Research Article

Adaptive Backstepping Controller Design for the Anti-Synchronization of Identical WINDMI Chaotic Systems with Unknown Parameters and its SPICE Implementation

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Abstract

This paper derives new results for the adaptive backstepping controller design for the anti-synchronization of identical WINDMI systems (Wind-Magnetosphere-Ionosphere models) with unknown parameters and also details the SPICE implementation of the proposed adaptive backstepping controller. In the anti-synchronization of chaotic systems, the sum of the outputs of master and slave systems is made to converge asymptotically to zero with time. The adaptive controller design for the anti-synchronization of identical WINDMI systems with unknown parameters has been established by applying Lyapunov stability theory. MATLAB simulations have been shown for the illustration of the adaptive anti-synchronizing backstepping controller for identical WINDMI chaotic systems. Finally, the proposed controller has been implemented using SPICE and circuit simulation results have been detailed.

Keywords: Chaos, chaotic systems, anti-synchronization, WINDMI system, stability, SPICE simulation.

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1. Introduction

Chaotic systems can be defined as nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and also with dense periodic orbits [1]. The sensitivity to initial conditions of a chaotic system is indicated by a positive Lyapunov exponent. A dissipative chaotic system is characterized by the condition that the sum of the Lyapunov exponents of the chaotic system is negative.

A great breakthrough in chaos theory occurred when Lorenz discovered a 3-D chaotic system, while he was studying weather patterns [2]. There are many paradigms of 3-D chaotic systems in the literature, such as Rössler system [3], Rabinovich system [4], ACT system [5], Sprott systems [6], Chen system [7], Lü system [8], Shaw system [9], Feeny system [10], Shimizu system [11], Liu-Chen system [12], Cai system [13], Tigan system [14], Colpitt's oscillator [15], WINDMI system [16], Zhou system [17], etc.

Recently, many 3-D chaotic systems have been discovered such as Li system [18], Elhadj system [19], Pan system [20], Sundarapandian system [21], Yu-Wang system [22], Sundarapandian-Pehlivan system [23], Zhu system [24], Vaidyanathan systems [25-31], Vaidyanathan-Madhavan system [32], Pehlivan-Moroz-Vaidyanathan system [33], Jafari system [34], Pham system [35], etc.

The study of chaos theory in the last few decades had a big impact on the foundations of Science and Engineering and has found several engineering applications.

Some important applications of chaos theory can be cited as oscillators [36-38], lasers [39,40], robotics [41-43], chemical reactors [44,45], biology [46,47], ecology [48,49], neural networks [50-52], secure communications [53-56], cryptosystems [57-61], economics [62-64], etc.

The phenomenon of anti-synchronization of chaotic systems can be stated as follows. If a particular chaotic system is called the master system and another chaotic system is called the slave system, then the idea of anti-synchronization is to use the output of the master system to control the output of the slave system so that the outputs of the master and slave systems have the same amplitude but opposite signs asymptotically. Thus, in the anti-synchronization of chaotic systems, the sums of the states of the master and slave systems are expected to converge to zero asymptotically with time. This is an important research problem with several applications in engineering [65].

In the chaos literature, many different methodologies have been proposed for the control of chaotic systems such as active control method [66-79], adaptive control method [70-76], backstepping control method [77-79], sliding mode control [80-81], etc.

In the chaos literature, many different methodologies have been also proposed for the synchronization and anti-synchronization of chaotic systems such as PC method [82], active control [83-93], time-delayed feedback control

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[94,95], adaptive control [96-107], sampled-data feedback control [108-111], backstepping control [112-118], sliding mode control [119-124], etc.

In this research paper, we design an adaptive backstepping controller for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters. WINDMI chaotic systems and Wind-Magnetosphere-Ionosphere models [16], which describe the energy flow through the solar wind-magnetosphere-ionosphere system.

We apply Lyapunov stability theory to establish the main result for the adaptive backstepping controller for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters. MATLAB simulations have been depicted to validate and illustrate the main results of this research work. Finally, we detail SPICE implementation of the adaptive backstepping controller proposed in this research work for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters.

2. Analysis on WINDMI Chaotic System

WINDMI chaotic system is one of the paradigms of 3-D chaotic systems. It is described by the following normalized state equations:

$$\begin{cases} \frac{dx_1}{dt} & = & x_2 \\ \frac{dx_2}{dt} & = & x_3 \\ \frac{dx_3}{dt} & = & -ax_3 - x_2 + b - \exp(x_1) \end{cases} \quad (1)$$

where x_1, x_2, x_3 are the states and a, b are positive parameters.

The WINDMI system (1) depicts a strange chaotic attractor when the parameter values are taken as:

$$a = 0.7 \text{ and } b = 2.5 \quad (2)$$

Also, we take the initial conditions as:

$$x_1(0) = 1.4, x_2(0) = 0.8, \ x_3(0) = 2.5 \quad (3)$$

The 3-D phase portrait of the WINDMI chaotic attractor is shown in Fig. 1.

The Lyapunov exponents of the WINDMI chaotic system are numerically obtained as:

$$L_1 = 0.0845, L_2 = 0, L_3 = -0.7870 \quad (4)$$

Thus, the maximal Lyapunov exponent (MLE) of the WINDMI chaotic system is given by $L_1 = 0.0845$.

The spectrum of the Lyapunov exponents of the WINDMI chaotic system (1) is depicted in Fig. 2.

The Lyapunov dimension of the WINDMI system is calculated as follows:

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1074 \quad (5)$$

which is fractional.

Since the sum of the Lyapunov exponents in (4) is negative, the WINDMI chaotic system is a dissipative chaotic system.
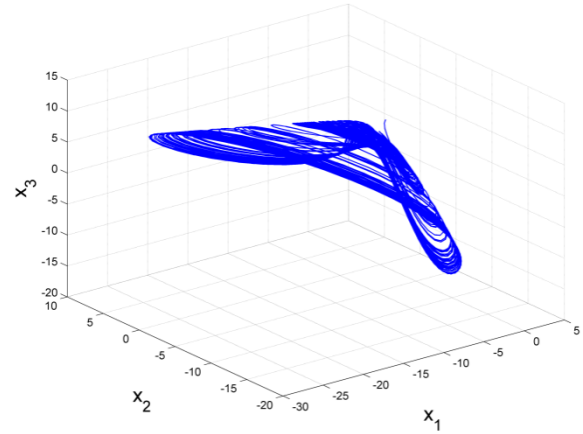

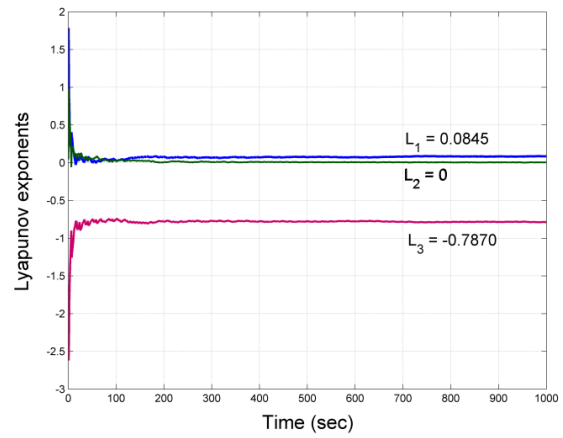**Fig. 1.**The strange attractor of the WINDMI chaotic system.

Fig. 2. The spectrum of the Lyapunov exponents of the WINDMI chaotic system.

3. Adaptive Anti-Synchronization of WINDMI Chaotic Systems via Backstepping Control

In this section, new results are derived for the anti-synchronization of WINDMI chaotic systems with unknown parameters via adaptive backstepping control method.

As the master system, we consider the WINDMI system of Eq.(1), while as the slave system, the following controlled WINDMI system is considered.

$$\begin{cases} \frac{dy_1}{dt} & = & y_2 \\ \frac{dy_2}{dt} & = & y_3 \\ \frac{dy_3}{dt} & = & -ay_3 - y_2 + b - \exp(y_1) + u \end{cases} \quad (6)$$

In (6), the parameters a and b are unknown, and $u(t)$ is a feedback control to be determined using the states of (1) and (6), and estimates $A(t)$ and $B(t)$ of the unknown parameters a and b, respectively.

The anti-synchronization error between the states of the WINDMI systems (1) and (6) is defined by

$$\begin{cases} e_1(t) & = & y_1(t) + x_1(t) \\ e_2(t) & = & y_2(t) + x_2(t) \\ e_3(t) & = & y_3(t) + x_3(t) \end{cases} \quad (7)$$

A simple calculation yields the error dynamics as:

$$\begin{cases} \frac{de_1}{dt} &= e_2 \\ \frac{de_2}{dt} &= e_3 \\ \frac{de_3}{dt} &= -ae_3 - e_2 + 2b - \exp(y_1) - \exp(x_1) + u \end{cases} \tag{8}$$

We define the parameter estimation errors as:

$$\begin{cases} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \end{cases} \tag{9}$$

In (10), $A(t)$ and $B(t)$ are estimates for the unknown parameters a and b respectively, which will be determined using adaptive control theory.

It follows from (9) that

$$\begin{cases} \frac{de_a}{dt} &= -\frac{dA}{dt} \\ \frac{de_b}{dt} &= -\frac{dB}{dt} \end{cases} \tag{10}$$

Next, we shall state and prove the main result of this section.

Theorem 1. *The identical WINDMI chaotic systems (6) and (7) with unknown parameters a and b are globally and exponentially anti-synchronized by the adaptive backstepping control law*

$$\begin{aligned} u(t) = &-2B(t) - 3e_1 - 4e_2 - kz_3 \\ &+ (A(t) - 3)e_3 + \exp(y_1) + \exp(x_1) \end{aligned} \tag{11}$$

where $k > 0$ is a gain constant, with

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{12}$$

and the update law for the parameter estimates is given by

$$\begin{cases} \frac{dA}{dt} &= -e_3 z_3 \\ \frac{dB}{dt} &= 2z_3 \end{cases} \tag{13}$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2}z_1^2 \tag{14}$$

where

$$z_1 = e_1 \tag{15}$$

Differentiating V_1 along the dynamics (8), we get

$$\frac{dV_1}{dt} = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{16}$$

Now, we define

$$z_2 = e_1 + e_2 \tag{17}$$

Using (17), we can simplify (16) as:

$$\frac{dV_1}{dt} = -z_1^2 + z_1 z_2 \tag{18}$$

Next, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2}z_2^2 = \frac{1}{2}(z_1^2 + z_2^2) \tag{19}$$

Differentiating V_2 along the dynamics (9), we obtain

$$\frac{dV_2}{dt} = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{20}$$

Now, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{21}$$

Using (21), we can simplify (20) as:

$$\frac{dV_2}{dt} = -z_1^2 - z_2^2 + z_2 z_3 \tag{22}$$

Finally, we define a quadratic Lyapunov function

$$V(\mathbf{z}, e_a, e_b) = V_2(z_1, z_2) + \frac{1}{2}z_3^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 \tag{23}$$

From (23), it is clear that V is a positive definite function on R^5.

The time-derivative of V is calculated as:

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \frac{dA}{dt} - e_b \frac{dB}{dt} \tag{24}$$

where

$$S = z_3 + z_2 + \frac{dz_3}{dt} = z_3 + z_2 + 2\frac{de_1}{dt} + 2\frac{de_2}{dt} + \frac{de_3}{dt} \tag{25}$$

Simplifying the equation (25), we obtain

$$\begin{aligned} S = &2b + 3e_1 + 4e_2 + (3 - a)e_3 - \exp(y_1) \\ &- \exp(x_1) + u \end{aligned} \tag{26}$$

Substituting the control law (11) into (26), we get

$$S = 2(b - B(t)) - (a - A(t))e_3 - kz_3 \tag{27}$$

Using (9), we can simplify the equation (27) as:

$$S = 2e_b - e_a e_3 - kz_3 \tag{28}$$

Substituting the value of S from (28) into (24), we obtain

$$\begin{aligned} \frac{dV}{dt} = &-z_1^2 - z_2^2 - (1 + k)z_3^2 + e_a\left(-z_3 e_3 - \frac{dA}{dt}\right) \\ &+ e_b\left(2z_3 - \frac{dB}{dt}\right) \end{aligned} \tag{29}$$

Substituting the update law (13) into (29), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - (1 + k)z_3^2 \tag{30}$$

Thus, it is clear that $\frac{dV}{dt}$ is a negative semi-definite function on R^5.

From (30), it follows that the vector $z(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t))$, are globally bounded, *i.e.*

$$[z_1(t) \quad z_2(t) \quad z_3(t) \quad e_a(t) \quad e_b(t)] \in \boldsymbol{L}_\infty \qquad (31)$$

Also, it follows from (30) that

$$\frac{dV}{dt} \leq -z_1^2 - z_2^2 - z_3^2 = -\|\boldsymbol{z}\|^2 \qquad (32)$$

That is,

$$\|\boldsymbol{z}\|^2 \leq -\frac{dV}{dt} \qquad (33)$$

Integrating the inequality (34) from 0 to t, we get

$$\int_0^t \|\boldsymbol{z}(\tau)\|^2 d\tau \leq V(0) - V(t) \qquad (34)$$

From (49), it follows that $z(t) \in \boldsymbol{L}_2$, while from (8), it can be deduced that $\frac{dz}{dt} \in \boldsymbol{L}_\infty$.

Thus, using Barbalat's lemma [125], we can conclude that $\boldsymbol{z}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $z(0) \in \boldsymbol{R}^3$.

Hence, it is immediate that the anti-synchronization error $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in \boldsymbol{R}^3$.

Thus, it follows that WINDMI chaotic systems (1) and (6) are globally and exponentially anti-synchronized for all initial conditions $x(0), y(0) \in \boldsymbol{R}^3$.

This completes the proof. ∎

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (1), (6) and (13), when the adaptive control law (11) is applied.

The parameter values of the WINDMI chaotic systems (1) and (6) are taken as in the chaotic case, *viz.* $a = 0.7$ and $b = 2.5$. The positive gain constant k is taken as $k = 6$.

Furthermore, as initial conditions of the master system (1), we take $x_1(0) = 3.8$, $x_2(0) = -4.5$ and $x_3(0) = 6.2$.

As initial conditions of the slave system (7), we take $y_1(0) = -7.4$, $y_2(0) = -3.6$ and $y_3(0) = 4.9$.

Also, as initial conditions of the estimates $A(t)$ and $B(t)$, we take $A(0) = 5.1$ and $B(0) = 6.4$.

In Figs. 3-5, the anti-synchronization of the states of the master system (1) and slave system (6) is depicted, when the adaptive control law (11) and parameter update law (13) are implemented. In Fig. 6, the time-history of the anti-synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$ is depicted.
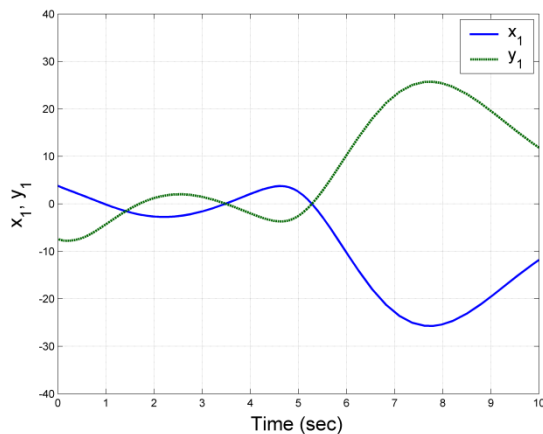

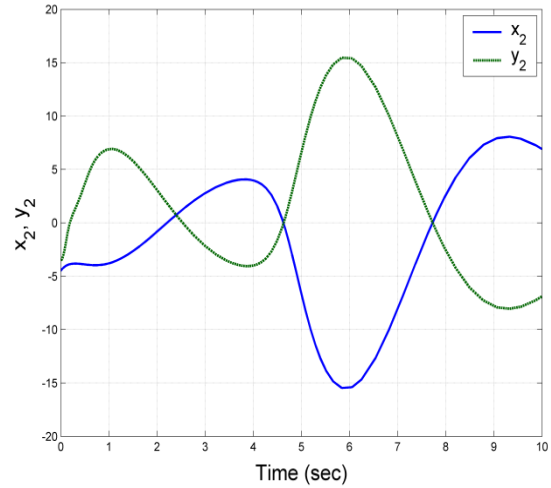**Fig. 4.** Anti-Synchronization of the states $x_2(t)$ and $y_2(t)$.

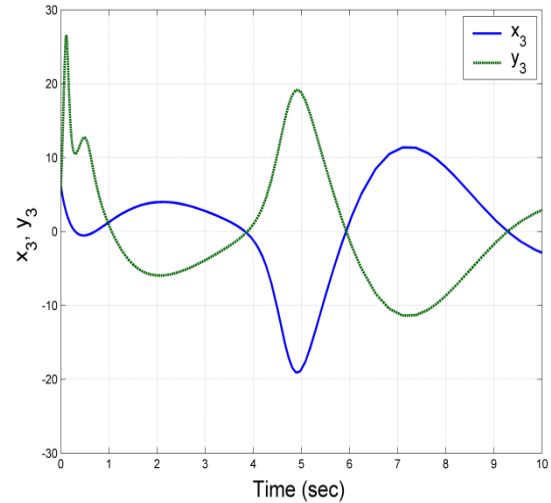**Fig. 5.** Anti-Synchronization of the states $x_3(t)$ and $y_3(t)$.

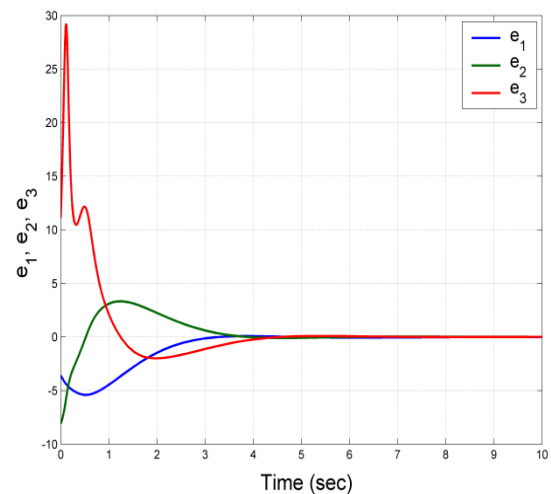**Fig. 3.** Anti-Synchronization of the states $x_1(t)$ and $y_1(t)$.

Fig. 6. Time-history of the anti-synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$.

6. Circuit Realization of the Adaptive Backstepping Controller Design for the Anti-Synchronization of WINDMI Chaotic Systems

In this section, we design an electronic circuit modellingWINDMI system (1) and the controller (11) used for synchronization. The circuits in Figs. 7 & 8 has been designed following an approach based on operational amplifiers [59,60,70] where the state variables x_1, x_2, and x_3 of the system (1) are associated with the voltages across the capacitors C_1, C_2, and C_3, respectively. In Figs. 7 & 8, there are five operational amplifiers, which are connected as integrators (U_1, U_2, U_3, U_4 and U_5) and one as inverting amplifier (U_6). The nonlinear equations for the electronic circuit are derived as follows:

$$\begin{cases} \dfrac{dx_1}{dt} &= \dfrac{1}{R1C1}x_2 \\ \dfrac{dx_2}{dt} &= \dfrac{1}{R2C2}x_3 \\ \dfrac{dx_3}{dt} &= -\dfrac{1}{R3C3}x_3 - \dfrac{1}{R4C3}x_2 + \dfrac{1}{R5C3} - \dfrac{1}{R6C3}\exp(x_1) \end{cases}$$
(35)

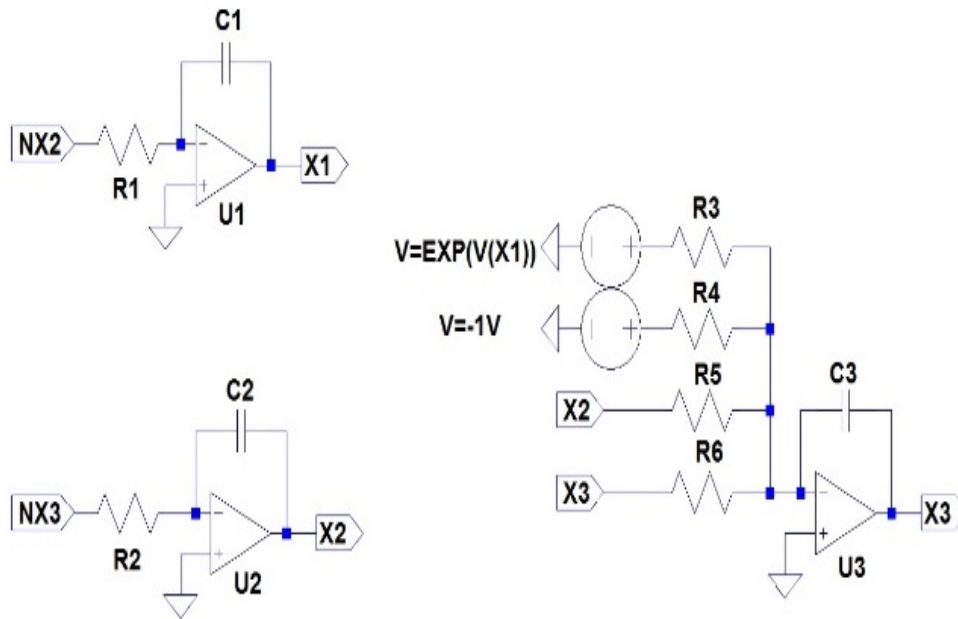where the values of components are chosen as: $R_1 = R_2 = R_3 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = 1\text{k}\Omega$, $R_4 = 0.4\text{k}\Omega$, $R_6 = 1.428\text{k}\Omega$ and $C_1 = C_2 = C_3 = C_4 = C_5 = 1\text{mF}$. The power supplies of all active devices are $\pm 15\text{V}_{DC}$.

Figures 9, 10 & 11 show the state portraits of x_1 versus x_2, x_2 versus x_3 and x_3 versus x_1, respectively.

7. Conclusion

In this work the adaptive backstepping controller design for the anti-synchronization of identical third-order chaotic systems with unknown parameters was studied. For this reason, an interesting system, such as the Wind-Magnetosphere-Ionosphere model (WINDMI) was chosen. The proposed controller design has been established by applying the Lyapunov stability theory. The simulation results,with MATLAB, confirmed the effectiveness of the adaptive anti-synchronization controller in the case of identical chaotic systems. Finally, the circuitries of the chosen dynamical system and the system's controller have been realized and the very satisfactory agreement of the SPICE results with those of MATLAB simulations has been observed.
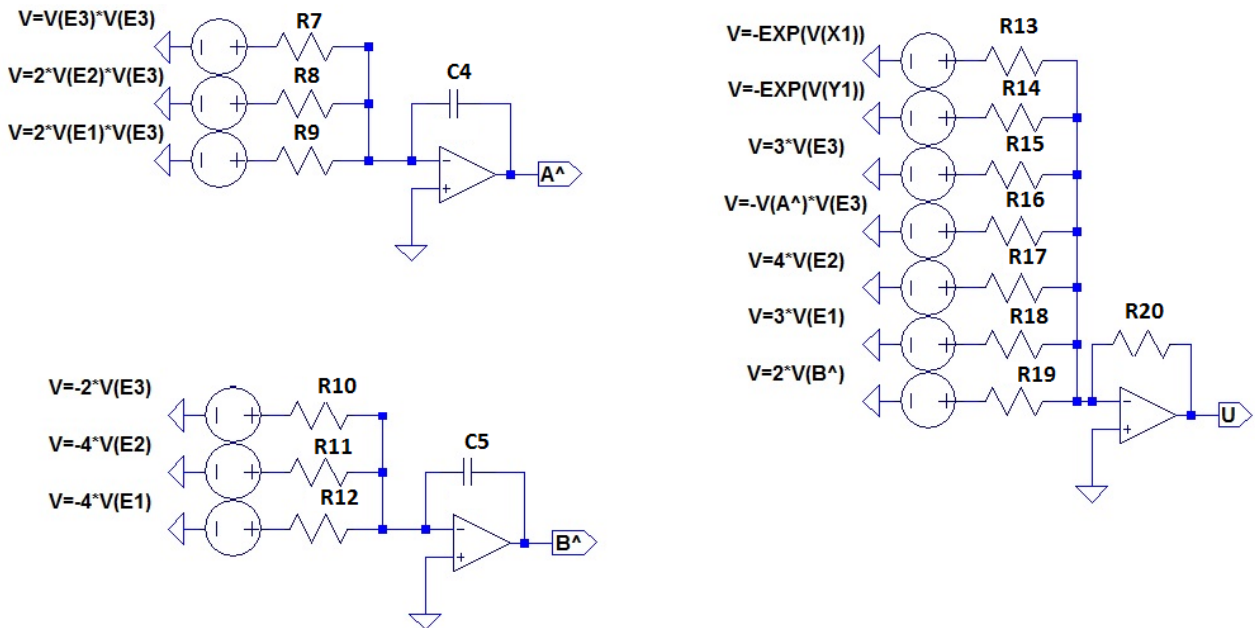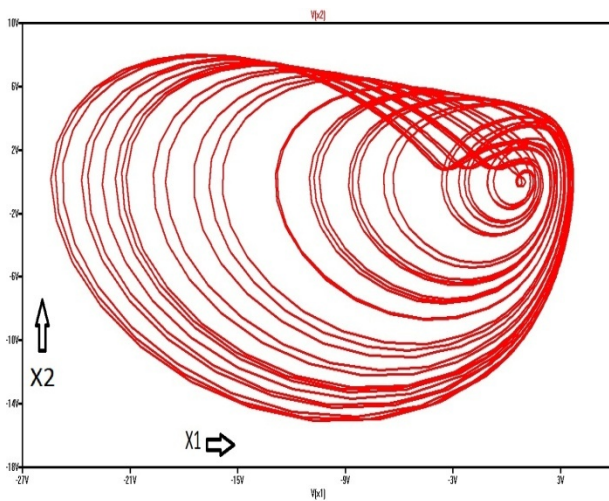

Fig.7.LTSpice schematic for master system.

Fig.8. LTSpice schematic for controller.

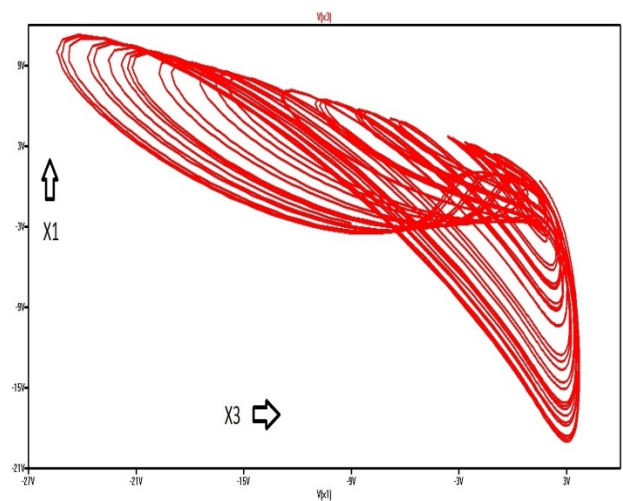**Fig.9**. Phase portrait of x_1 vs. x_2.

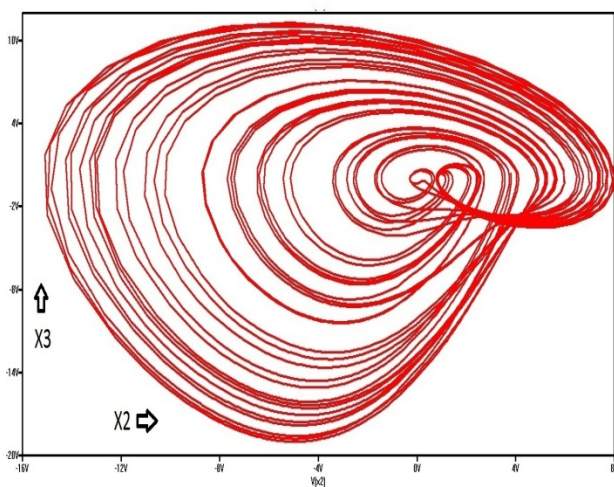**Fig.11.** Phase portrait of x_3 vs. x_1.

Fig.10. Phase portrait of x_2 vs. x_3.

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