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Chaotic Analysis of Gold Price Index

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Abstract

This paper applies non linear methods to analyze the weekly gold prices close index .The aim of the analysis is to quantitatively show if the corresponding time series is a deterministic chaotic one and its predictable. For this purpose the correlation and minimum embedding dimensions of the corresponding strange attractor are calculated. Also the maximum Lyapunov exponent is estimated.

Keywords: Gold price index, forecasting , chaos, Lyapunov exponent

1. Introduction

The use of Physical laws in Economics as well as the use of Physical principles in human behavior will give us additional tools to solve economic problems, model economic systems and predict economic time series as the weekly gold price index

Recent evidence on implication of chaos theory in financial time series prediction comes from works [1-2] Exchange and empirically shows that it has informational efficiency in semi-strong form. In the case that a system presents deterministic chaotic behavior we can find the number of first order differential equations that described its evolution.. For this purpose we have applied the method proposed by Grassberger and Procaccia to evaluate the invariant parameters of weekly gold price close index, as the correlation and minimum embedding dimensions. Also the maximum Lyapunov exponent is estimated in order to determine the predictability horizon.

2. Gold price index.

The weekly Gold price index is presented as a signal x=x(t) as it shown at Fig – 1. It covers N=1807 data from 29/12/1990 to 09/08/2013. The sampling rate was Δt=1 week. The data corresponds to closed value.

3. Chaotic Analysis.

In order to evaluate the afore mentioned time series we have used the method proposed by Grassberger and Procaccia [3,4] and successfully applied in similar cases [1,5]. According to Takens [6] the measured time series was used to reconstruct the original phase space. For this purpose we calculated the correlation integral, for the recorded signal, defined by the following relation as Kantz and Schreiber [7] proposed, for $r \rightarrow 0$ and $N \rightarrow \infty$,

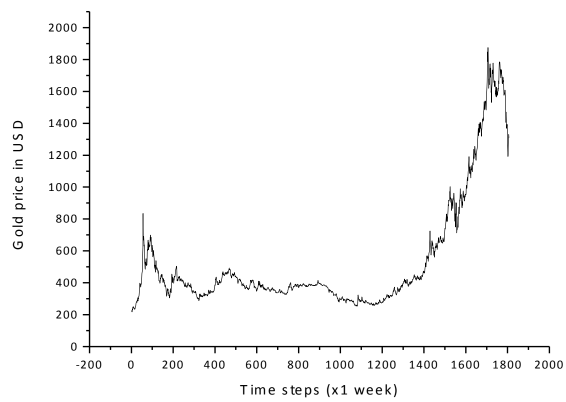

Fig 1. Time Series of Gold price index.

$$C(r) = \frac{1}{N_{pairs}} \sum_{\substack{l=1,\\ j=l+W}}^{N} H\left(r - \left\|\vec{X}_l - \vec{X}_j\right\|\right) \quad (1)$$

where

N is the number of points,
H is the Heaviside function,

$$N_{pairs} = \frac{2}{(N-m+1)(N-m+W+1)},$$

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m is the embedding dimension
The summation counts the number of pairs $(\vec{X}_i, \vec{X}_j)$ for which the distance, (Euclidean norm), $\|\vec{X}_i - \vec{X}_j\|$ is less than r, in an m dimensional Euclidean space and W is the Theiler window. As Theiler pointed out if temporally correlated points are not neglected, spuriously low dimension estimate may be obtained.
In the above equation N is the number of the record data (weekly gold price index), here N=1807, $\vec{X}_i$ is a vector in the m dimensional phase space given by the following relation

$$\vec{X}_i = \{x_i, x_{i-\tau}, x_{i-2\tau}, \ldots, x_{i+(m-1)\tau}\} \qquad (2)$$

The vector $\vec{X}_i = \{x_i, x_{i-\tau}, x_{i-2\tau}, \ldots, x_{i+(m-1)\tau}\}$, represents a point to the m dimensional phase space in which the attractor is embedded each time, where τ is the time delay determined by the first minimum of the mutual information. In our case τ=37 time steps as shown in Figure - 2.

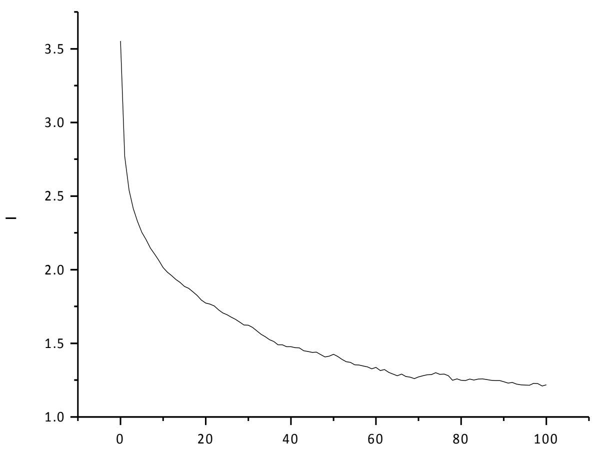

Fig 2. Mutual Information I vs time delay τ

Using value τ=37 as an optimum delay time for the reconstruction of phase space in eq (1) In order to estimate the Theiler window we use the space time separation plots [8]. The idea is that in the presence of temporal correlations the propability that a given pair of points has a distance smaller than r does not only depends on r but also on the time that has elapsed between the two measurements.This dependence can be detected by plotting the number of pairs as function of two variables the time separation Δt (orbit lag) and the spatial distance r (spatial seperation). In Figure 3 the time space separation plots are shown . From the curve's plateau [7,9] we estimate the Theiler windows to be W=90.

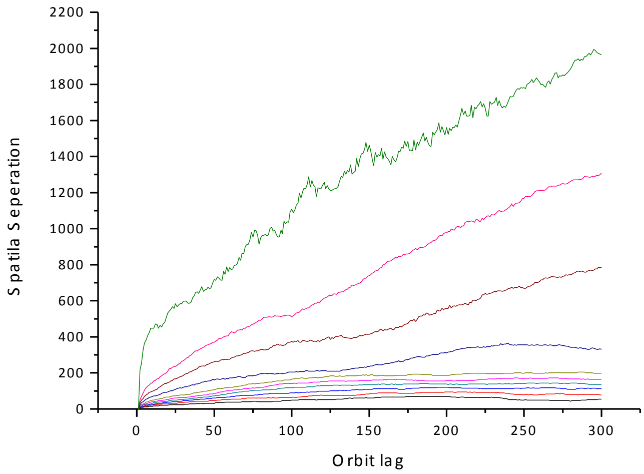

Fig 3. Space Time seperation plots

The next step is to divide this space into hyper cubes with a linear dimension r and we count all points with mutual distance less than r.

It has been proven by [9 -11] that if our attractor is a strange one, the correlation integral is promotional to r^v where v is a measure of the dimension of the attractor, called the correlation dimension. Then the correlation function is related to the radius with a power law $C(r) \sim r^{\nu}$ and v is the slope of the lnC(r) versus logr plot. Since the data set will be continuous, r cannot get to close to zero. To handle this situation, one plots lnC(r) versus logr and selects the apparently linear portion of the graph. The slope of this portion will approximate ν. Practically, one computes the correlation integral for increasing embedding dimension m and calculates the related ν(m) in the scaling region.

Using value τ=37 as an optimum delay time and W=90, we reconstruct the phase space. The correlation integral C(r), by definition is the limit of correlation sum of equation 1 for embedding dimensions m in the range from 1 up to 10 and shown in Fig- 4.

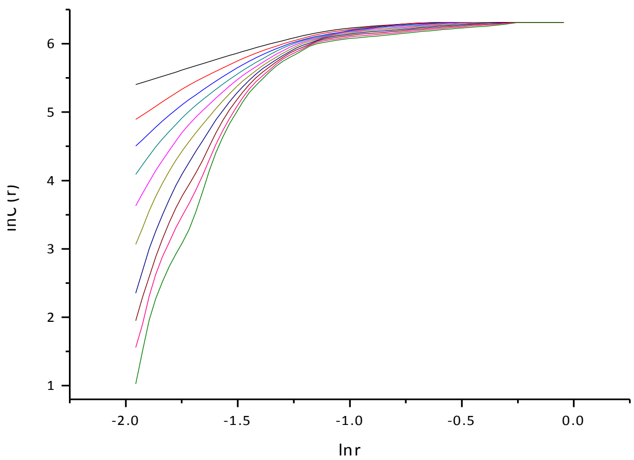

Fig 4. Correlation integral v radius r for different embedding dimensions m. The values of m increased from top to bottom.

In Fig-5 the corresponding average slopes v are given as a function of the embedding dimension m, indicating that for high values of m, v tends to saturate at the non integer value of v=2.1. The decimal value of v as well as the saturated behavior as a function of m reveals chaotic behavior of the Gold price index.

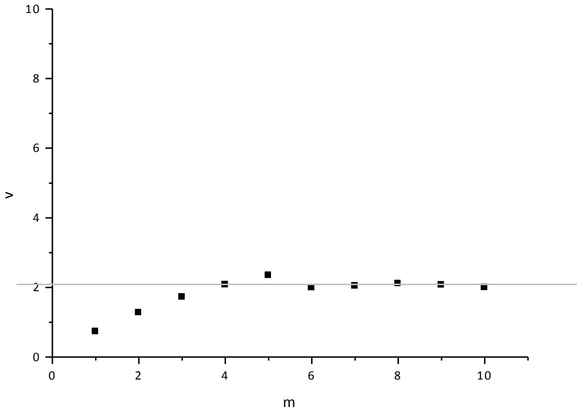

Fig 5. Correlation dimension v vs embedding dimension m.

According [9], the closest integer above the correlation dimension provides with the proper minimum embedding dimension $m_{\min}$, which in this case possesses the value $m_{\min}=3$. This minimum embedding dimension is referred to the system's attractor under the specific conditions and it reveals the essential dimension of the corresponding dynamical system phase space (and the number of the essential variables) necessary to model the dynamics of the attractor.

On the other hand, the sufficient phase space dimension, necessary to fully describe the global dynamics of the system, can also be experimentally identified in Fig. 5, by identifying the embedding dimension where the correlation exponent reaches its saturation value [12]. In this case it is apparent that this happens after the 6th embedding dimension. Thus, the sufficient phase space embedding dimension for the attractor, describing the gold index global dynamics is equal to 6.

In order to describe the predictability of the system we calculated the average local Lyapunov exponents using as time delay the value $\tau=37$,using three dimensional models. . Applying the method proposed by [9], from Figure 6 we estimate the maximum Lyapunov to be $\lambda_{max}= 0.33$. This means that the predictable horizon $1/\lambda_{max}$ [9] is about 3 weeks ahead.

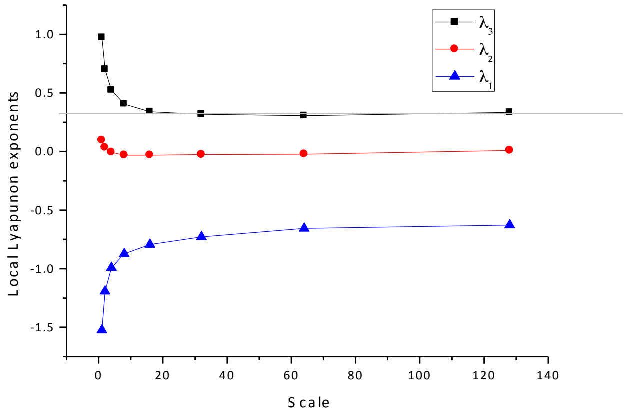

Fig 6. The average local Lyapunov exponents for weekly gold price data. We used local three dimensional models

4. Conclusion

From the previous analysis the conclusion is that the gold price index is a deterministic chaotic system To full describe its behaviour we need 6 differential equations but we can capture its dynamic using three of them. the positive max Lyapunov exponent allows to predict 3 times steps ahead i.e. three weeks ahead the gold price value.

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