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Sunspot numbers: data analysis, predictions and economic impacts

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Abstract

We analyze the monthly sunspot number (*SSN*) data from January 1749 to June 2013. We use the Average Mutual Information and the False Nearest Neighbors methods to estimate the suitable embedding parameters. We calculate the correlation dimension to compute the dimension of the system's attractor. The convergence of the correlation dimension to its true value, the positive largest Lyapunov exponent and the Recurrence Quantitative Analysis results provide evidences that the monthly SSN data exhibit deterministic chaotic behavior. The future prediction of monthly SSN is examined by using a neural network-type core algorithm. We perform ex-post predictions comparing them with the observed SSN values and the predictions published by the Solar Influences Data Analysis Center. It is shown that our technique is a better candidate for the prediction of the maximum monthly SSN value. We perform future predictions trying to forecast the maximum SSN value from July 2013 to June 2014. We show that the present cycle 24 is yet to peak. Finally, the negative economic impacts of maximum solar activity are discussed.

Keywords: yearly sunspots number; grand solar minimum; Maunder Minimum; solar activity predictions; deterministic chaos

1. Introduction

The Sun's magnetic field appears concentrated in flux tubes or ropes that appear on the surface of the photosphere as sunspots, pores, plages, and surface networks. Sunspots have a magnetic field, which results from the solar dynamo that includes all solar motions from rotation to turbulent convection (Ruzmaikin, 2001; Rogachevskii & Kleeorin, 2007). In particular, the sunspots are transient features in the photosphere. They have vertically directed magnetic fields of the order of 1000 to about 4000 Gauss (de Jager, 2005). Some basic characteristics of the sunspots according to de Jager (2005) are the following:

a. At the location of the fields the convective motions are inhibited; hence less energy is carried upward than elsewhere in the photosphere. This results in the darker appearance of the spots. Yet the spots are not dark; their effective temperature is still as high as 4200°K.
b. The majority of the spots do not live longer than 2 days. The average lifetime is 6 days. Large spots may live for weeks and in rare cases even for months.
c. Typically spot diameters range from 2,000km to more than 40,000km. While motions are practically totally inhibited inside spots, there is a complicated velocity field under and around them.

Sunspots, by themselves, do not emit radiation or particles that could interact in some way with the Earth, but sunspots are markers of the Centres of Activity (de Jager, 2005). Hence the variation of the sunspot number shows the activity level of the Sun. The knowledge of the level of solar activity years ahead is important to Earth. Edmund Halley, following the spectacular auroral display in Europe in March 1716, made the first step of understanding the Sun-Earth connection. He suggested that charged particles moving along the Earth's magnetic field lines are the cause of the aurora (Halley, 1692). The radiation environment of the Earth's atmosphere is very dynamic and consists of several components of ionizing radiation: galactic cosmic rays, solar energetic particles and radiation belt particles. Galactic cosmic rays reach their maximum intensity when the Sun is least active and are at a minimum intensity during solar maximum. In contrast, during maximum solar activity an increased number of Coronal Mass Ejections (CMEs) and solar flares produce high-energy solar particles (O'Sullivan, 2007). Beyond the protective shield of the Earth's atmosphere and magnetosphere, there are sources of radiation that can be a serious hazard to humans and electronic equipment. These effects can have severe negative economic impacts on our society.

After 17 years of sunspot observations, the apothecary Schwabe (1843, confirmed in 1851) found that the solar activity, measured by the number of sunspots, varies in time and shows an 11-year periodicity (de Jager, 2005). The last recorded solar cycle lasted 12.6 years (1996 – 2008). In that order the current cycle (2008 –) is number 24. In Fig.1 we

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isualize[1] the plot of the monthly sunspot number (*SSN*) data for the last 24 sunspot cycles[2] (January 1749 – June 2013).

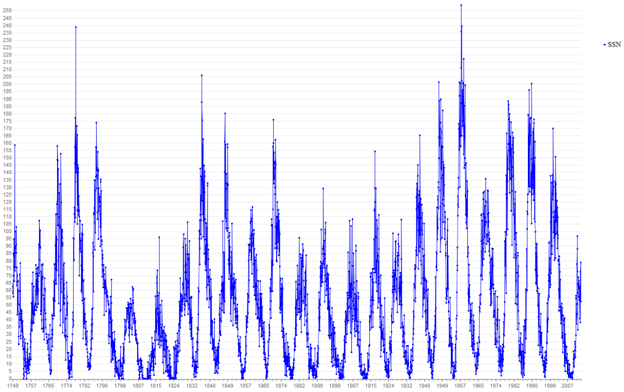

Fig. 1: The time series for the last 24 sunspot number cycles.

The 11-year periodicity rule is not strict; as we observe in Fig.1 there are short and long cycles, weak and strong ones. Moreover we observe that the most active *SSN* cycle since 1749 is the cycle 19; its maximum value deviates the most. Furthermore, we observe that the data trend supports Waldmeier (1961) hypothesis that lower activity cycles rise to peak latter in time. Observing the 10 prior cycles (14-23) the observed timelines fall into 2 categories (Ahluwalia & Jackiewicz, 2012): (a) the cycles 14, 15, 17, 20 and 23 are slow risers like the cycle 24, (b) the cycles 16, 18, 19, 21 and 22 rise relatively steeply and exhibit above average activity. Further, Gnevyshev & Ohl (1948) found that there exists good correlation between the properties of the even and the next following odd cycle, and not the preceding odd one. Beginning with cycle 10, Gnevyshev & Ohl (1948) noted that there is a pattern such that even cycles of the even-odd pairing are less active; this pattern disappears after cycle 21 (the even-odd symmetry in *SSN* cycles broke down with cycle 22). The physical cause for this pattern is unknown. They might reappear in the future.

2. Sunspot Number (*SSN*) data analysis

The behavior of solar activity dynamics has been investigated by many researchers. The daily sunspot numbers, the monthly means and yearly means may be from a stochastic process (Siscoe, 1976) or from a deterministic chaotic process (Feynman & Gabriel, 1990). Morfill et al. (1991) analyzed the sunspot record over time scales of weeks or months. They consider a stochastic model, a heuristic model and a Lorenz model to represent the data. They showed that the deterministic chaos model (Lorenz) provides the best fit of the data. Mundt et al. (1991) studied the variable solar activity over the time period from January 1749 to May 1990 using 2897 monthly sunspot numbers. They showed that the attractor does not fill the space and is a sheet much like the Rössler and Lorenz attractors with a dimension ~2.3. The solar dynamo can be expressed with three differential equations identical to the Lorenz equations. Thus the solar cycle appears to be chaotic of low dimension and can only be predicted for a short term. Zhang (1996) performed a nonlinear analysis of the smoothed monthly sunspot numbers to obtain nonlinear parameters to predict the numbers. The analysis of the monthly smoothed numbers from January 1850 to May 1992 indicates the numbers are chaotic and of low dimension described by three to seven parameters.
Ostryakov & Usokin (1990) examined the structural character and inherent stochastic behavior of the monthly mean sunspot numbers. They calculated that the fractal dimension for the periods 1749 – 1771, 1792 – 1828 and 1848 – 1859 is 4.3, 3.0 and 4.0 respectively. Zhang (1994, 1995) calculated the fractal dimension $D = 2.8 \pm 0.1$ and the largest Lyapunov exponent $\lambda_{\max} = 0.023 \pm 0.004$ bits/month, for the monthly mean sunspot numbers for the period January 1850 to May 1992 using the methods given by Grassberger & Procaccia (1983b) and Wolf et al. (1985).
In this section we analyze[3] the sunspot record over time scales of months using 3174 monthly sunspot numbers (January 1749 – June 2013). Packard et al. (1980) outline a simple method (time lag) developed by Ruelle & Takens (1971) for reconstructing a phase space from one dynamical variable: let $x_1, x_2, \ldots, x_N$ be measurements of a physical variable at the time $t_i = t_0 + i\Delta t$, $i = 1, \ldots, N$. From this sequence one can construct a set of m-dimensional vectors $u_i = 1, \ldots, N - (m-1)T$, of the form:

$$u_i = \left(x_i, x_{i+T}, x_{i+2T}, \ldots, x_{i+(m-1)T} \right) \quad (1)$$

where the time delay (time lag), *T*, is an integer multiple of Δ*t*. This method fills the other dimensions with lagged versions of one dynamical variable. Thus in order to examine the dynamics of our system in space defined by delayed vectors of dimension *m*, we have to estimate the embedding parameters (i.e. the time delay and the embedding dimension).

2.1 Estimation of the suitable time delay and embedding dimension

Fraser & Swinney (1986) suggested the Average Mutual Information (AMI) as a method to determine a reasonable delay for nonlinear systems. The AMI is a generalization from the correlation function, which measures the linear correlations, to the case of nonlinear correlations between measurements. So, in order to find the suitable time delay τ for the embedding, we use the Average Mutual Information as a function of the time delay $I = I(T)$ (Fig.2). The lag at which the first minimum of the AMI function occurs is $T_m = 29$ and it is chosen as a delay time $\tau = 29$.

In order to find the suitable dimension (*m*), we use the False Nearest Neighbors (*FNN*) method, which has been first introduced by Kennel et al. (1992) as a convenient method to determine the minimal sufficient embedding dimension. Fig.3 illustrates the *FNN* as a function of the embedding dimension *m*. The suitable embedding dimension should not be smaller than the first dimension at which the number of false nearest neighbors drops to zero. Thus the suitable embedding dimension to unfold dynamics is estimated to be

[1] The data visualization has been exhibited using the software package GMDH Shell 3.0.1 (http://www.gmdhshell.com/).
[2] The monthly sunspot number data were taken from the Solar Influences Data Analysis Center (SIDC) (http://sidc.oma.be/sunspot-data/).
[3] The data analysis has been integrated by using the software package Auguri 3.14.354.1482 (http://aag-auguri.com/index.html).

about m = 7, 8 (i.e. at an embedding of 7 to 8 dimensions the attractor of the sunspot series is unfolded).

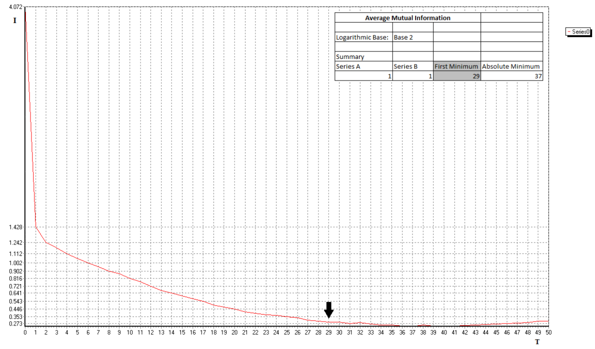

Fig. 2: The Average Mutual Information as a function of the time delay $I = I(T)$ for $T = 0,1,...,50$.

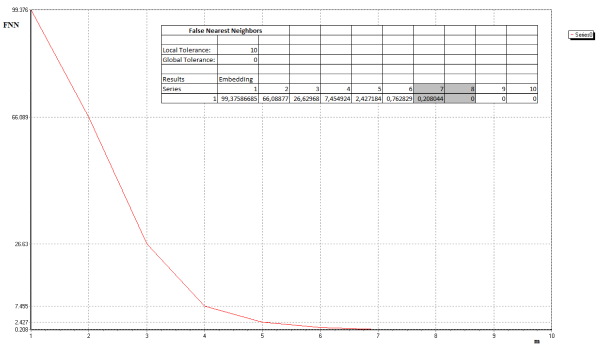

Fig. 3: The False Nearest Neighbors as a function of the embedding dimension $FNN = FNN(m)$ for $m = 1,2,...,10$ embedding steps.

2.2 Estimation of the correlation dimension

The correlation dimension has been introduced by Grassberger & Procaccia (1983a, 1983b) to compute a fractal dimension measurement of an attractor. In particular Grassberger & Procaccia (1983a) for several model systems showed that $\nu \simeq D_c$, where D_c is the dimensional of system's attractor and ν is the correlation dimension. So ν is expected to be a good estimate of the exact dimensionality

$$D_c : D_c(m) \propto \lim_{r \to 0} \frac{\ln C^{(m)}(r)}{\ln r} \tag{2}$$

where $\ln C^{(m)}(r)$ is the logarithm of the correlation integral for m dimension; $\ln r$ is the logarithm of the distance in phase space. We plot the graph of $\ln C^{(m)}(r)$ as a function of $\ln r$ for various embedding dimensions m for the monthly *SSN* data (Fig.4). By finding the slope of $\ln C^{(m)}(r)$ versus $\ln r$, through least squares regression, we estimate the correlation dimension $D_c(m)$ for the embedding dimension m.

We calculate the correlation dimension as a function of the embedding dimension $D_c = D_c(m)$ in order to determine the fractal dimension of the attractor of the monthly sunspot numbers (Fig.5). By increasing m, the correlation dimension $D_c(m)$ will eventually converge to its true value $\nu \simeq D_c$. In Fig.5, we observe that, when the embedding dimension exceeds $m_c \approx 7$ the correlation dimension converges to the value $D_c \approx 3.8$. The embedding dimension value at which the convergence begins is about twice the attractor dimension $m_c \approx 2D_c$. Thus, the dimension of the attractor is estimated to be about $D_c \approx 3.8$ (low dimensional). The attractor's dimension defines the number of variables of the system. Therefore, the time series of the monthly sunspot numbers can be described by $\nu = 4$ independent variables.

The convergence of the correlation dimension $D_c(m)$ with increasing values of the embedding dimension m to its true value D_c is an indication of chaotic behavior. A correlation dimension that does not converge corresponds to a white noise signal.

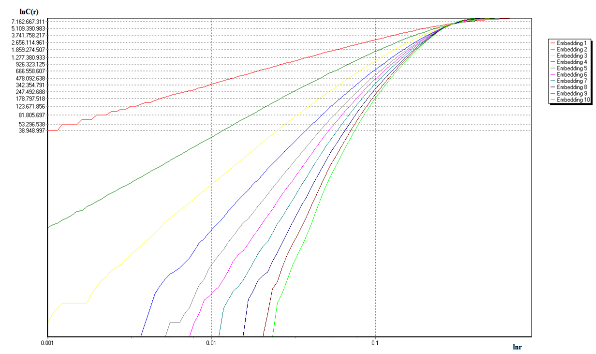

Fig. 4: The graph of $\ln C^{(m)}(r)$ for increasing values of $\ln r$ for $m = 1,2,...,10$ embedding dimensions.

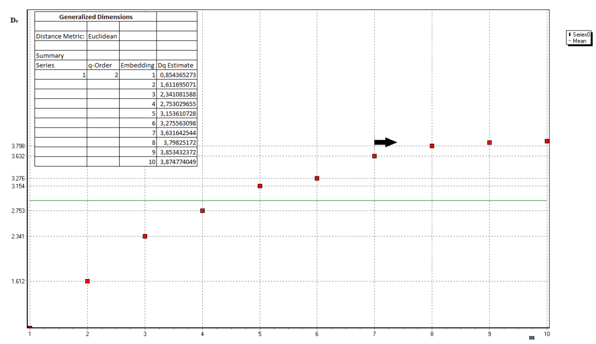

Fig. 5: The Correlation dimension as a function of the embedding dimension $D_c = D_c(m)$ for $m = 1,2,...,10$ embedding steps.

2.3 The Largest Lyapunov Exponent

In order to determine the presence of a deterministic chaos in the time series, we calculate the largest Lyapunov exponent $(\lambda_{\max})$. In order to calculate $\lambda_{\max}$, we used the Kantz algorithm, which calculates the largest Lyapunov exponent by searching for all neighbors within a neighborhood of the reference trajectory and computes the average distance between neighbors and the reference trajectory as a function of time (Kantz, 1994). The largest Lyapunov exponent is $\lambda_{\max} \simeq 0.0195 > 0$; the positive value of the largest Lyapunov exponent indicates the presence of chaos in solar activity dynamics. In this case we have the so-called exponential instability where two arbitrary close trajectories will diverge apart exponentially; that is the hallmark of chaos (Bershadskii, 2009). The small largest Lyapunov exponent value indicates that the chaos for the monthly sunspot numbers is weak. The predictive power can be estimated by $\Delta t = 1/\lambda_{\max}$ (Sprott, 2003). So the upper limit of the theoretical time scale on which the monthly sunspot number can be used to make deterministic predictions is $\Delta t = 1/\lambda_{\max} = 1/0.0195 \simeq 51$ months. We observe that long-term predictions are not possible most likely due to the fact that the chaotic nature of the system results a high sensitive dependence on initial conditions for the monthly sunspot numbers.

2.4 Recurrence Quantitative Analysis

Recurrence Plots (RPs) and Recurrence Quantitative Analysis (RQA) are numerical analysis methodologies that can be used in order to inform about the dynamic properties of a time series (Fabretti & Ausloos, 2005). RPs are 2D graphs, which are based on the phase space reconstruction introduced by Eckmann et al. (1987), in order to visualize the recurrences of trajectories of dynamical systems. RQA is a statistical quantification of RPs introduced later by Zbilut & Webber (1992) and Webber & Zbilut (1994) in order to quantify the diagonal (and vertical) line structures in recurrence plots.

We plot the RP (Fig.6) and use the RQA to study the recurrent patterns that exist within the time series of the monthly *SSN* data. Table 1 summarizes the RQA results. The high values of %DET, Maxline and ENT indicate the deterministic chaotic behavior of our system. In particular, the high value of Determinism[4] ($\%DET = 98.31\%$) indicates that most of the recurrent points are found in deterministic structures. The high value of the variable Maxline[5] ($L_{\max}$ = 247) is consistent with the small value of the largest Lyapunov exponent ($\lambda_{\max} \simeq 0.0195$) indicating that the signal of the system's attractor is only slightly chaotic and the system is more stable. Moreover, the value of Trend[6] (*TND* = – 1.99) does not deviate significantly from zero, indicating the system's stationarity. The large value of Entropy[7] (*ENT* = 4.8694) indicates the high complexity of the deterministic structure in the recurrence plot. Finally, the small value of Recurrence rate[8] (%*REC* = 21.76%) indicates that the monthly *SSN* data exhibit aperiodic dynamics.

[4] The Determinism allows distinguishing between dispersed recurrent points and those that organized in diagonal patterns (Belaire-Franch et al., 2002); it is a measurement of determinism.

[5] The Maxline is the length of the longest diagonal line in the recurrence plot (Marwan et al., 2007). This quantity is proportional to the inverse of the largest Lyapunov exponent (Trulla et al., 1996).

[6] Trend provides information about the non-stationarity of the time series (Marwan et al., 2007). High values of Trend are associated with a non-stationary process having strong trend (Fabretti & Ausloos, 2005).

[7] The Entropy reflects the complexity of the RP in respect of the diagonal lines (Marwan et al., 2007). A high Entropy value indicates that much information are required in order to identify the system (Fabretti & Ausloos, 2005).

[8] The Recurrence rate measures the recurrence density (Marwan et al., 2007).

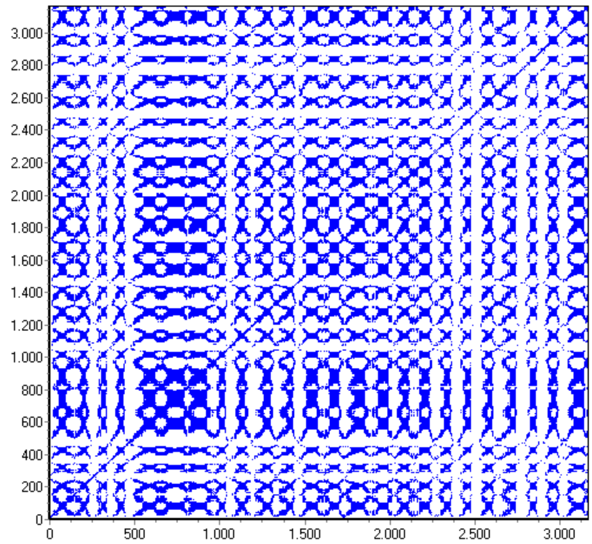

Fig. 6: The Recurrence Plot for the monthly *SSN* data (January 1749-June 2013).

Table 1: Recurrence Quantitative Analysis results for the monthly *SSN* data (January 1749 – June 2013).

Recurrence Analysis

Series	Epoch	DIS	REC	DET	ENT	MAXLINE	TREND
1	Entire Series	135.4786567	0.21759432	0.983094045	4.869374122	247	-1.990353611

Thus the results of our data analysis indicate that the monthly sunspot numbers for the period January 1749 to June 2013 is a system of low dimensional deterministic chaos. Although chaotic systems are theoretically unpredictable in the long term, their underlying deterministic nature allows accurate short-term predictions (Mundt et al., 1991).

3. Sunspot numbers prediction

heng (1993) used the leap-step threshold autoregressive model and technique in nonlinear time series to obtain predictions from 1 month to 12 months ahead. De Meyer (2003) attempted to forecast the values of sunspot numbers (R_Z) on the basis of his model of the solar cycle consisting of a sequence of independent overlapping events. He predicted that the 24[th] cycle would start in 2007 to reach in 2011 a peak height in the range 95 – 125. Predicting solar activity is quite challenging, but there are indications that solar activity may decrease in coming decades. Clilverd et al. (2003) suggested a nearly constant level of solar activity till about 2050 and a slow decrease thereafter. However, this approach was criticized by Tobias et al. (2004): "*The future of such a chaotic system is intrinsically unpredictable*".

In this chapter we try to forecast the peak cycle-24 activity. The results have been analyzed by using the software

GMDH Shell (GS). *GS* is a predictive modeling tool that produces mathematical models and makes predictions. As a model selection criterion we define the Root-Mean-Square Error (RMSE), which selects models with the lowest difference between values predicted by a model and the values actually observed. We also select ranking variables according to their ability to predict testing data (variables ranking by error). As a core algorithm we define the polynomial neural networks. A core algorithm generates models from simple to complex ones until the testing accuracy increases. The GMDH-type neural networks algorithm iteratively creates layers of neurons with two or more inputs. Every neuron in the network applies to a transfer function that allows exhaustive combinatorial search. Thus, the transfer function is suitably chosen that can predict testing data most accurately. We use two input variables and a linear transfer function for neurons.

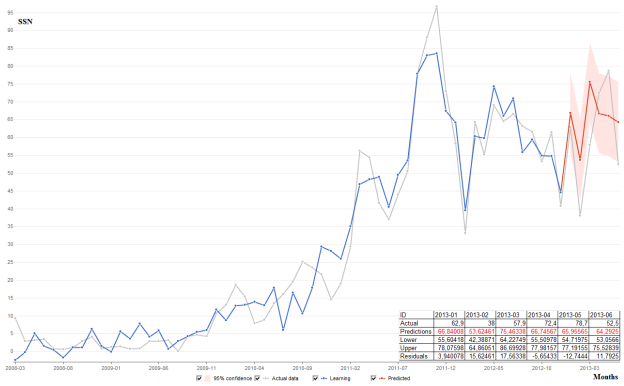

Fig. 7: Ex-post predictions of 6 steps (months) back in time (January 2013 – June 2013); the grey curve corresponds to the actual monthly *SSN* data; the blue curve corresponds to the proposed neural network model; the red curve corresponds to the ex-post forecasts.

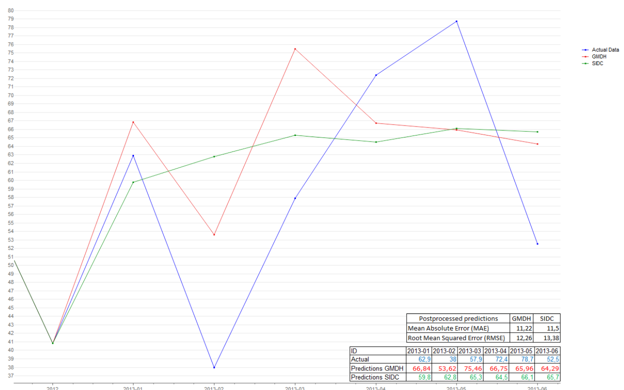

Fig. 8: Time series of the actual *SSN* data (blue curve), predictions of the proposed neural network model (red curve) and predictions published by SIDC (green curve) from January 2013 to June 2013.

3.1 Ex-post sunspot numbers predictions

In order to reduce further the drop of predictions' accuracy caused by chaotic behavior, we perform ex-post simulations and we are taking into account the Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE) obtained by comparing the predicted to the observed values. In particular we perform ex-post predictions of 6 steps (months) back in time (January 2013 – June 2013) and we compare them with the observed values (Fig.7) and the corresponding predictions given by the Solar Influences Data Analysis Center[9] (SIDC) (Fig.8).

In Fig.7 we observe that the differences between the predicted and observed values (residuals) are quite small. So the ex-post predictions of the proposed neural network model fit the values of the actual data quite well. Moreover, in Fig.8 we observe that the values of MAE[10] and RMSE[11] for the post-processed predictions of the proposed neural network model (MAE = 11.22 and RMSE = 12.26) are consistent with those of the predictions published by SIDC (MAE = 11.5 and RMSE = 13.38).

However, the oscillations in the observed *SSN* values seem to be predicted better by the proposed neural network model than the method used by SIDC (Fig.8); i.e. the time series of our neural network model predictions (red curve) fits better the actual sunspot data (blue curve) than the SIDC predictions (green curve). Furthermore, we observe that the maximum observed value from January 2013 to June 2013 was 78.7 in May 2013. Although the SIDC predicted maximum was in May 2013, the predicted value was 66.1, while the predicted maximum of our neural network model was 75.46 (much closer to the observed value 78.7) in March 2013. Thus our proposed neural network model seems to predict better the maximum *SSN* value (with a deviation of ±2 months) than the method used by SIDC.

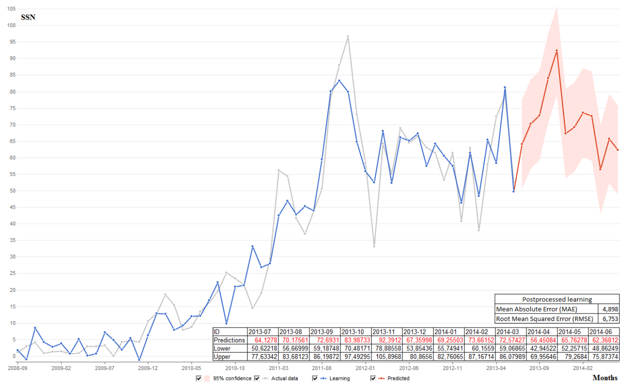

Fig. 9: Future predictions of 12 time steps (months) ahead (July 2013 – June 2014); the grey curve corresponds to the actual monthly SSN data; the blue curve corresponds to the proposed neural network model; the red curve corresponds to the future forecasts.

3.2 Future sunspot numbers predictions

Finally, we perform future predictions in order to forecast the maximum sunspot number value during the next 12 months. The predictions of 12 time steps (months) ahead, for the period from July 2013 to June 2014, are illustrated in Fig.9. The proposed neural network model of the 12-months-ahead prediction is:

$$\begin{aligned}
SSN[t] &= 3.03787 - 0.319168 \cdot SSN[t-1428] + 1.3856 \cdot N116 \\
N116 &= -1.13723 + 0.613612 \cdot N154 + 0.41996 \cdot N264 \\
N154 &= -4.40757 + 0.294369 \cdot SSN[t-273] + 0.157676 \cdot SSN[t-2098] \\
N264 &= -1.11764 + 0.18513 \cdot SSN[t-771] + 0.584038 \cdot SSN[t-1154]
\end{aligned} \qquad (3)$$

SSN[*t*] is the predicted value of the monthly sunspot number at time period *t* (for *t* = 3175, 3176, …, 3186); *SSN*[*t*

[9] We are using the predictions of the monthly *SSN* published by the Solar Influences Data Analysis Center (SIDC) based on the Combined Method (CM); a regression technique combining a geomagnetic precursor (aa index) with a least-square fit to the actual profiles of the past 24 solar cycles (http://sidc.oma.be/sunspot-data/).

[10] MAE is the average over the verification sample of the absolute values of the differences between forecast and the corresponding observation.

[11] RMSE is calculated by taking the difference between forecast and corresponding observed values each squared and then averaged over the sample. Finally the square root of the average is taken.

− 1428], $SSN[t − 273]$, $SSN[t − 2098]$, $SSN[t − 771]$ and $SSN[t − 1154]$ are the observed values of SSN at 1428, 273, 2098, 771 and 1154 steps back in time, before the time period t respectively (e.g. for the next period, $t = 3175$, July 2013, the $SSN[t − 1428] = SSN[3175 − 1428] = SSN[1747]$ corresponds to the observed value in July 1894).

According to the predictions of the proposed neural network model (Eq.2), the predicted maximum value of SSN for the next 12 months for cycle 24 is expected to be 92.4 in November 2013 (± 2 months). Moreover, the Mean Absolute Error and the Root Mean Square Error are relatively small (MAE = 4.898 and RMSE = 6.753) indicating the high predictive accuracy of our neural network model. Thus the SSN cycle 24 is yet to peak.

4. Discussion

Space weather has severe impacts on satellites (Hastings & Garrett, 1996) and GPS/navigation (Wellenhof et al., 2001). During solar events and geomagnetic storms the system may give navigators information that is inaccurate by as much as several km (Filjar, 2008). The problems of geomagnetically induced currents in power lines and flowing in high voltage transformers have been recognized at least since the early 1970s (Albertson et al., 1974), but were brought into serious consideration by the widespread failure of the Hydro-Quebec power grid (Canada) resulting from severe geomagnetic storm on March 13-14, 1989 (Blais & Metsa, 1993).

The first observations of space weather effects on technological systems were made in telegraph equipment more than 150 years ago (Barlow, 1849). Many times since then, systems have suffered from peak overvoltages, interruptions in the operations and even fires caused by Geomagnetically Induced Currents (GICs) flowing through the equipment (Boteler et al., 1998). During a magnetic storm in July 1982, such an effect made traffic lights turn red without any train coming, in Sweden (Wallerius, 1982). Submarine telephone cables lying on the ocean floors form a special category of systems affected by geomagnetic disturbances (Root, 1979). Another consequence of space weather is its effect on humans and biological systems in space and on aircraft (Baker et al., 2006). Solar proton events (SPEs) can knock electrons from cell molecules and damage them, especially from the skin, eye and blood-forming organs. These damaged cells are unrepairable (Crosby et al., 2006). If DNA (deoxyribo nucleic acid) is damaged, then cell reproduction is hampered and even the effect could be passed to the next generations. Biological effects can also be in the form of severe burns, sterilization, cancer and damage to other organs.

Therefore, trying to predict the maximum of solar cycles becomes more and more of an urge, since it can minimize economic losses and help society save hundreds of millions of money each year.

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