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Research Article

Research on the Transmission Performance of Multi-hop Cognitive System

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Abstract

By analyzing the multi-hop cooperative diversity cognitive network in Underlay Sharing(US) mode, an approximate outage probability formulation and diversity order has been derived for decode-and-forward(DF) protocol in high signal-to-noise ratio. As DF protocol cannot achieve space diversity gain, modified multi-hop selective decode-and-forward(SDF) protocol is proposed and an approximate outage probability formulation and diversity order are deduced for SDF cognitive protocol to improve system performance. Theoretical analysis and simulation result show that the SDF protocol can decrease cognitive network outage probability and achieve the full space diversity gain contrasted with DF protocol.

Keywords: cognitive network; outage probability; space diversity; diversity gain

1. Introduction

In the scene of the long distance from source to destination, both signal-hop and even the two-hop can't transmit data to the destination correctly, so multi-hop with the advantage of expanding coverage can solve the above problem effectively, but at the same time it will lead to the large delay and increase the complexity of the algorithm. In [1-5], multi-hop relay system performance constituted by the source, the relay and the destination has been studied, the relay only receives the data transmitted by the preceding fixed node, so performance loss is more serious and system can't achieve the space diversity gain. In order to take full advantage of the broadcast nature of wireless channel and achieve diversity gain, Boyer proposed multi-hop diversity model in [6] which each relay can detect and receive broadcasting signal from all transmitting nodes before it to achieve diversity gain. In [6-7], outage probability and bit error ratio(BER)of fixed route DF and AF multi-hop diversity system are analyzed respectively, system performance has a significant improvement compared with series link multi-hop relay system. Fixed and selective relay multi-hop diversity system are analyzed and an approximate outage performance are achieved in [8].

With the development of cognitive radio technology, multi-hop cognitive transmission network attracts much attention of scientists over the world. In [9-11], the maximum average throughput of cognitive system is analyzed with low complexity routing and dynamic power control strategy of multi-hop cognitive system in Underlay Sharing(US) mode.

In this paper, we mainly study the transmission performance of multi-hop diversity cognitive system. An approximate outage probability formulation and diversity gain has been derived for DF protocol multi-hop cognitive diversity system in US mode. Due to DF diversity protocol failure of achieving space diversity gain, modified multi-hop SDF diversity protocol is proposed and an approximate outage probability formulation and diversity gain are deduced for SDF cognitive system. Both theory and simulation show that SDF performance has more significant improvement than DF and SDF can achieve the full space diversity gain.

2. System Model

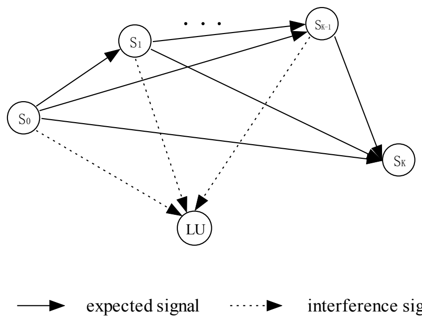

Fig. 1. Multi-hop cognitive system mode

Considering a K-hop cognitive relay system has K-1 relay node as shown in Figure 1, relay $\{S_1, S_2, \cdots, S_{K-1}\}$ providing assistance for cognitive

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source node S_0 to transmit information to destination node S_K. The source node,relay node and destination node are all half-duplex mode and this mode applies to end the *K*-hop communication network, that is, each node can't receive and transmit the same frequency information at the same time. LU is expressed as licensed user. The entire collaborative communication process is divided into *K* stages and cognitive users $\{S_0, S_1, \cdots, S_{K-1}\}$ operating in a fixed TDMA mode for sending data. We assuming that all channels in the system are narrowband frequency non-selective slow fading channel, which channel fading coefficients $h_{n,l}$ follows cyclic symmetry complex Gaussian random distribution with mean 0 and variance $\lambda_{n,l}$.

3. The instantaneous signal-to-noise ratio of multi-hop DF cognitive system

In *m*-th phase$(m = 1,2,\cdots K)$, signal x_{m-1} is transmitted by S_{m-1} node, the received signal of node $S_m, S_{m+1} \cdots S_K$ is given by

$$y_j^{(m-1)} = \sqrt{P_{m-1}} h_{m-1,j} x_{m-1} + n_{m-1,j},\ j = m,\cdots,K \tag{1}$$

where the received signal $y_j^{(m-1)}$ is modeled as data that S_j node receives from S_{m-1} node, P_{m-1} represents transmitted signal from S_{m-1}, $h_{m-1,j}$ denotes channel fading coefficients between S_{m-1} and S_j, $n_{m-1,j}$ is zero-mean and σ_n^2 - variance complex Gaussian noise which are pairwise independent.

Finally S_j combines the received signal from $S_0,\cdots,S_{j-1}$ in maximum ratio, instantaneous signal-to-noise ratio(SNR) of S_m node received by perceived node S_j is

$$\gamma_{k,j} = \frac{P_k \left|h_{k,j}\right|^2}{\sigma_n^2}, k = 0,\cdots, j-1 \tag{2}$$

In order to share frequency spectrum with licensed user, cognitive users cannot interfering with the normal communication of LU, so transmit power P_k of cognitive node S_k needs to satisfy the interference limit of LU, that is $P_k \left|h_{k,p}\right|^2 \le Q$,that is $P_k \left|h_{k,p}\right|^2 \le Q$,where Q is modeled as interference constraints, $h_{k,p}$ denotes channel fading coefficients between S_k and LU .

The above equation is given by

$$\gamma_{k,j} = \frac{\left|h_{k,j}\right|^2}{\left|h_{k,p}\right|^2} \bar{\gamma} \tag{3}$$

where $\bar{\gamma} = Q/\sigma_n^2$ represents average signal-to-noise ratio.

In DF mode, S_j combines the received signal from different stages in maximum ratio, so instantaneous signal-to-noise ratio(SNR) received by perceived node S_j is

$$\gamma_j = \sum_{k=0}^{j-1} \gamma_{k,j} \tag{4}$$

4. The interrupt performance and diversity analysis of multi-hop DF cognitive system

The maximum average mutual information of S_j is

$$I_j = \frac{1}{K} \log\left(1 + \gamma_j\right) \tag{5}$$

Interrupt event occurs in the situation where each relay node can't be decoded correctly in multi-hop path. If P_{out_j} represents outage probability of S_j , the total outage probability of multi-hop cognitive system is

$$P_{out,DF} = 1 - \prod_{j=1}^{K} \left(1 - P_{out_j}\right) \tag{6}$$

Theorem: Under the condition of high signal-to-noise ratio $\left(\bar{\gamma} \to \infty\right)$, the outage probability of cognitive relay system multi-hop DF can be approximately expressed as

$$P_{out,DF} \cong \sum_{j=1}^{K} \frac{\varsigma^j}{j!} \prod_{i=0}^{j-1} \frac{1}{\eta_{i,j}} \tag{7}$$

where, $\varsigma = \left(2^{KR} - 1\right)/\bar{\gamma}$。

Proof: See Appendix.

Definition: Diversity order is given by

$$d = -\lim_{\gamma \to \infty} \frac{\lg P_{out}(\gamma)}{\lg \gamma} \tag{8}$$

Diversity order represents robustness fading of system.

According to(7), the outage probability of multi-hop DF cognitive system can be expressed as

$$P_{out,DF}\left(\bar{\gamma}\right) \cong \sum_{j=1}^{K} \frac{\varsigma^j}{j!} \prod_{i=0}^{j-1} \frac{1}{\eta_{i,j}} = \sum_{j=1}^{K} P_{out,DF}\left(\bar{\gamma}, j\right) \tag{9}$$

where $P_{out,DF}\left(\bar{\gamma}, j\right) = \frac{\varsigma^j}{j!} \prod_{i=0}^{j-1} \frac{1}{\eta_{i,j}}$。

Obviously, if $\bar{\gamma} \to \infty$, $P_{out}\left(\bar{\gamma}, 1\right)$ plays a major role in outage probability expression, while others can be approximately thought as higher order infinitesimal of

$1/\bar{\gamma}$,so the multi-hop DF cognitive system diversity order of fixed gain relay is given by

$$d_{DF} = -\lim_{\bar{\gamma}\to\infty} \frac{\lg P_{out,DF}(\bar{\gamma})}{\lg \bar{\gamma}} = -\lim_{\bar{\gamma}\to\infty} \frac{\lg P_{out,DF}\left(\bar{\gamma},1\right)}{\lg \bar{\gamma}} = 1 \quad (10)$$

5. The interrupt performance and diversity analysis of modified multi-hop SDF cognitive system

In multi-hop DF cognitive system, as each relay node has to decode ,receive, merge, re-encode and resend the information sent by the preceding node ,signal-to noise ratio of each hop has close relation with its preceding component channel , the whole link will interrupt as long as a relay can't decode correctly. In response to this shortcoming, [10-13]raise SDF mode ,if relay node can decode signal successfully, the relay will forward information, otherwise the source node will transmit information directly. This can avoid interrupt event occurring at relay node influencing the performance of the whole system.

In SDF mode, the interrupt event of destination node S_K can be equivalent to

$$\bigcup_{i=0}^{K-1}\left(\hat{I}_i \le R\right) \quad (11)$$

Where $\hat{I}_i$ represents the maximum average mutual information of the destination node when there are i relays can decode signal correctly, that is

$$\hat{I}_i = \frac{1}{K}\log\left[1+\left(K-i\right)\gamma_{0,K} + \sum_{i\in\psi_i}\gamma_{i,K}\right] \quad (12)$$

Where,ψ_i is relay collection which has i relays can decode signal correctly. According to(11)and(12), outage probability of K hop SDF cognitive system can be simplified as

$$\begin{aligned} P_{out,SDF} &= \sum_{i=0}^{K-1}\Pr\left(\hat{I}_i \le R\right) \\ &= \sum_{i=0}^{K-1}\Pr\left(\left(K-i\right)\gamma_{0,K} + \sum_{j\in\psi_i}\gamma_{j,K} \le 2^{KR}-1\right) \quad (13) \\ &= \sum_{i=0}^{K-1}\Pr\left(\left(K-i\right)X_{0,K} + \sum_{j\in\psi_i}X_{j,K} \le \varsigma\right) \end{aligned}$$

Set ψ_i has total $C_{K-1}^{i} = \frac{\left(K-1\right)!}{i!\left(K-i-1\right)!}$ compound modes and different compound modes will achieve different interrupt performances, then

$$\begin{aligned} &\Pr\left(\left(K-i\right)X_{0,K} + \sum_{j\in\psi_i}X_{j,K} \le \varsigma\right) \quad (14) \\ &= \sum_{\psi_i}\Pr\left(\left(K-i\right)X_{0,K} + \sum_{j\in\psi_i}X_{j,K} \le \varsigma \mid \psi_i\right)\Pr\left(\psi_i\right) \end{aligned}$$

In the above formula, $\Pr\left(\psi_i\right)$ represents the probability of relay collection ψ_i and the first item of summation formula represents interrupt probability of relay collection ψ_i which can be divided into

$$\begin{aligned} &\Pr\left(\left(K-i\right)X_{0,K} + \sum_{j\in\psi_i}X_{j,K} \le \varsigma \mid \psi_i\right) \quad (15) \\ &= \frac{\varsigma}{K-i}\int_0^1 \Pr\left(\sum_{j\in\psi_i}X_{j,K} \le \varsigma x \mid \psi_i\right) f_{X_{0,K}}\left(\frac{\varsigma\left(1-x\right)}{K-i}\right)dx \end{aligned}$$

Where $x = 1-\left(K-i\right)X_{0,K}/\varsigma$ 。 While according to the conclusion of (5-36),under the condition of high signal-to-noise ratio,the first item of the above integral can be approximately expressed as

$$\Pr\left(\sum_{j\in\psi_i}X_{j,K} \le \varsigma x \mid \psi_i\right) \cong \frac{\left(\varsigma x\right)^j}{j!}\prod_{j\in\psi_i}\frac{1}{\eta_{j,K}} \quad (16)$$

While

$$\lim_{\varsigma\to 0} f_{X_{0,K}}\left(\frac{\varsigma\left(1-x\right)}{K-i}\right) = \frac{1}{\eta_{0,K}} \quad (17)$$

According to(16)and(17), under the condition of high signal-to noise ratio ,we can obtain the approximate expression of (15),that is

$$\begin{aligned} &\Pr\left(\left(K-i\right)X_{0,K} + \sum_{j\in\psi_i}X_{j,K} \le \varsigma \mid \psi_i\right) \cong \quad (18) \\ &\frac{\varsigma^{i+1}}{\eta_{0,K}\left(K-i\right)\left(i+1\right)!}\prod_{j\in\psi_i}\frac{1}{\eta_{j,K}} \end{aligned}$$

Nextly, we will calculate the probability $\Pr\left(\psi_i\right)$ of relay collection ψ_i .First of all, we analyze the state of each relay node, if relay node $S_k\left(k=1,\cdots,K-1\right)$ wants to participate in cooperation, it must can decode the signal received from the preceding k stages correctly, so its current state mainly depends on all its preceding nodes condition and channel transmission characteristic. The probability of relay node under different states can be expressed as

$$\theta_{k,i} = \begin{cases} \Pr\left(\sum_{l=0}^{k-1}\xi_l X_{l,k} > \varsigma\right), & S_k \in \psi_i \\ \Pr\left(\sum_{l=0}^{k-1}\xi_l X_{l,k} \le \varsigma\right), & S_k \notin \psi_i \end{cases} \quad (19)$$

Where, $\xi_l \in \left\{0,1\right\}\left(l=1,\cdots,k-1\right)$ represent current state of cognitive node S_l . $\xi_l = 1$ represents that S_l can decode the signal correctly, so S_l will participate and forward data in the $l+1$ stage; while $\xi_l = 0$ represents that S_l can't decode the signal correctly and data will be transmitted directly by source node S_0 . $\xi_0 = k - \sum_{l=1}^{k-1}\xi_l$

represents the number of the signal sent directly from the source node of the preceding k stages. Under the condition of independent non-related Rayleigh distribution, we can obtain formula (20)by deducing formula (19)

$$\theta_{k,i}=\begin{cases}1-\dfrac{\varsigma^{\sum_{l=1}^{k-1}\xi_l+1}}{\eta_{0,k}\xi_0\left(\sum_{l=1}^{k-1}\xi_l+1\right)!}\prod_{l\in\psi_i}\dfrac{1}{\eta_{l,k}}, & S_k\in\psi_i\\ \dfrac{\varsigma^{\sum_{l=1}^{k-1}\xi_l+1}}{\eta_{0,k}\xi_0\left(\sum_{l=1}^{k-1}\xi_l+1\right)!}\prod_{l\in\psi_i}\dfrac{1}{\eta_{l,k}}, & S_k\notin\psi_i\end{cases} \tag{20}$$

Therefore ,probability $\Pr(\psi_i)$ is the product of each relay node state probability of the relay collection ψ_i , that is

$$\Pr(\psi_i)=\prod_{k=1}^{K-1}\theta_{k,i} \tag{21}$$

We can draw the following conclusion by substituting formula (18) and (21)into formula (14) Under the condition of high signal-to-noise ratio ,the outage probability of cognitive multi-hop relay system can be approximately expressed as

$$P_{out,SDF}\cong\sum_{i=0}^{K-1}\frac{\varsigma^{i+1}}{\eta_{0,K}(K-i)(i+1)!}\sum_{\psi_i}\prod_{j\in\psi_i}\frac{1}{\eta_{j,K}}\prod_{k=1}^{K-1}\theta_{k,i} \tag{22}$$

According to(22)，assuming that the outage probability of multi-hop SDF cognitive system is $P_{out,SDF}\left(\bar{\gamma}\right)\cong\sum_{i=0}^{K-1}P_{out,SDF}\left(\bar{\gamma},i\right)$, where，

$$P_{out,SDF}\left(\bar{\gamma},i\right)=\frac{\varsigma^{i+1}}{\eta_{0,K}(K-i)(i+1)!}\sum_{\psi_i}\prod_{j\in\psi_i}\frac{1}{\eta_{j,K}}\prod_{k=1}^{K-1}\theta_{k,i}\text{。}$$

If $i=0$,set ψ_i has only one compound mode, that is all relays don't participate in cooperative communications, $P_{out,SDF}\left(\bar{\gamma},0\right)$ and ς^K are positively correlated; while $i\neq 0$,set ψ_i has C_{K-1}^{i} compound modes, $P_{out,SDF}\left(\bar{\gamma},i\right)$ is the sum of function C_{K-1}^{i} ,each function and higher-order function of ς (exponent is greater than K) are positively correlated. When $\bar{\gamma}\to\infty$, $\varsigma\to 0$,minimal exponent number function about variable ς of sum formula will play the major role, and others can be approximately thought of as high-order infinitesimal, while ς the minimal state appears in the situation where all relays can't decode correctly, at the same time the outage probability and $1/\bar{\gamma}^{K}$ are positively correlated, so we can draw the following conclusion:

$$\begin{aligned}d_{SDF}&=-\lim_{\gamma\to\infty}\frac{\lg P_{out,SDF}\left(\bar{\gamma}\right)}{\lg\bar{\gamma}}\\&=-\lim_{\gamma\to\infty}\frac{\lg P_{out,DF}\left(\bar{\gamma},0\right)}{\lg\bar{\gamma}}=K\end{aligned} \tag{23}$$

6. Simulation results and analysis

Build a path loss (not considering the shadow attenuation channel) model for verifying the above theoretical analysis, that is, the channel variance from S_m to

S_n $(m=0,1,\cdots K-1;n=1,\cdots K)$ is

$$\Gamma_{m,n}=d_{m,n}^{-\mu} \tag{24}$$

Where $d_{m,n}$ represents the distance between S_m and S_n , μ represents channel path loss factor.

Simulation parameters are set as the following:

- Using BPSK modulation;
- The coordinate of cognitive source node S_0 and destination node S_K are (0,0) and (12m,0);
- $K-1$ cognitive relays equally spaced between source node and destination node, then the corresponding coordinate of relay S_i is $\left(\frac{12i}{K}\text{m},0\right)$;
- The coordinate of the node received by licensed user is (4m,4m);
- The pass loss factor is $\mu=3$;
- $R=1\text{bit/s/Hz}$.

Figure 2 and 3 shows the outage probability performance of DF and SDF cognitive system respectively under the condition of different average signal-to-noise ratio and different hops. Approximation of figure 2 represents the result derived from theory of formula (7) ;Approximation of figure (3) represents the conclusion of formula (22),and simulation is the result simulated by computer Monte Carlo. We can know from the figure ,the outage probability of cognitive system decreases as the increase of average signal-to-noise ratio. The theoretical scope curve accords with the simulation curve. After the logarithmic coordinates of figure 2 ordinate ,both the down slope of 2-hop and 3-hop are 1/10,so we verify the diversity of DF cognitive system is 1. While the logarithmic coordinates of figure 3 ordinate ,the down slope of 2-hop SDF outage probability is 1/5,and the down slope of 3-hop is 3/10,therefore we verify the diversity of SDF cognitive system is K .At the same time, comparing figure 2 with 3 ,we can know that the SDF interrupt performance of multi-hop cognitive system improves more obviously than DF. For example, when the system outage probability is 10^{-2}, the signal-to-noise ratio of 3-hop SDF cognitive system improves more obviously with almost 4.3dB, while the signal-to-noise ratio of 2-hop SDF cognitive system improves more obviously with almost 5.2dB.

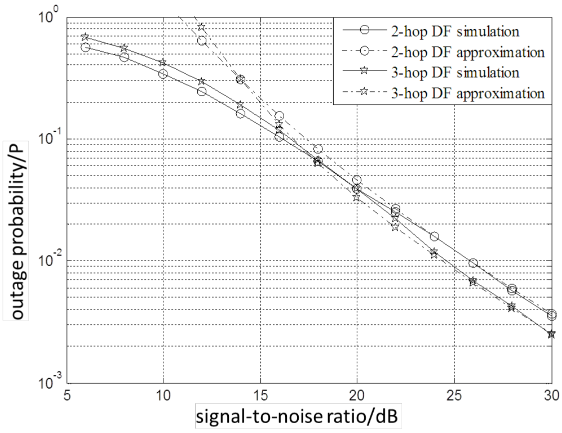

Fig. 2. Interrupt performance of multi-hop DF cognitive relay system

8. Conclusions

In this text ,the interrupt performance of multi-hop DF wireless cognitive system in US mode has been studied and an approximate outage probability formulation of DF cognitive system in high signal-to-noise ratio has been derived .If there is a relay of multi-hop DF mode not decoding correctly, it will lead to the system performance decrease of all links, and it can't achieve the full space diversity gain .In response to this shortcoming ,an approximate outage probability formulation of SDF cognitive system in improved high signal-to-noise ratio has been derived .Though simulation comparison, we can know SDF can achieve the full space diversity and the system performance has been greatly improved contrasted with DF protocol.

9. Acknowledge

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Appendix

The outage probability of S_j node is

$$\begin{aligned} P_{out_j} &= \Pr\left(I_j \le R\right) \\ &= \Pr\left(\sum_{k=0}^{j-1}\gamma_{k,j} \le 2^{KR}-1\right) \\ &= \Pr\left(\sum_{k=0}^{j-1}X_{k,j} \le \varsigma\right) \end{aligned} \tag{25}$$

where $X_{k,j} = \left|h_{k,j}\right|^2 / \left|h_{k,p}\right|^2$,

$\varsigma = \left(2^{KR}-1\right)/\bar{\gamma}$ .Considering $Y_j = \sum_{k=0}^{j-1}X_{k,j}$,

according to [15,17], (25) can be simplified as

$$\begin{aligned} \Pr\left(\sum_{k=0}^{j-1}X_{k,j} \le \varsigma\right) &= \Pr\left(Y_j \le \varsigma\right) \\ &= \int_0^{\varsigma} f_{Y_j}(x)dx = \varsigma\int_0^1 f_{Y_j}(\varsigma s)ds \end{aligned} \tag{26}$$

Where $s = x/\varsigma$, $f_Z(x)$ is the probability density function of function Z, where $Y_j = \sum_{k=0}^{j-2}X_{k,j} + X_{j-1,j}$, According to probability theory, the probability density function $f_{Y_j}(x)$ can be

$$\begin{aligned} f_{Y_j}(x) &= \int_0^x f_{\sum_{k=0}^{j-2}X_{k,j}}(x-r)f_{X_{j-1,j}}(r)dr \\ &= x\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[x(1-y)\right]f_{X_{j-1,j}}(xy)dy \end{aligned} \tag{27}$$

w here $y = r/x$。

Under the condition of high signal-to-noise ratio $\left(\bar{\gamma}\to\infty\right)$, $\varsigma\to 0$.

According to formula(25)and(26)，we can know

$$\lim_{\varsigma\to0}\frac{P_{out_j}}{\varsigma^j} = \lim_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds \tag{28}$$

First of all, try to obtain the lower bound of $\lim_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds$, according to Famous lemma [16]

We can know

$$\liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds \ge \int_0^1 \liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}f_{Y_j}(\varsigma s)ds \tag{29}$$

According to(27)and(29)，we can know

$$\begin{aligned} &\int_0^1 \liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}f_{Y_j}(\varsigma s)ds \\ &= \int_0^1 \liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}\left\{\varsigma s\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right]\right. \\ &\left. f_{X_{j-1,j}}(\varsigma s y_1)dy_1\right\}ds \\ &= \int_0^1 s\liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-2}}\left\{\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right] f_{X_{j-1,j}}(\varsigma s y_1)dy_1\right\}ds \\ &\ge \int_0^1 s\int_0^1\left\{\liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-2}}f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right]\right. \\ &\left.\liminf_{\varsigma\to0}f_{X_{j-1,j}}(\varsigma s y_1)\right\}dy_1 ds \end{aligned} \tag{30}$$

According to[14],we can know $f_{X_{n,l}}(x) = \frac{\eta_{n,l}}{\left(x+\eta_{n,l}\right)^2}$,

$\eta_{n,l} = \lambda_{n,l}/\lambda_{n,p}$。

So, $\liminf_{\varsigma\to0}f_{X_{j-1,j}}(\varsigma s y_1) = \frac{1}{\eta_{j-1,j}}$ (31)

Substitute (31) into (30) ,we can know

$$\begin{aligned} &\int_0^1 \liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-1}}f_{Y_j}(\varsigma s)ds \ge \\ &\frac{1}{\eta_{j-1,j}}\int_0^1 s\left\{\liminf_{\varsigma\to0}\frac{1}{\varsigma^{j-2}}\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right]dy_1\right\}ds \end{aligned} \tag{32}$$

And so on, we can finally get

$$\liminf_{\varsigma\to 0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds \geq \prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}}\int_0^1 s^{j-1}\int_0^1(1-y_1)^{j-2}\cdots\int_0^1(1-y_{j-1})^0 dy_{j-1}\cdots dy_1 ds = \frac{1}{j!}\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}} \tag{33}$$

While for the upper bound of $\lim_{\varsigma\to 0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds$

$$\begin{aligned}&\limsup_{\varsigma\to 0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds\\ &= \limsup_{\varsigma\to 0}\frac{1}{\varsigma^{j-2}}\int_0^1 s\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right]f_{X_{j-1,j}}(\varsigma s y_1)dy_1 ds\\ &\leq \limsup_{\varsigma\to 0}\frac{1}{\varsigma^{j-2}}\int_0^1 s\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right]\\ &\quad\times\limsup_{\varsigma\to 0} f_{X_{j-1,j}}(\varsigma s y_1)dy_1 ds\end{aligned} \tag{34}$$

While according to $f_{X_{n,l}}(x)=\frac{\eta_{n,l}}{\left(x+\eta_{n,l}\right)^2}\leq\frac{1}{\eta_{n,l}}$, the following formula can be changed into

$$\begin{aligned}&\limsup_{\varsigma\to 0}\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds\\ &\leq\frac{1}{\eta_{j-1,j}}\limsup_{\varsigma\to 0}\frac{1}{\varsigma^{j-2}}\int_0^1 s\int_0^1 f_{\sum_{k=0}^{j-2}X_{k,j}}\left[\varsigma s(1-y_1)\right]dy_1 ds\\ &\leq\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}}\int_0^1 s^{j-1}\int_0^1(1-y_1)^{j-2}\cdots\int_0^1(1-y_{j-1})^0 dy_{j-1}\cdots dy_1 ds\\ &=\frac{1}{j!}\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}}\end{aligned} \tag{35}$$

Combine formula(33)and(35) , we can draw the following conclusion:

In high signal-to-noise ratio,

$$\frac{1}{\varsigma^{j-1}}\int_0^1 f_{Y_j}(\varsigma s)ds \cong \frac{1}{j!}\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}} \tag{36}$$

So the interrupt probability of S_j is

$$P_{out_j}=\frac{\varsigma^j}{j!}\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}} \tag{37}$$

Substitute (37) into (6) ,we can get:

Under the condition of high signal-to-noise ratio $\left(\bar{\gamma}\to\infty\right)$, the interrupt probability of multi-hop DF cognitive relay system can be approximately expressed as

$$P_{out,DF}\cong 1-\prod_{j=1}^{K}\left(1-\frac{\varsigma^j}{j!}\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}}\right) \tag{38}$$

Because of $\varsigma\to 0$, (38)can be further approximately expressed as

$$P_{out,DF}\cong\sum_{j=1}^{K}\frac{\varsigma^j}{j!}\prod_{i=0}^{j-1}\frac{1}{\eta_{i,j}} \tag{39}$$

References

1. Wu H, Qiao C, Tonguz O K. “A new generation wireless system with integrated cellular and mobile relaying technologies,” *In Proc. of WAS’2000*, Dec. 2000, pp.55-62.
2. Hasna M O, Alouini M-S. “Outage probability of multihop transmissions over Nakagami fading channels,” *In Proc. of the IEEE ISWC’2002*, Sardinia, Italy, Sep.2002, pp.207-208.
3. Hasna M O, Alouini M S. “A performance study of dual-hop transmissions with fixed gain relays,” *IEEE Transactions on Wireless Communications*, vol. 3, no. 6, Nov. 2004, pp.1963-1968.
4. Lin Y, Hasna M O, Alouini M-S. “Average outage duration of multihop communication systems with regenerative relays,” *IEEE Transactions on Wireless Communications*, vol.4, no.4, July. 2005, pp.1366-1371.
5. Karagiannidis G K. “Performance bounds of multihop wireless communications with blind relays over generalized fading channels,” *IEEE Transactions on Wireless Communications*, vol.5, no.3, March. 2006, pp. 498-503.
6. Boyer J, Falconer D D, and Yanikomeroglu H, “Multi-hop diversity in wireless relaying channels,” *IEEE Transactions on Wireless Communications*, vol. 52, Oct. 2004, pp. 1820-1830.
7. Boyer J, “Multihop wireless communications channels,” *Master’s thesis*, Carleton University, Ottawa, ON, Canada, 2001.
8. Golnaz F, Norman C. B, “Fixed relaying versus selective relaying in multi-hop diversity transmission system,” *IEEE Transactions on Communications*, vol.58, no.3, Mar. 2010, pp.956-965.
9. Shi Y, Hou Y T, “A distributed optimization algorithm for multi-hop cognitive radio networks,” *In Proc. of the IEEE INFOCOM ’2008*, Phoenix, AZ, USA, Apr.2008, pp.1292-1300.
10. Zhenquan Qin, Wenjing Liu, Huizhen Jiang, Weifeng Sun. "A diversity-based multi-antenna cooperative detection model in cognitive radio network", Journal of Digital Information Management, vol.11, n3,June 2013, pp. 220-226.
11. Ruan L Z, Lau K N, “Decentralized dynamic hop selection and power control in cognitive multi-hop relay systems,” *IEEE Transactions on Wireless Communications*, vol.9, no.10, October. 2010, pp.3024-3030.
12. Aitmagambetov A.Z., Jakipbayeva Z.M., "Cognitive radio networks is the next step in communication technology" Computer Modelling and New Technologies, vol.16, n3,2012, pp. 63-67.
13. Laneman J N and Wornell G W, "Cooperative diversity in wireless networks: efficient protocols and outage behavior", IEEE *Transactions on Information Theory*, vol. 50, no. 12, Dec. 2004, pp.3062-3080.
14. Farhadi G and Beaulieu N C，“Fixed relaying versus selective relaying in multi-hop diversity transmission,” *IEEE Transactions on Wireless Communications*, vol. 58, no. 3, Mar. 2010 , pp.956-965.
15. Ekpenyong M.E., Eromosele G., Isabona J., "On the performance modeling of outage probability in CDMA wireless networks”, Journal of Engineering Science and Technology Review, vol.4, no.2, 2011, pp. 174-182.
16. Laneman J N and Wornell G W，"Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks", *IEEE Transactions on Information Theory*, vol. 49, no. 10, Oct. 2003, pp.2414-2425.
17. Tang Jingmin，Luo tao，Le Guangxin，“Study of the cognitive system performance based on relay selection,” *Beijing University of Posts and Telecommunications*, Vol.34 (3), 2011.