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Tsallis *q*-triplet and Stock Market Indices: The cases of S & P 500 and TVIX

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Abstract

In this study we present results of the evaluation of $q-$ triplet of Tsallis non-extensive statistics concerning two time stock market time series, Standart & Poor's 500 (S & P) 500 and TVIX. The analysis of the results, support the hypothesis in which economic dynamics from physical point of view correspond to far from equilibrium spatial distributed non-linear dynamics. In particular, the analysis of the stock market time series revealed underlying complex dynamics, indicating clearly that the statistics of the dynamics in the multifractal phase space can be described by Tsallis distribution functions of power law and heavy tails forms. The non-extensive character of the underlying dynamics is related to the existence of long range interactions in space and time, as well as the interaction in many scales. In addition, the detailed analysis of S & P index unraveled the existence of non-equilibrium phase transitions depicted clearly in the variations of Tsallis *q*-triplet values. These phase transitions are connected with non-equilibrium stationary states of economical dynamics derived from processes of strong self organization which correspond to local maxima of Tsallis entropy, while the changes in the control parameters can induce new phase transitions and shifts to new metaequilibrium steady states of maximizing Tsallis entropy. These phase transitions lead to changes in Tsallis q-triplet values which correspond to multi-fractal changes in the formation of the phase state and an alteration in the phenomenology of the economical system. Finally, these characteristics also indicate the existence of fractional dynamics in the phase space which can be described through fractional Fokker-Planck equations and anomalous diffusion equations. The solutions of these equations are fractional spatiotemporal functions and non-Gaussian distributions functions which fall into the category of Levy distributions and Tsallis distributions.

Keywords: Tsallis non-extensive statistics, Tsallis q-triplet, Stock Market Indexes, Economic Complexity

1. Introduction

Economy is a vast and complicated set of arrangements and actions wherein agents—consumers, firms, banks, investors, government agencies—buy and sell, speculate, trade, oversee, bring products into being, offer services, invest in companies, strategize, explore, forecast, compete, learn, innovate, and adapt. And from all this concurrent behavior markets form, prices form, trading arrangements form, institutions and industries form. Aggregate patterns form (Arthur, 2013). Thus, the economy can be thought as a complex system with heterogeneous interacting agents who collectively organize themselves to generate aggregate phenomena which cannot be regarded as the behavior of some average or representative individual. Some of these phenomena are bubbles and crashes, herd behavior, the transmission of information and the organization of trade etc (Kirman, 2004). In addition, various economic systems can exhibit ubiquitous complex dynamics evidenced by large-amplitude and aperiodic fluctuations in economic and financial variables such as foreign exchange rates, gross domestic product, interest rates, production, stock market prices and unemployment. Large-amplitude fluctuations in economic and financial systems are indications that these systems are driven far away from the equilibrium whereby the non-linearity dominates the system behavior (Chian A.C.-L. et al., 2006).

Far from equilibrium, complex systems can exhibit a rich variety of self-organized critical point phenomena such as: long range correlations, far from equilibrium phase transitions, reduction of dimensionality, chaotic dynamics, self organized criticality, intermittent turbulence, anomalous diffusion, power law distributions, nonextensive statistics (Pavlos et al., 2013). Many of the above phenomena are expected to be found in economic systems since non-equilibrium dynamics is a natural property for the economic dynamics which are always open to reaction. For example, Mantegna and Stanley (1995) showed that the scaling of the probability distributions of the Standard & Poor 500 index can be described by a non-Gaussian process with dynamics that, for the central part of the distribution, corresponds to that predicted for a Levy stable process. This isn't only due to outside shocks or external influences, but also because

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nonequilibrium arises endogenously in the economy (e.g. fundamental uncertainties, technological innovation etc)(Mantegna and Stanley, 1997; Mantegna & Stanley, 2000; Stanley et al., 2002; Gabaix et al., 2003; Tsallis et al., 2003; Scalas, 2006; Tambakis, 2008).

Extreme events in economical complex systems are observed much more often than is suggested by the standard assumption in applied econometric work that data follow the Gaussian, or normal, distribution. The frequency and sizes of financial asset prices and even economic recessions are characterized by highly non-Gaussian outcomes (Farmer et al., 2012). Indeed, the heavy tails of real price fluctuations, under-predicted by the Gaussian distribution resulting from accumulation of an uncorrelated random walk, can lead to disastrous mis-estimates of risk. More recent work by physicists extends analytic methods for pricing options to take the heavy tails and volatility bursts of real prices into account. The non-extensive statistical mechanics pioneered by the Tsallis group offers a consistent theoretical framework, based on a generalization of Boltzmann – Gibbs entropy, to describe such far from equilibrium non-linear complex dynamics which reveal strong self-organization and development of bifurcation processes causing long range correlations and multi-fractal strange attractors [Tsallis, 2009]. There are examples supporting the non-extensive behavior of different complex systems ([Tsallis, 2011; Tsallis, 2012] and refs therein). In addition, some of the authors have successfully applied Tsallis non-extensive statistics in various physical systems such as the earthquakes, Earth's magnetosphere, solar dynamics etc. [Pavlos et al., 2011; Iliopoulos et al. 2012; Karakatsanis et al., 2013; Pavlos et al. 2014 and references quoted therein]. Concerning economic complexity recent applications of the ideas associated with nonextensive statistical mechanics to phenomena in economics, namely a simple trading model which takes into account risk aversion, a generalization of the Black–Scholes equation for pricing options, and a phenomenological description of distributions of returns and volumes in the real market (Tsallis et al., 2003 and references within). Other applications of nonextensive statistics concern the connection between the ARCH model and Tsallis statistics (Queirȯs, 2004), the analysis of financial market observables, specifically returns and traded volumes (Queirȯs et al., 2007), interacting-agent modeling of stock markets (Kaizoji, 2006) the Athens Stock Exchange General Index (Stavroyiannis et al., 2010) etc.

In this study we estimate the Tsallis q-triplet for two time series, namely the Standard & Poor's 500 (S&P 500) index and its corresponding TVIX index. The S&P 500 is a stock market index based on the market capitalizations of 500 leading companies publicly traded in the U.S. stock market, as determined by Standard & Poor's. The TVIX index is a ETF product that is related to the volatility of the S&P 500. The index was designed to provide investors with exposure to one or more maturities of futures contracts on the VIX, which reflects implied volatility of the S&P 500 index at various points along the volatility forward curve.

2. The methodology of time series analysis

In the following first we describe the mathematical framework concerning the algorithm used for the stock market time series analysis and then we provide the results of the analysis.

2.1 The q-triplet estimation

The non-extensive statistical theory is based mathematically on the nonlinear equation:

$$\frac{dy}{dx} = y^q, \; (y(0) = 1, q \in \Re) \tag{1}$$

with solution the $q-$exponential function such as: $e_q^x = [1+(1-q)x]^{\frac{1}{1-q}}$. The solution of this equation can be realized in three distinct ways included in the $q-$triplet of Tsallis: ($q_{sen}, q_{stat}, q_{rel}$). These quantities characterize three physical processes which are summarized here, while the $q-$triplet values characterize the attractor set of the dynamics in the phase space of the dynamics and they can change when the dynamics of the system is attracted to another attractor set of the phase space. The equation (1) for $q=1$ corresponds to the case of equilibrium Gaussian Boltzmann-Gibbs (BG) world (Tsallis, 2009). In this case of equilibrium BG world the $q-$ triplet of Tsallis is simplified to ($q_{sen}=1, q_{stat}=1, q_{rel}=1$).

According to previous analysis concerning the q- triplet of Tsallis, we estimate the ($q_{sen}, q_{stat}, q_{rel}$) as follows:

The q_{sen} index is given by the relation:

$$q_{sen} = 1 + \frac{a_{\max} a_{\min}}{a_{\max} - a_{\min}} \tag{2}$$

The $a_{\max}, a_{\min}$ values correspond to the zeros of multifractal spectrum function $f(a)$, which is estimated by the Legendre transformation $f(a) = \bar{q}a - (\bar{q}-1)D_{\bar{q}}$, where $D_{\bar{q}}$ describes the Rényi generalized dimension of the solar wind time series in accordance with the relation:

$$D_{\bar{q}} = \frac{1}{q-1} \cdot \lim \left(\log \sum p_i^q \Big/ \log r \right) \text{ for } r \to 0 \tag{3}$$

The q_{stat} values are derived from the observed Probability Distribution Functions (PDF) according to the Tsallis q-exponential distribution:

$$PDF[\Delta Z] \equiv A_q \left[1 + (q-1)\beta_q (\Delta Z)^2\right]^{\frac{1}{1-q}} \tag{4}$$

where the coefficient A_q, β_q denote the normalization constants and $q \equiv q_{stat}$ is the entropic or non-extensivity factor ($q_{stat} \le 3$) related to the size of the tail in the distributions. Our statistical analysis is based on the algorithm as described in Ferri (2010). We construct the $PDF[\Delta Z]$ which is associated to the first difference $\Delta Z = Z_{n+1} - Z_n$ of the experimental sunspot time series, while the ΔZ range is subdivided into little ``cells'' (data binning process) of width δZ, centered at Z_i so that one can assess the frequency of ΔZ-values that fall within each cell/bin. The selection of the cell-size δZ is a crucial step of the algorithmic process and its equivalent to solving the binning problem: a proper initialization of the bins/cells can speed up the statistical analysis of the data set and lead to a convergence of the algorithmic process towards the exact solution. The resultant histogram is being properly normalized and the estimated *q*-value corresponds to the best linear fitting to the graph $\ln_q(p(Z_i))\, vs\, Z_i^2$, where $\ln_q(p(Z_i))$ is the so-called *q*-logarithm: $\ln_q(x) = (x^{1-q} - 1)/(1-q)$. Our algorithm estimates for each $\delta_q = 0{,}01$ step the linear adjustment on the graph under scrutiny (in this case the $\ln_q(p(Z_i))\, vs\, Z_i^2$ graph) by evaluating the associated correlation coefficient *(CC)*, while the best linear fit is considered to be the one maximizing the correlation coefficient. The obtained q_{stat}, corresponding to the best linear adjustment is then being used to compute the following equation:

$$G_q(\beta, Z) = \frac{\sqrt{\beta}}{C_q} e_q^{-\beta z^2} \tag{5}$$

where $C_q = \sqrt{\pi} \cdot \Gamma(\frac{3-q}{2(q-1)}) / \sqrt{q-1} \cdot \Gamma(\frac{1}{q-1})$, $1 < q < 3$ for different *β*-values. Moreover, we select the *β*-value minimizing the $\sum_i [G_{q_{sstat}}(\beta, Z_i) - p(Z_i)]^2$, as proposed in Ferri (2010).

Finally the q_{rel} index is given by the relation: $q_{rel} = (s-1)/s$ where *s* is the slope of the log-log plotting of mutual information $I(\tau)$ given in Fraser and Swinney (1986) by the relation:

$$I(\tau) = -\sum_{x(i)} P(x(i)) \log_2 P(x(i)) - \sum_{x(i-\tau)} P(x(i-\tau)) \log_2 P(x(i-\tau)) + \sum_{x(i)} \sum_{x(i-\tau)} P(x(i), x(i-\tau)) \log_2 P(x(i), x(i-\tau)) \tag{6}$$

2.2 Estimation of singularity spectrum $f(a)$

The singularity spectrum $f(a)$ can be estimated by using the Legendre transformation:

$$f(a) = \bar{q}a - \tau(\bar{q}), a = \frac{d\tau(\bar{q})}{d\bar{q}} \tag{7}$$

where $\tau(\bar{q})$ is the mass exponent related to the generalized dimension function $D(\bar{q})$ through the relation $\tau(\bar{q}) = (\bar{q} - 1)D_{\bar{q}}$. We also follow Consolini et al. (1996) for estimating the scaling exponent function $\tau(\bar{q})$ corresponding to the experimental time series $Z(t_i)$ after the estimation of the corresponding function $\Gamma(\bar{q}, \Delta t)$ according to the relation

$$\Gamma(\bar{q}, \Delta t) = \sum_{\Lambda_i} P_i(\Delta t)^{\bar{q}} \approx (\Delta t)^{\tau(\bar{q})}$$

(8)

where $P_i(\Delta t)$ is the probability coarse-grained weight for time segments Λ_i of the time size (Δt) of the experimental signal.

2.3 Theoretical Prediction of the singularity spectrum $f(a)$

According to Arimitsu and Arimitsu (2000, 2001) we can also estimate theoretically the singularity exponents spectrum $f(a)$ by using the relation $P(a) \approx \ln^{d-F(a)}$ as follows:

$$f(a) = D_0 + \log_2[1 - (1-q)\frac{(a-a_o)^2}{2X/\ln 2}] / (1-q)^{-1}$$

(9)

where a_0 corresponds to the *q*-expectation (mean) value of a through the relation:

$$<(a-a_0)^2>_q = (\int daP(a)^q (a-a_0)^q) / \int daP(a)^q \tag{10}$$

while the *q*-expectation value a_0 corresponds to the maximum of the function $f(a)$ as $df(a)/da \,|\, a_0 = 0$. For the Gaussian dynamics ($q \to 1$) we have mono-fractal spectrum $f(a_0) = D_0$.

For the estimation of the singular spectrum function $f(a)$ we need three parameters (a_0, q, X). According to Arimitsu and Arimitsu (2000) we can estimate these parameters by using the equations:

$$\tau(\bar{q} = 1) = 0, \qquad \mu = 1 + \tau(\bar{q} = 2) \qquad \text{and}$$
$$1/_{1-q} = 1/_{a_-} + 1/_{a_+} \tag{11}$$

where *μ* is the intermittency exponent (Frisch, 1996).

2.4 *p*-model prediction of the singularity spectrum $f(a)$

According to Meneveau and Sreenivasan (1987), the p-model is a one-dimensional model version of a cascade model of eddies, each breaking down into two new ones according to a generalized two-scale Cantor set with $l_1 = l_2 = 1/_2$. The *p*-model was introduced to account for the occurrence of intermittence in fully developed turbulence. The best nonlinear fit of the generalized dimension $D(\bar{q})$ function is represented by

$$D_{\bar{q}} = \log_2 \left[p^{\bar{q}} + (1-p)^{\bar{q}} \right] \Big/ 1-\bar{q} \qquad (12)$$

Where p is the probability of the fragmentation in the cascade process and q is the moment order.

3. Results

3.1 Time series

In Fig. 1a the Standard & Poor's 500 (S&P 500) index closing value time series is presented corresponding to the period from 1950 to 2012. Moreover, in Fig. 1b the corresponding TVIX index, which is related to the volatility of S & P 500 for the same period, is shown. As we can see in Fig. 1a the time series is non-periodic, nonstationary, with large and rapid fluctuations especially in the last part of the S&P 500 index. On the contrary, the TVIX index is smoother, however maintaining the nonperiodic and the rapid fluctuations character.

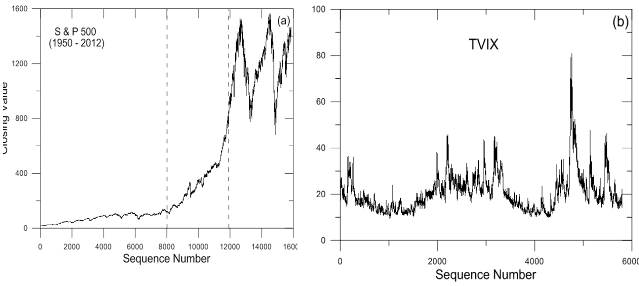


Fig. 1. a) The S & P 500 index for the period of 1950 – 2012. b) The corresponding TVIX index.

3.2 The non-extensive *q*-statistics of the S & P 500 and TVIX

Here, we present results concerning the estimation of q_{sens}, q_{stat} and q_{rel} indices for the S & P 500 and TVIX. The S&P 500 time series was divided in three segments in order to study in more detail the transition from low to high closing values.

3.2.1 Determination of q_{sens}

In Fig. 2(a,b) we present the multifractal (singularity) scaling spectra $f(a)$ from which the q_{sens} was estimated using the relation (2) and the corresponding generalized dimension spectra D_q, for the TVIX time series. As we can see in Fig. 2a the theoretical estimation is faithful with high precision on the left part of the experimental function $f(a)$. However, the fit of theoretical and experimental data are less faithful for the right part. A similar comparison of the theoretical prediction and the experimental estimation of the generalized dimensions function *D(q)* is shown in Fig. 2(b). In this figure the solid orange line corresponds to the *p*-model prediction according to paragraph 2.4, while the solid red line corresponds to the *D(q)* function estimation according to Tsallis theory (Arimitsu and Arimitsu, 2000, 2001). As it can be seen the theoretical model fits the generalized dimension spectrum D_q better than the *p*-model, indicating a deep connection between Tsallis nonextensive statistical mechanics and the underlying dynamics. The q_{sens} of the TVIX time series was found to be q_{sens}=0.1095 ± 0.0015. Furthermore, we estimated the difference between a_{min}, a_{max} values using a sixth- degree polynomial. These values correspond to the densest $\left[a_{min} = a\left(\bar{q} = +\infty\right)\right]$ and the sparsest $\left[a_{max} = a\left(\bar{q} = -\infty\right)\right]$ regions of the attractor and the difference was found to be: $\Delta\alpha = \alpha_{max} - \alpha_{min} = 0.9551$, $\Delta D_q = D_{q=-\infty} - D_{q=+\infty} = 0.625795$.

We also evaluated the values of q_{sens} and the corresponding differences of a_{min}, a_{max} for the three segments of the S&P 500 time series, presented in Fig. 3(a,b). The results indicate strong multi-fractal behavior of the underlying dynamics in all period.

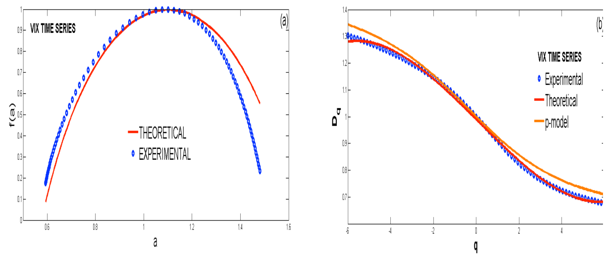


Fig. 2. (a) Multifractal spectrum of the TVIX time series (blue cycles). The solid red line corresponds to theoretical estimation of the multifractal spectrum.The index was found to be q_sen=0.1095 ± 0.0015. (b) The generalized dimension *D*(*q*) vs. *q* of the TVIX time series. The solid red line corresponds to theoretical estimation of generalized dimension, the blue dots to experimental estimation and the orange line to *p*-model estimation.

In addition, as we can notice in Figure 3 (a,b) the profile of the $f(a)$ spectra changes drastically as the dynamics evolve from the low closing values to the high closing values period. These results indicate the presence of non-equilibrium phase transition processes. As it is shown in Fig. 3(a,b) there is a significant shift both in q_{sens} and $\Delta\alpha$ corresponding to Adj2 segment, indicating an enhancement of the multi-fractal behavior of the dynamics in the second period pointing to strong instability of the system, before its relaxation. In particular, the estimated q_{sens} values were found to be: $q_{sens}(Adj1) = 0.316$, $q_{sens}(Adj2) = 0.7315$ and $q_{sens}(Adj3) = -0.229$. Furthermore, the multi-fractal ranges Δa initially increase (Adj2 segment) and then there is a rapid decrease corresponding to Adj2 segment time series. In particular the ranges $\Delta\alpha$ were found to be: $\Delta\alpha(Adj1) = 1.091289$, $\Delta\alpha(Adj2) = 1.286$ and $\Delta\alpha(Adj3) = 0.7241$.

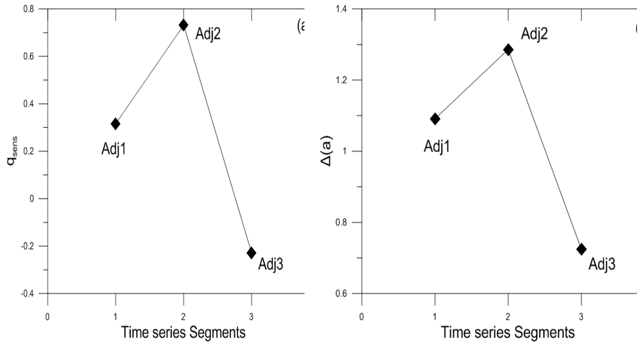


Fig. 3. (a) The q_{sens} values for the three time series segments of the S & P 500. (b) The corresponding difference of a_{min}, a_{max} values, $\Delta\alpha$, for the three time series segments of the S & P 500.

Finally, the parameter p of the p-model, as evaluated from the non-linear best fitting of D_q was found as follows: p(TVIX) = 0.66182, p(Adj1) = 0.68482, p(Adj2) = 0.71836 and p(Adj3)=0.63291. These values are different from the value p = 0.5 corresponding to the Gaussian turbulence indicating the presence of turbulence cascade in the underlying dynamics, as well as partial mixing and asymmetric (intermittent) fragmentation processes of the energy dissipation. The results of the estimation of p-model also results indicate the presence of non-equilibrium phase transition processes. Finally, the estimated values of q_{sens} and p-model for TVIX time series are closer to Adj1 segment of the S&P 500 time series.

3.2.2 Determination of the q_{stat}

In Figure 4a we present (by open blue circles) the experimental probability distribution function (PDF) $p(z)$ vs. z, where z corresponds to the $Z_{n+1} - Z_n, (n = 1, 2, ..., N)$ of the TVIX time series values. In Fig. 4b we present the best linear correlation between $\ln_q[p(z)]$ and z^2. The best fitting was found for the value of $q_{stat} = 1.56$. This value was used to estimate the q-Gaussian distribution presented in Fig. 4a by the solid red line.

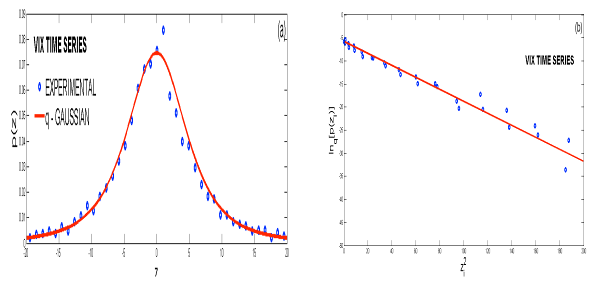


Fig. 4. (a) Normalized PDF $p(z)$ vs. z_i q Guassian function that fits P(z_i) for the TVIX time series (b) Linear Correlation between $\ln_q p(z_i)$ and $(z_i)^2$ where q_{stat} = 1.56 for the TVIX time series.

We also evaluated the values of q_{stat} for the three segments of the S&P 500 time series, presented in Fig. 5. The q_{stat} values were all found to be greater than 1: $q_{stat}(Adj1) = 1.59$, $q_{stat}(Adj2) = 1.48$, $q_{stat}(Adj3) = 1.44$. According to these results the following relation is satisfied: $q_{stat}(Adj1) > q_{stat}(Adj2) > q_{stat}(Adj3)$, indicating that there is a significant difference between the three closing value time periods as far as the nonequilibrium metastable stock market dynamics is concerned. This difference is clearly depicted in the gradual decrease of the index q_{stat} corresponding to the reduction of the non-Gaussian and Tsallis nonextensive character of the underlying dynamics, a result that indicates the relaxation of the phenomenon towards to another metastable stationary state and the presence of non-equilibrium phase transition processes. Finally, the estimated value of q_{stat} for TVIX time series is closer to Adj1 segment of the S&P 500 time series.

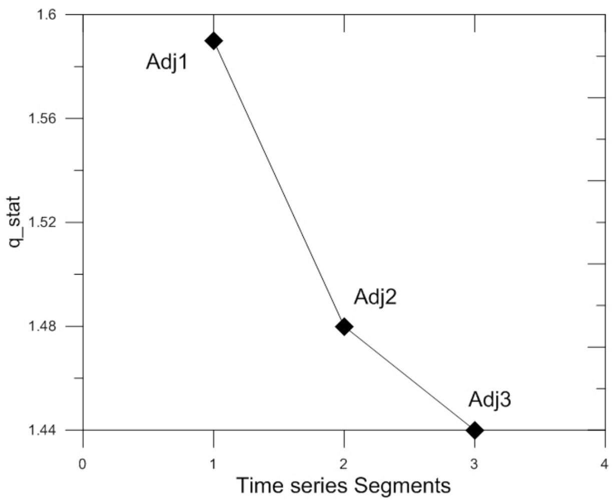
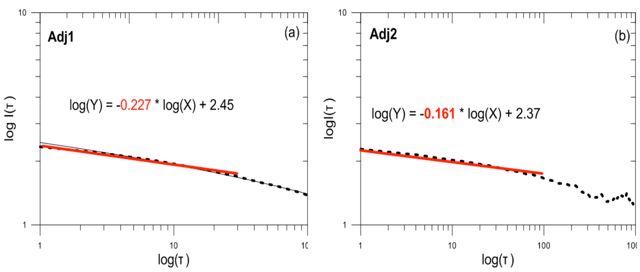


Fig. 5. The index q_{stat} for the three S&P 500 time series segments.

3.2.3 Determination of the q_{rel}

In Table 1, the best $\ln_q I(\tau)$ fittings of the mutual information functions $I(\tau)$ of the three time series segments of S&P 500 as well as TVIX indices is presented. The q_{rel} index was estimated by the relation $q_{rel} = (s-1)/s$ where s is the slope of the log-log plotting of mutual information $I(\tau)$ shown in Figure 6 with red color for the three S & P 500 time series segments. The results showed that the q_{rel} index was found to be q_{rel}(Adj1) = 15.706 for the first segment, q_{rel}(Adj2)= 10.71 for the second segment and q_{rel}(Adj3)= 4.125. Clearly, the q_{rel} decreases passing from the quiet to the shock period a result that reveals changing in the underlying dynamics and the presence of non-extensive character in all cases. Moreover, the value of q_{rel}(TVIX) = 4.56 was found to be closer to q_{rel}(Adj3). In addition, the values of q_{rel} clearly indicate non-equilibrium phase transition relaxation processes much different than the classical normal diffusion relaxation process of Boltzmann-Gibbs statistical theory.



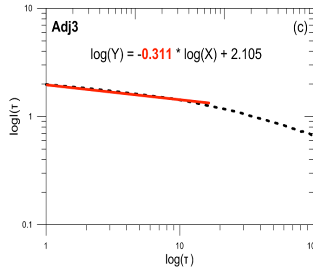


Figure 6: Log-log plot of the self-correlation coefficient $I(\tau)$ vs. time delay τ for the three segments of S & P 500 segments time series.

Table 1: The index q_{relax} for the four stock market time series.

q_{relax}			
Adj1	Adj2	Adj3	TVIX
15.706	10.71	4.125	4.56

4. Summary and Conclusions

In this study we presented results of the evaluation of $q-$triplet of Tsallis non-extensive statistics concerning two time stock market time series, S & P 500 and TVIX. The results of data analysis are summarized as follows:

- The non Gaussian (Tsallis q-Gaussian) and non-extensive statistics were found to be effective for all time series.
- The Tsallis $q-$triplet $(q_{sen}, q_{stat}, q_{rel})$ was found to verify according to the expected scheme $q_{sen} < 1 < q_{stat} < q_{rel}$. Moreover the Tsallis $q-$triplet estimation showed clear distinction between the dynamics underlying the S & P 500 time series.
- The multifractal character was verified for all the stock market time series. In addition there is a significant shift in the q_{sens} and Δa values concerning the second segment of the S&P 500 time series, indicating non-equilibrium phase transition processes.
- Efficient agreement between D_q and $p-$model was discovered indicating intermittent (multi-fractal) turbulence. In addition there is a significant shift in the p-values concerning the second segment of the S&P 500 time series.
- The q_{rel} index and the q_{stat} index follow a gradually decreasing profile, as far as the S&P 500 time series is concerned.
- Tsallis q_{sens} and q_{stat} of the TVIX time series are comparable to the first segment of S & P 500 (Adj1), while for the q_{rel} the value was comparable to the third segment (Adj3).

As we have shown the experimental results of this study indicate clearly the non-Gaussian Tsallis q-Gaussian character of the probability distributions of the S & P 500 and TVIX time series. This result is in contrast with the mainstream hypothesis that economic dynamics can be studied through the Newtonian approach which treats the economic fluctuations as linear perturbations near the equilibrium, while the dynamics follow random walks and the statistics are Gaussian, independent and identically distributed (Chian A.C.-L. et al., 2006; Stavroyiannis et al., 2010). Moreover, Tsallis q-triplet analysis showed that the Tsallis non-extensive q-statistics have been verified both for the S & P 500 and TVIX time series indicating non-extensivity as well as the multi-fractal and multi-scale dynamics of the stock market underlying dynamics. The Tsallis q-statistics reveal the presence of long range correlations and strong self-organization processes in the underlying stock market dynamics.

In addition, the detailed analysis of S & P 500 index unraveled the existence of phase transitions depicted clearly in the variations of Tsallis q-triplet values. These phase transitions are connected with non-equilibrium stationary states of economical dynamics derived from processes of strong self organization which correspond to local maxima of Tsallis entropy, while the changes in the control parameters can induce new phase transitions and shifts to new metaequilibrium steady states of maximizing Tsallis entropy. This phase transition leads to a multifractal change in the formation of the phase state and an alteration in the phenomenology of the economical system. Finally, the existence of Tsallis distributions indicate the existence of fractional dynamics in the phase space which can be described through fractional Fokker-Planck equations and anomalous diffusion equations, since the solutions of these equations are fractional spatiotemporal functions and non-Gaussian distributions functions which fall into the category of Levy distributions and Tsallis distributions (Tsallis and Lenzi, 2002; Lenzi et al., 2003).

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