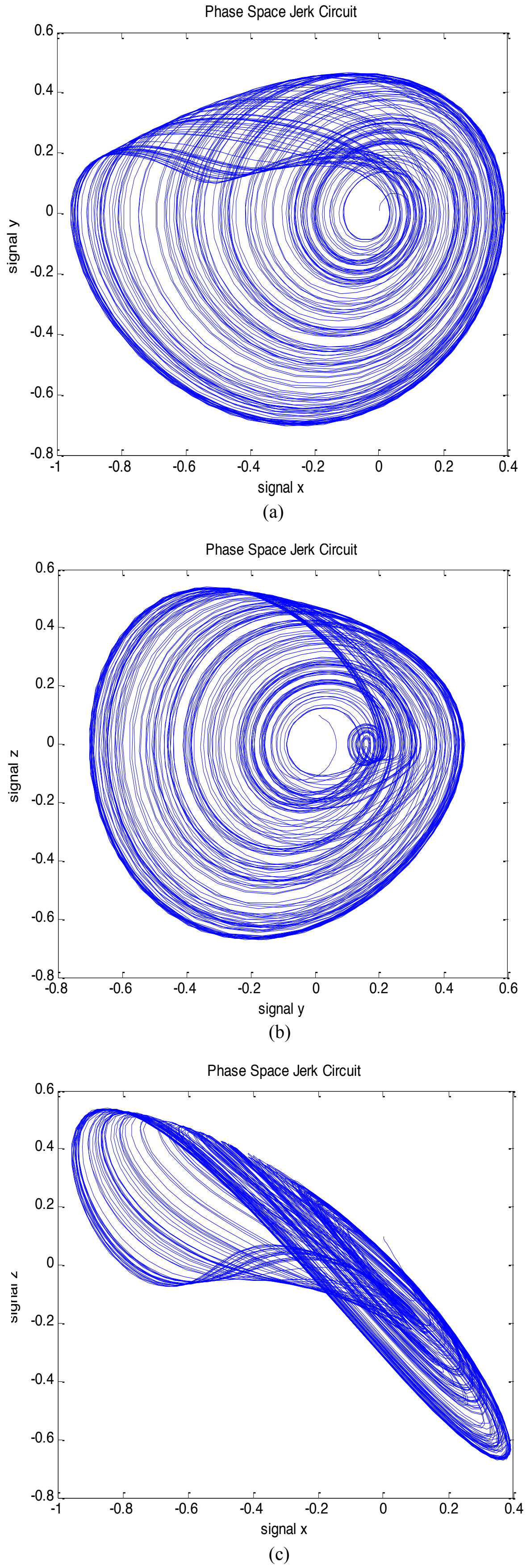
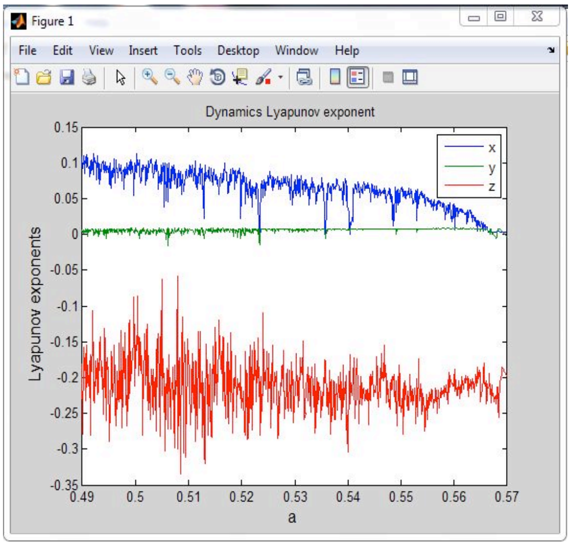
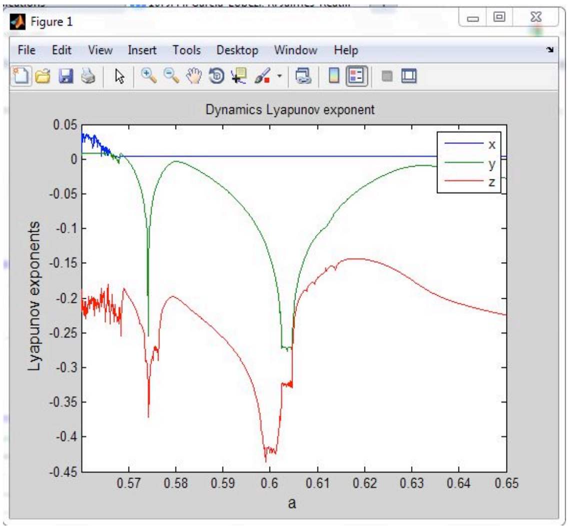


Fig.1. Numerical simulation results using MATLAB 2010, for $a = 0.5$, $b = 0.125$, in (a) *x-y* plane, (b) *y-z* plane, (c) *x-z* plane.



(a)



(b)

Fig. 2. Nonlinear dynamics of system (1) for specific values set b=0.125. (a) Lyapunov exponents versus the parameter control $a \in$ [0.49-0.57] (b) Lyapunov exponents versus the parameter control $a \in$ [0.56-0.65], with MATLAB 2010.

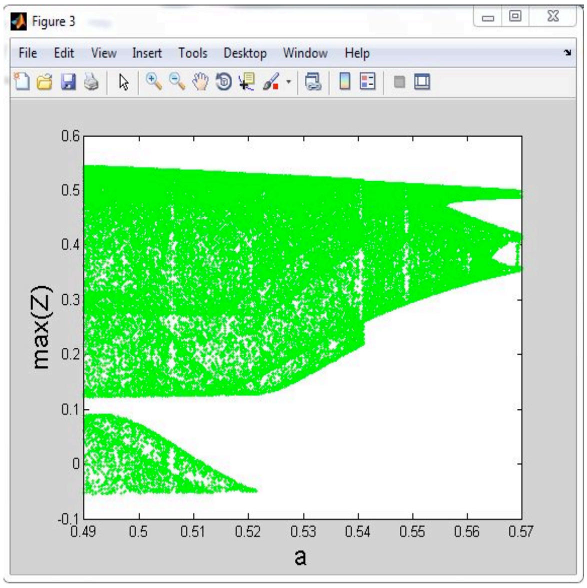
2.2 Analog Circuit Simulation Using MultiSIM

A simple electronic circuit is designed that can be used to study chaotic phenomena. The circuit employs simple electronic elements, such as resistors, capacitors, multiplier and operational amplifiers. In Fig. 5, the voltages of C_1, C_2, C_3 are used as x, y and z, respectively. The nonlinear term of system (1) are implemented with the analog multiplier. The corresponding circuit equation can be described as:

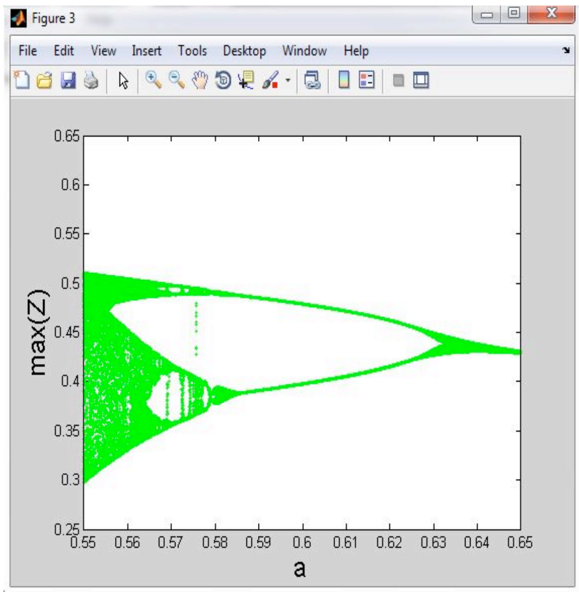
$$\left.\begin{aligned} \dot{x} &= \frac{1}{C_1R_1}y \\ \dot{y} &= \frac{1}{C_2R_4}z \\ \dot{z} &= -\frac{1}{C_3R_7}x - \frac{1}{C_3R_8}y - \frac{1}{C_3R_9}z - \frac{1}{10C_3R_{10}}x^2 \end{aligned}\right\} \quad (2)$$

We choose R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = 100 kΩ, R_{10} = 80 kΩ. C_1 = C_2 = C_3 C_4 = 1 nF. The circuit has three integrators (by using Op-amp TL082CD) in a feedback loop and a multiplier (IC AD633). The supplies of all active devices are ±9 V. With MultiSIM 10.0, we obtain the experimental observations of system (1) as shown in Fig.6. As compared with Fig.1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Jerk circuit is confirmed. The parameter variable a of system (1) is changed by adjusting the resistor R_9, and obeys the following relation:

$$a = \frac{1}{C_3R_9} \quad (3)$$

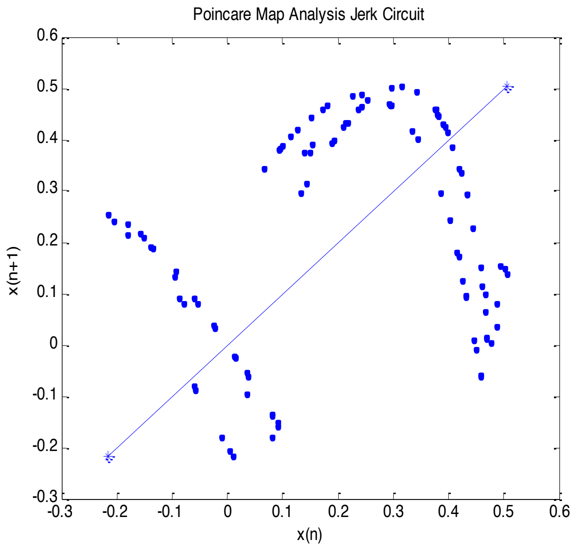


(a)

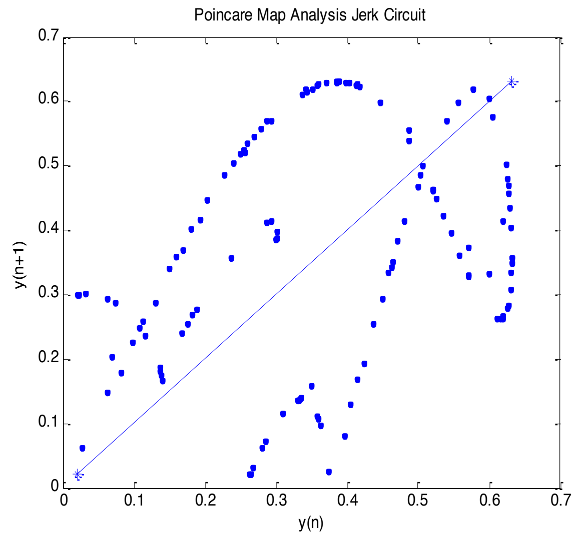


(b)

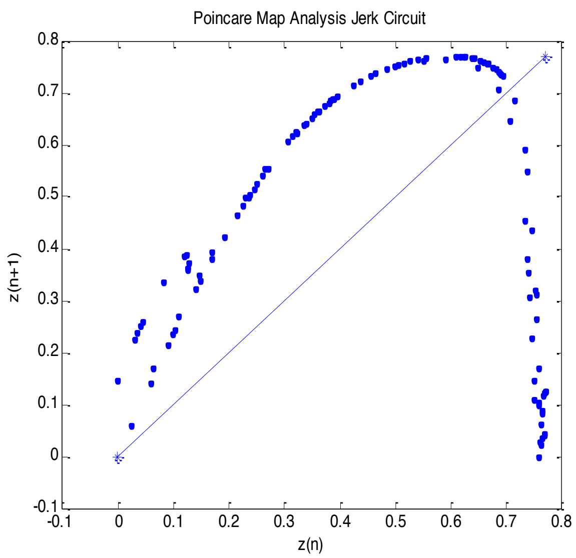
Fig.3. Nonlinear dynamics of system (1) for specific values set b = 0.125. (a) Bifurcation diagram of z vs. the control parameter $a \in$ [0.49-0.57], (b) Bifurcation diagram of z vs. the control parameter a $a \in$ [0.56-0.65],, with MATLAB 2010.



(a)

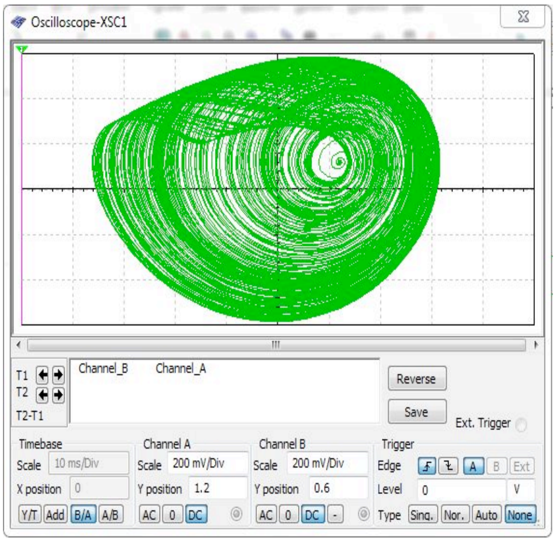


(b)

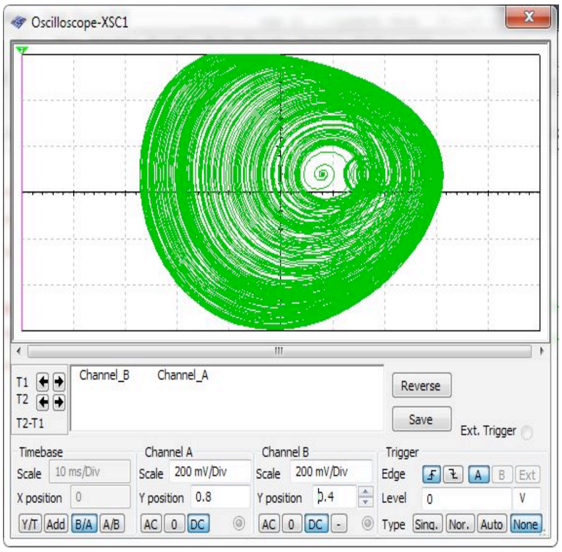


(c)

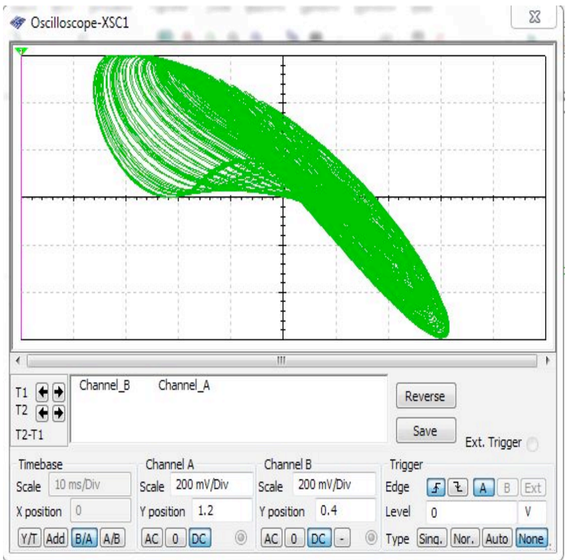
Fig. 4. A gallery of Poincare maps for system (1), for $a = 0.5$, $b = 0.125$. The plots give the maxima of (a) $x(n + 1)$ versus those of $x(n)$; (b) $y(n + 1)$ versus those of $y(n)$; (c) $z(n + 1)$ versus those of $z(n)$, obtained with MATLAB 2010.



(a)



(b)



(c)

Fig. 6. Various projections of the chaotic attractor using MultiSIM 10.0, for $a = 0.5$, $b = 0.125$ (a) x-y plane (b) y-z plane (c) x-z plane.

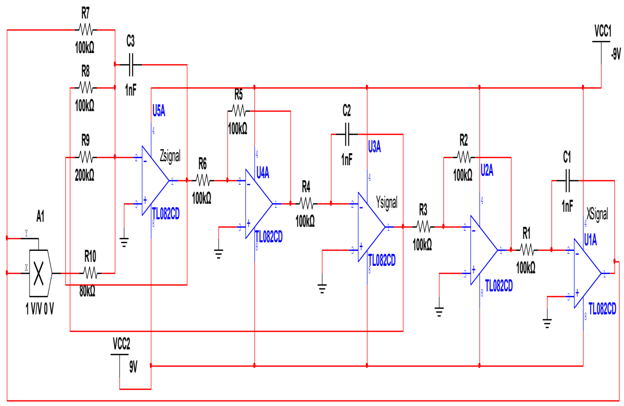


Fig. 5. Schematic of the proposed Jerk circuit by using MultiSIM 10.0.

3. Bidirectional Coupling Scheme of Jerk Circuits

Synchronization between coupled chaotic systems has received considerable attention and led to communication applications. With coupling and synchronizing identical chaotic systems method, a message signal sent by a transmitter system can be reproduced at a receiver under the influence of a single chaotic signal through synchronization. This work presents the study of numerical simulation of chaos synchronization of coupled chaotic Jerk circuits.

Synchronization of chaotic motions among coupled dynamical systems is an important generalization from the phenomenon of the synchronization of linear systems, which is useful and indispensable in communications. The idea of the method is to reproduce all the signals at the receiver under the influence of a single chaotic signal from the driver. Therefore, chaos synchronization provides potential application to communications and signal processing. However, for the realization of secure communication systems, some other important factors, need to be considered [33].

The following bidirectional coupling configuration, is described below:

$$\left.\begin{aligned} \dot{x}_1 &= y_1 \\ \dot{y}_1 &= z_1 + g_c(y_2 - y_1) \\ \dot{z}_1 &= -x_1 - y_1 - az_1 - bx_1^2 \\ \dot{x}_2 &= y_2 \\ \dot{y}_2 &= z_2 + g_c(y_1 - y_2) \\ \dot{z}_2 &= -x_2 - y_2 - az_2 - bx_2^2 \end{aligned}\right\} \tag{4}$$

The coupling coefficient g_c is present in the equations of both systems, since the coupling between them is mutual.

Numerical simulations of system (4), by using the fourth-order Runge-Kutta method, are used to describe the dynamics of chaotic synchronization of bidirectionally coupled Jerk circuits. In bidirectional (mutual) coupling, both coupled systems are connected in such a way that they mutually influence each other's behavior. Synchronization numerically appears for a coupling factor $g_c \geq 0.3$ as shown in Figs.6(a) & (c), with error $e_x = x_1 - x_2 \rightarrow 0$, which implies the complete synchronization, in contrary to Figs. 6(b) & (d).

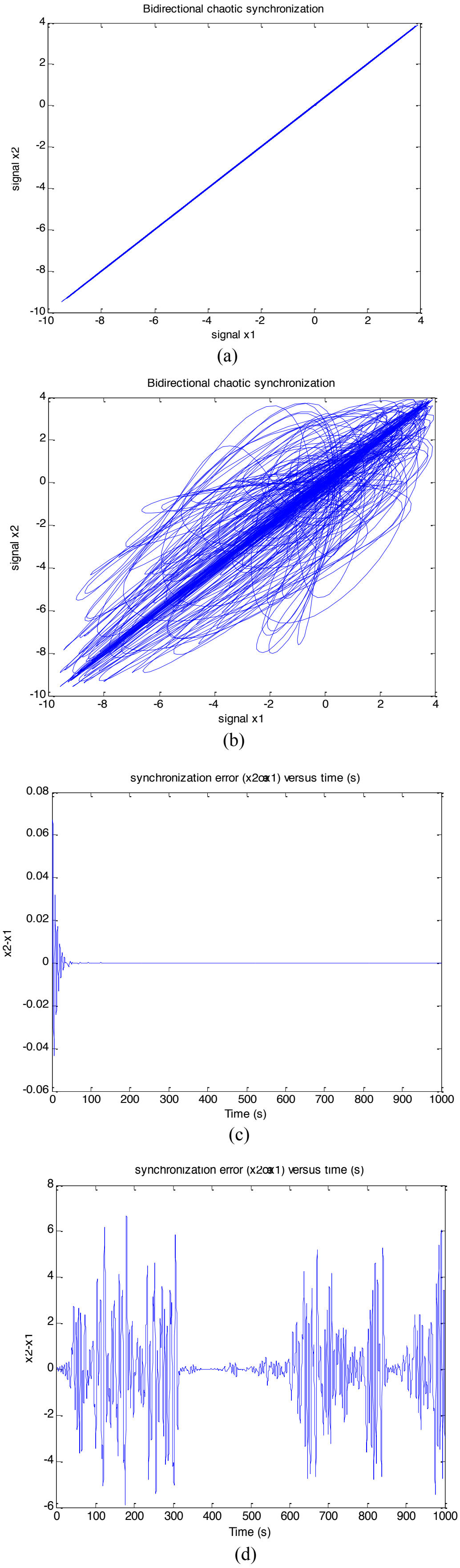


Fig. 6. Phase portrait of x_2 vs. x_1 and error $x_1 - x_2$ in the case of bidirectionally coupled Jerk circuits, for (a) g_c = 0.3 (full synchronization), (b) g_c = 0.2 (full desynchronization), (c) synchronization eror ($x_2 - x_1$) for g_c = 0.3 and (d) synchronization eror ($x_2 - x_1$) for g_c = 0.2.

4. Application to Secure Communication Systems

Two fundamental characteristics of chaos in physical systems are the complexity of the dynamics and the sensitivity of the time evolution to small perturbations. The sensitivity of chaos to small perturbations has been seen for a long time as merely a barrier to prediction, and not as a useful property. Major developments in the area of controlling chaos using small perturbations have proved otherwise: the sensitivity to small perturbations exhibited by chaotic systems allows to control them using electrical signals with a power far below the one produced by the chaotic system itself. Thus, the complexity of chaos and its sensitivity to small perturbations can be combined harmoniously by using the sensitivity to control (and take advantage of) the complexity. As a consequence, it is currently recognized by many engineers that the fact that chaos provides complex behavior from simple systems can be exploited to obtain technological advantages over conventional means for information transmission [34].

To study the effectiveness of signal masking approach in the Jerk system, we first set the information-bearing signal $m_s(t)$ in the form of sinusoidal wave:

$$m_s(t)= A \sin (2\pi f)t \tag{5}$$

where A and f are the amplitude and the frequency of the sinusoidal wave signal respectively.

The sum of the signal $m_s(t)$ and the chaotic signal $m_{\text{Jerk\_circuit}}(t)$, produced by the Jerk circuit, is the new encryption signal $m_{\text{encryption}}$, which is given by Eq.(6).

$$m_{encryption}(t) = m_s(t) + m_{\text{Jerk\_circuit}}(t) \tag{6}$$

The signal $m_{\text{Jerk\_circuit}}(t)$ is one of the parameters of equation (1). After finishing the encryption process the original signal can be recovered with the following procedure.

$$m_{\text{New\_Signal}}(t) = m_{\text{encryption}}(t) - m_{\text{Jerk\_circuit}}(t) \tag{7}$$

So, $m_{\text{New\_Signal}}(t)$ is the original signal and must be the same with $m_s(t)$. Due to the fact that the input signal can be recovered from the output signal, it turns out that it is possible to implement a secure communication system using the proposed chaotic system.

Information signal is added to the chaotic signal at transmitter and at receiver the masking signal is regenerated and subtracted from the receiver signal. For synchronization of transmitter and receiver, bidirectional coupling method of full synchronization technique is used. Figures 8(a) - (c) show the MATLAB 2010 numerical simulation results for the proposed chaotic masking communication scheme.

Also, in the proposed masking scheme, the sinusoidal wave signal of amplitude 1V and frequency 4 KHz is added to the synchronizing driving chaotic signal in order to regenerate the original driving signal at the receiver. Thus, as it can be shown from Fig.8(c), the message signal has been perfectly recovered by using the signal masking approach through the synchronization of chaotic Jerk circuits. Furthermore, simulation results with MATLAB 2010 have shown that the performance of chaotic Jerk circuits in chaotic masking and message recovery is very satisfactory.

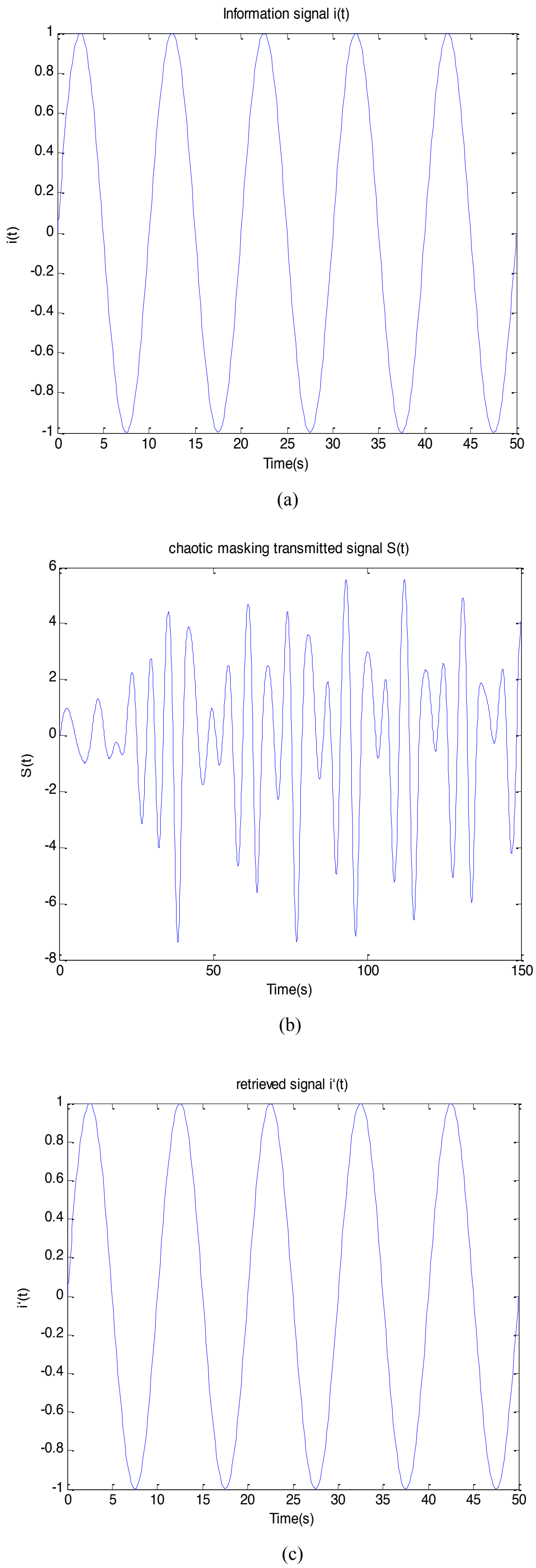


Fig. 8. MATLAB 2010 simulation of Jerk circuit masking communication system when amplitude is 1V and frequency 1KHz: (a) Information signal, (b) Chaotic masking transmitted signal and (c) Retrieved signal.

3. Conclusion

In this paper we have demonstrated how two identical chaotic systems can be synchronized using bidirectional coupling. Chaos synchronization and chaos masking were realized by using MATLAB 2010. The performance of the proposed method was demonstrated by using a Jerk system. We conclude that the suggested scheme can be effectively used in secure communications.

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