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Research Article

Nonlinear dynamics and load sharing of double-mesh helical gear train

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Abstract

Dynamic behaviours and load sharing of double-mesh helical gear trains have been studied in this paper. A nonlinear dynamic model of double-mesh helical gear train was established, including torsional vibration, axial vibration, time-varying mesh stiffness and backlashes. The governing equations were solved by Runge-Kutta method. Frequency responses and influences of main parameters to frequency responses were analyzed to determine the vibration characteristics of the system. The dynamic factor and load sharing in two mesh pairs were studied in two configuration cases and the result shows that case II has better dynamic response and load sharing.

Keywords: Double-mesh helical gear; nonlinear dynamics; backlash; load sharing

1. Introduction

Helical gears are one of the most important components in many mechanical transmission systems for its advantages such as smooth transmission, high carrying capacity and rarely tooth interference. The disadvantage of helical gear is the big axial force. However, axial force could be eliminated in double-mesh helical gear trains, so they are widely used in high power density systems such as propulsion systems of rotorcrafts and warships. Since their performances heavily affect the fatigue life and concealment of the transmission system, it's quite necessary to research on dynamic characteristics of double-mesh helical gear trains.

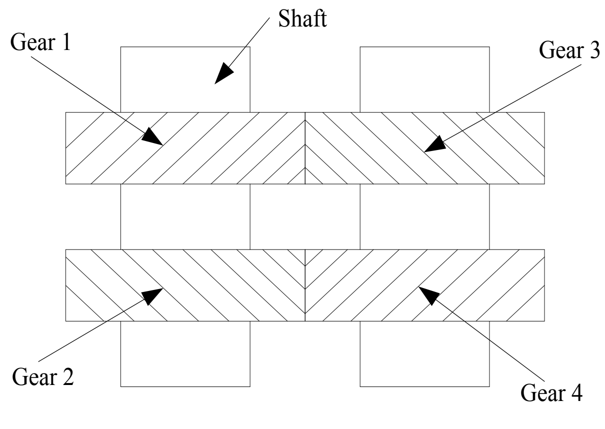

Fig.1 Double-mesh helical gear train

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Most of the literatures [1,2,3,4] were limited to a single-mesh gear system. Choi et al. [1] investigated the dynamics of a single helical gear and studied the forced vibration and eigenvalue. Blankenship et al. [2] established a dynamic model of helical gear and analyzed the influence of transmission error on its vibration response. Amezketa et al. [3] established a nonlinear model of a helical gear pair involving backlash and angel-varying mesh stiffness. Kahraman [4] investigated the effect of axial vibration to a helical gear pair. A few literatures [5,6] studied multi-mesh helical gears. Kahraman [5] studied the dynamic behavior of a multi-mesh single helical gear train, analyzed natural modes, sensitivity and forced responses. Li et al. [6] studied structure vibration characteristics of two stage double helical tooth planetary gear trains. Double helical gears were also studied by few researches [7,8]. Handschuh et al. [7] tested transmission performances of double helical gears using high speed gearing systems of tiltrotor aircraft. Wang et al. [8] established a model for double helical gear transmission and studied the dynamic behavior, but the model ignored torsional displacement of adjacent gears, time-varying mesh stiffness and backlash which are important to dynamic characteristics. Helical gears have been studied a lot, but research on vibration characteristics of double-mesh helical gear trains is obviously inadequate, especially its nonlinear vibration properties and load sharing.

In this paper, a nonlinear dynamic model of double-mesh helical gear train (as shown in Fig.1) was established, including torsional vibration, axial vibration, time-varying mesh stiffness and backlashes. The specific objectives of this study are (i) to develop a nonlinear dynamic model, (ii) to investigate characteristics of nonlinear dynamic responses, (iii) to analyze influences of parameters to the frequency responses and (iv) to investigate the load sharing between two mesh pairs.

2. Mesh stiffness, backlash and damping

2.1 Mesh stiffness

The time varying mesh stiffness is periodic according to the mesh frequency f_m. The time varying mesh stiffness can be expressed as a periodic square wave function [10] as shown in Fig.2. The periodic square wave function can be expressed as Fourier series. Since the first few series are enough for the computation precision, time varying mesh stiffness in Fig.2 are defined by

$$k_i(t) = k_{ai} + \sum_{j=1}^{N}\left(k_{vj}\sin(\omega_j t + \varphi_j)\right) \tag{1}$$

where $k_i(t)$ is time varying mesh stiffness of the ith pair teeth, k_{ai} is the average value of mesh stiffness, k_{vi} is varying part of the mesh stiffness amplitude, ω_j and φ_j are the frequencies of the jth Fourier series, respectively, N is the number of the chosen Fourier series, κ is defined as time varying stiffness (TVS) coefficient and $\kappa_i = k_{vi}/k_{ai}$, i=1, 2, j=1, 2, …, N.

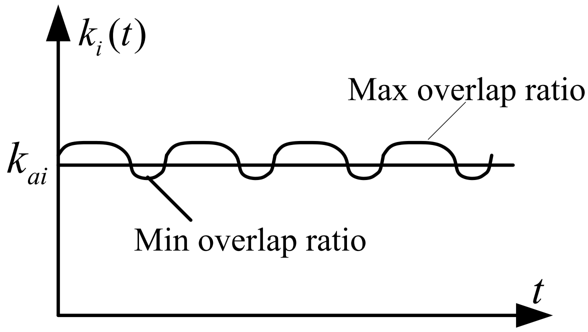


Fig.2 Time-varying mesh stiffness of a helical gear pair

2.2 Backlash and damping

$f(u_i)$ is a piecewise nonlinear function having three domains. Each teeth contact is defined by three cases of teeth configurations and they are shown in Fig.3. $f(u_i)$ is expressed as

$$f(u_i) = \begin{cases} u_i - w_i, \; u_i > w_i \\ 0, \quad -w_i < u_i < w_i \\ u_i + w_i, \; u_i < -w_i \end{cases} \tag{2}$$

where u_i and w_i are the mesh displacement and the half backlash between the contact teeth, respectively.

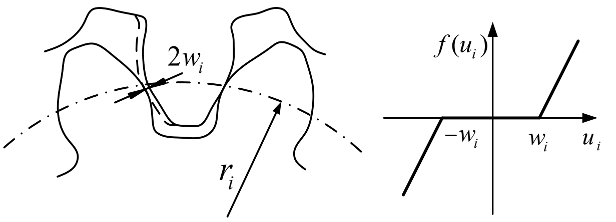


Fig.3 Backlash and its piecewise function

The viscous damping was considered to reduce the level of the vibrations. And the expression of the equivalent viscous damping follows reference [9].

3. Dynamic modeling

The double-mesh helical gear train is shown in Fig.4 including four gears a_1, a_2, b_1, b_2. Two adjacent helical gears (a_1 and a_2) connect together in reverse helical direction having a torsional displacement. They mesh with another two adjacent helical gears (b_1 and b_2) respectively. The input torque is applied to gear a_1 and load torque is applied to gear b_1.

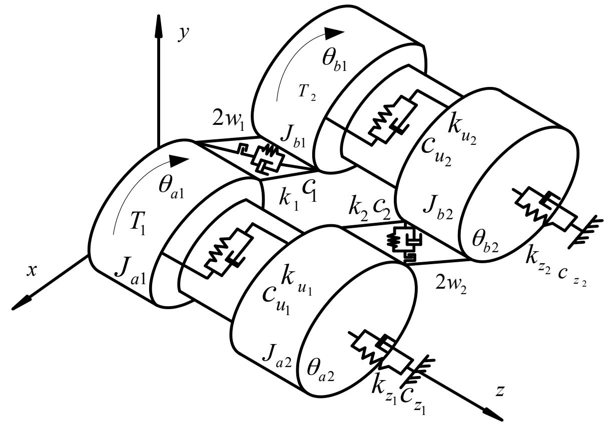


Fig.4 Dynamic model of double-mesh helical gear train

Two adjacent gears were considered as one in z direction. The clockwise rotation direction of each helical gear was assigned as positive direction. Positive directions of mesh displacements, torsional displacements and z-axis displacements were assigned as spring compression direction.

The governing equations of the system are as follows:

$$\begin{aligned}
&J_{a1}\ddot{\theta}_{a1} + c'_{u1}(\dot{\theta}_{a1} - \dot{\theta}_{a2}) + k'_{u1}(\theta_{a1} - \theta_{a2}) + r_1(c_1\dot{f}(u_1) \\
&\qquad + k_1(t)f(u_1))\cos\beta = T_1 \\
&J_{a2}\ddot{\theta}_{a2} - c'_{u1}(\dot{\theta}_{a1} - \dot{\theta}_{a2}) - k'_{u1}(\theta_{a1} - \theta_{a2}) + r_1(c_2\dot{f}(u_2) \\
&\qquad + k_2(t)f(u_2))\cos\beta = 0 \\
&J_{b1}\ddot{\theta}_{b1} + c'_{u2}(\dot{\theta}_{b1} - \dot{\theta}_{b2}) + k'_{u2}(\theta_{b1} - \theta_{b2}) + r_2(c_1\dot{f}(u_1) \\
&\qquad + k_1(t)f(u_1))\cos\beta = T_2 \\
&J_{b2}\ddot{\theta}_{b2} - c'_{u2}(\dot{\theta}_{b1} - \dot{\theta}_{b2}) - k'_{u2}(\theta_{b1} - \theta_{b2}) + r_2(c_2\dot{f}(u_2) \\
&\qquad + k_2(t)f(u_2))\cos\beta = 0 \\
&m_1\ddot{z}_1 + c_{z1}\dot{z}_1 + k_{z1}z_1 + (c_1\dot{f}(u_1) + k_1(t)f(u_1))\sin\beta \\
&\qquad - (c_2\dot{f}(u_2) + k_2(t)f(u_2))\sin\beta = 0 \\
&m_2\ddot{z}_2 + c_{z2}\dot{z}_2 + k_{z2}z_2 - (c_1\dot{f}(u_1) + k_1(t)f(u_1))\sin\beta \\
&\qquad + (c_2\dot{f}(u_2) + k_2(t)f(u_2))\sin\beta = 0
\end{aligned} \tag{3}$$

where J represents moment of inertia for each gear by different subscript, θ represents angular displacement of each gear by different subscript, c'_{u1} and c'_{u2} represent damping of the two group of adjacent gears, k'_{u1} and k'_{u2} represent stiffness between adjacent gears, r_1 and r_2 represent radius of mesh gear pair respectively, c_1 and c_2 represent mesh damping of two mesh gear pairs respectively,

k_1 and k_2 represent mesh stiffness of two mesh gear pairs respectively, β is helical angle, z and m represent z-axis displacements and masses of two groups of adjacent gears by different subscript, c_z and k_z represent z-axis damping and stiffness of two group of gears by different subscript, respectively.

The nonlinear dynamic model above includes 6 degrees of freedom. Two teeth deflection u_1 and u_2 were defined as displacements of each mesh gear pair and two relative deflection u_3 and u_4 were defined as displacements of the adjacent gears, and $u_1=\theta_{a1}r_1\cos\beta+\theta_{b1}r_2\cos\beta+z_1\sin\beta-z_2\sin\beta$, $u_2=\theta_{a2}r_1\cos\beta+\theta_{b2}r_2\cos\beta-z_1\sin\beta+z_2\sin\beta$, $u_3=r_1(\theta_{a1}-\theta_{a2})$, $u_4=r_2(\theta_{b1}-\theta_{b2})$.

Then, Eq.(3) was deduced by u_1, u_2, u_3 and u_4 and its dimensionless form was used for a numerical simulation with time dimension $t=\omega_h t'$ and size dimension w_1, where $\omega_h=\sqrt{k_{a1}/2m_{a1}}$ and w_1 is half of the backlash between gears a_1 and b_1.

4 Results and discussion

Four order Runge-Kutta algorithm was used to solve the strong nonlinear dimensionless dynamic equations. Main parameters of the example system are listed in Table 1.

Table 1: Parameters of double-mesh helical gear train

Name	Value
Helical angle [°]	β=15
Number of teeth	Z_{a1}=55, Z_{a2}=55, Z_{b1}=55, Z_{b2}=55
Modulus [mm]	m=4
Stiffness [N/m]	$k_a=5\times10^8$, $k_u=k_z=1\times10^9$
Moment of inertia [kgm^2]	J_{a1}=0.09, J_{a2}=0.09, J_{b1}=0.09, J_{b2}=0.09
Radius [mm]	r_1=100; r_2=100
Mass[Kg]	m_1=36, m_2=36

4.1 Nonlinear frequency responses

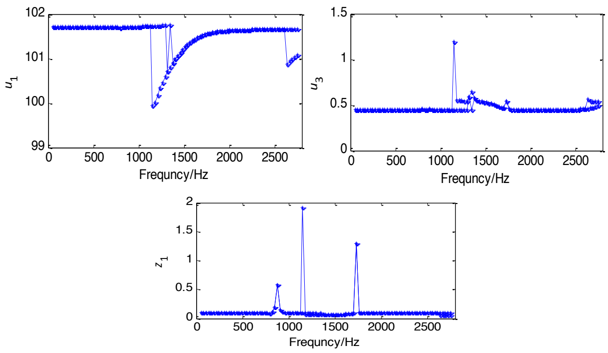


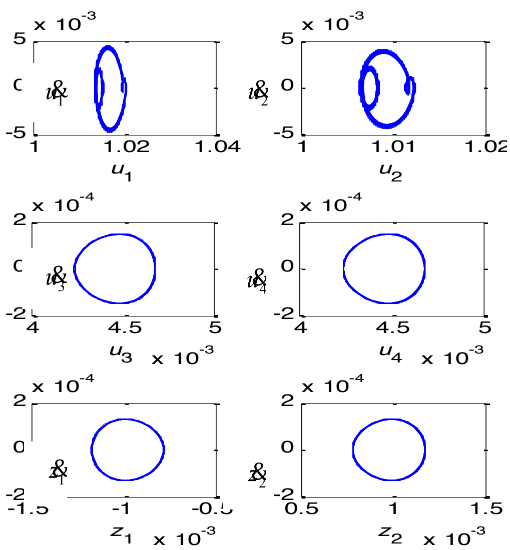
Fig.5 Frequency responses in condition $w_1=w_2$=0.08mm, $T_1=T_2$=100Nm, $\kappa_1=\kappa_2$=0.2

Fig.5 is frequency responses in the conditions $w_1=w_2$= 0.08mm, $T_1=T_2$=100Nm, $\kappa_1=\kappa_2$=0.2. Multi-value and amplitude jump exist in frequency responses, which are the typical characteristics of nonlinear vibration. The domain of multi-value is about 1125Hz to 1730Hz and 2610Hz to 2750Hz in u_1, u_2, which means the nonlinear vibration area and the other frequency domains mean the linear vibration areas. The frequency responses of u_3, u_4, z_1 and z_2 also have multi-value and amplitude jump phenomenon caused by the coupling with the mesh pairs for their backlashes. Since the same type displacements have the similar vibration characteristics, Fig.5 only shows the displacements of u_1, u_3 and z_1.

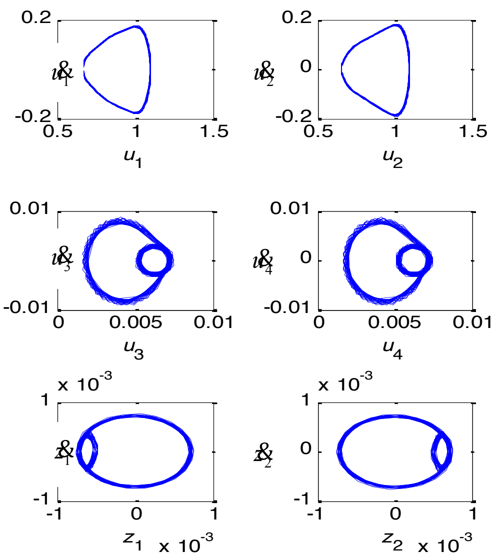
Fig.6a are phase diagrams corresponding to mesh frequencies 550Hz and 1155Hz. Two mesh pairs are both in contact status when there is no amplitude jump as shown in Fig.5 and tooth separation appears in Fig.6b when there is amplitude jump. Vibration amplitudes of torsional displacements in Fig.6b are all larger than those in Fig.6a, which means the nonlinear vibration increases the torsional vibration between the adjacent gears in the same shaft. Otherwise, the two axial displacements vibrate inversely in both Fig.6a and Fig.6b, and they vibrate in the same direction in Fig.6a but in bi-directions in Fig.6b which will result in the axial alternating loads in the mesh pairs and axial bearings. The tooth separation will cause impact between mesh pairs and result in large vibration and extra noise. The dynamic factors in nonlinear area are much larger than those in linear area. The maximum dynamic factor in mesh pair 1 in mesh frequency 550Hz are 8.1, while it's only 1.63 in mesh frequency 1155Hz. So designers should try to avoid nonlinear vibration area to reduce the dynamic load.

4.2 Effect of main parameters

Since the parameters affect the same type displacements similarly, only u_1, u_3 and u_5 (corresponding to z_1) are illustrated in the following analysis. Fig.7a shows the influence of helical angle to frequency responses. The increase of helical angle will lower the amplitude jump frequency of mesh pairs but change little. The bigger helical angle could result in much more amplitude jump to the torsional and axial displacements. In high frequency area (>2700Hz), the big helical gear will cause very complex nonlinear vibration in mesh pairs. Fig.7b shows that big axial stiffness could restrain the axial and torsional vibrations significantly, but have less influence to the vibrations of mesh pairs. In the area of about 900Hz to 1600Hz, nonlinear vibration areas in mesh pairs will reduce, but the amplitudes in high frequency area (bigger than 2700Hz) change a lot. With the increase of axial stiffness, it has less influence to the torsional vibrations but has great influence to the axial vibrations. Torsional stiffness has the same influence to the frequency responses as axial stiffness. Fig.8a shows that big TVS coefficient could increase the nonlinear areas and its amplitude greatly, but almost have no influence to the linear areas.



(a)



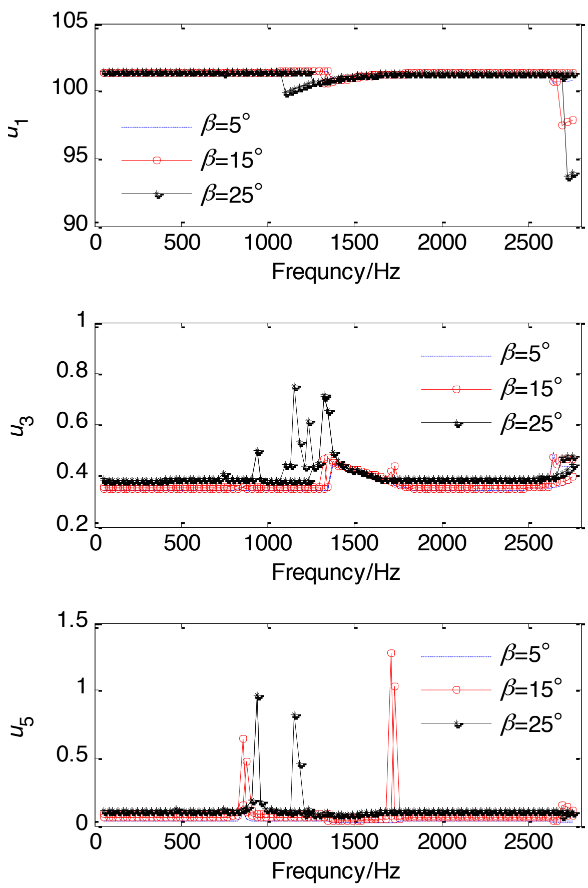
(b)

Fig.6 Phase diagram in mesh frequency (a) 550Hz, (b) 1155Hz

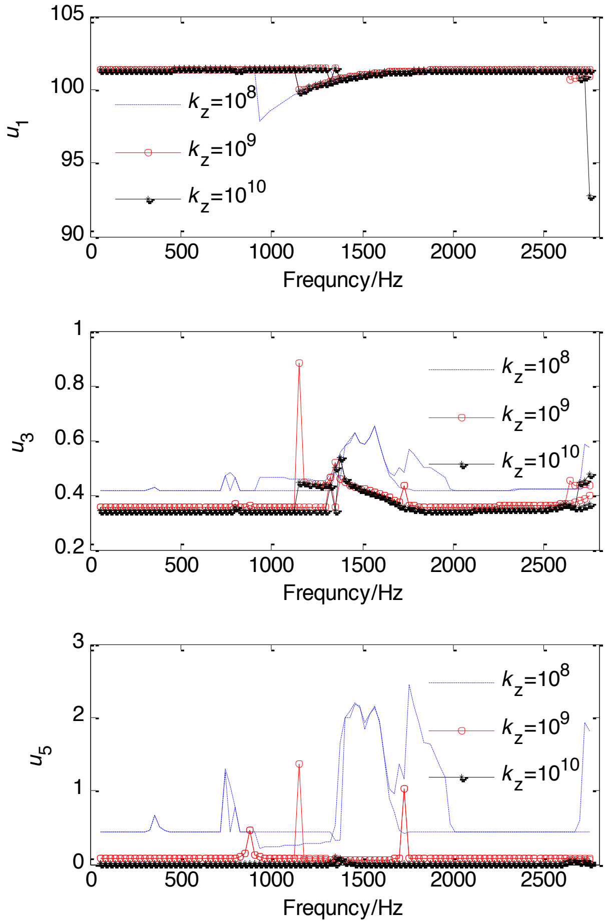
Fig.8b shows influences of backlash to frequency responses. It could observe that although backlashes increase, the amplitudes decrease instead. This is because the amplitude is dimensionless, the absolute amplitude of all displacements increase actually. This means the big backlash could result in heavy vibrations.

4.3 Load sharing

There are two cases for the input and output arrangement in Fig.4. Case I is shown in Fig.4 taking a_1 as input and b_1 as output; case II makes a_1 as input and b_2 as output.



(a)

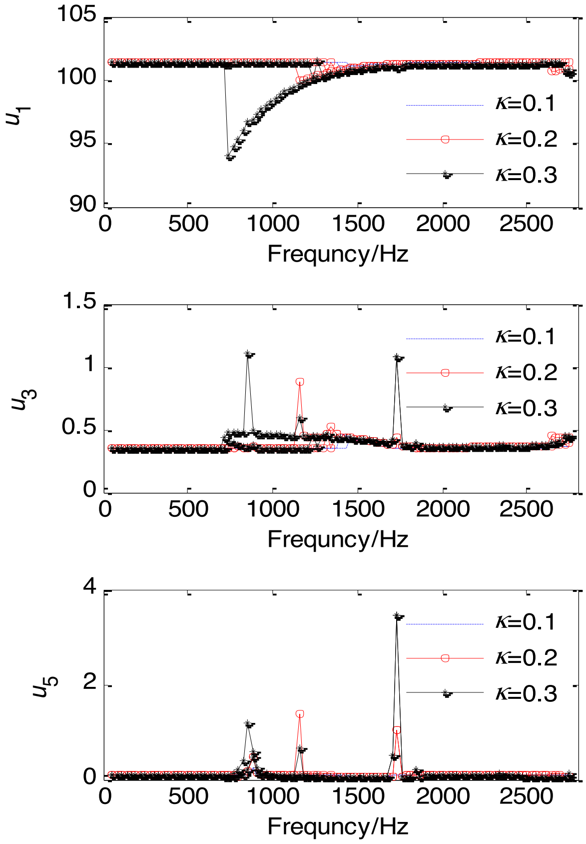


(b)

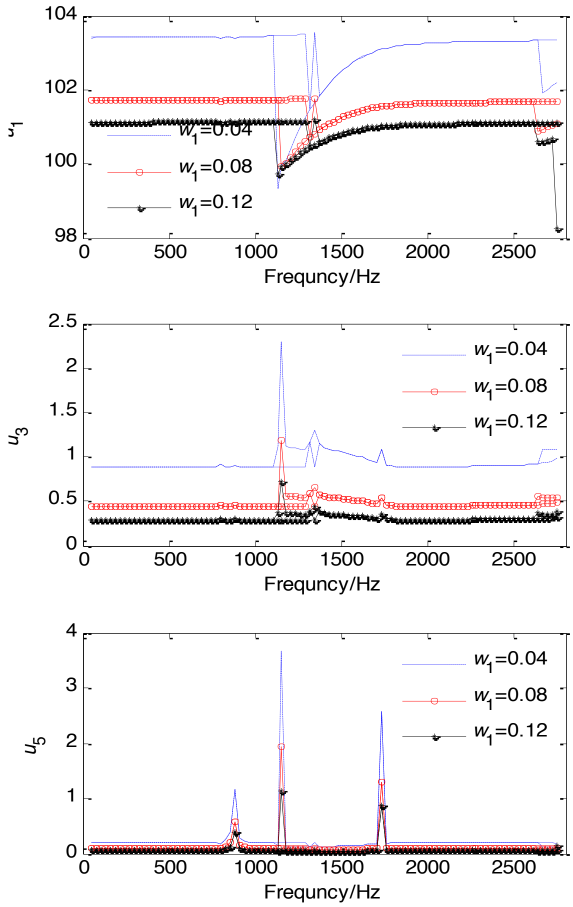
Fig.7 Influence of (a) helical angle and (b) axial stiffness in the conditions w_1=w_2=0.08mm, T_1=T_2=100Nm, κ_1=κ_2=0.2

Fig.9a and Fig.9b are dynamic factors in linear (550Hz) and nonlinear areas (1155Hz) in case I using the parameters as Fig.5. The dynamic factors in mesh pair 1 is much bigger than those in mesh pair 2 in linear vibration status as show in Fig.9a and they are almost equal in nonlinear vibration status as shown in Fig.9b. The maximum dynamic factors in u_1 and u_2 are1.63 and 0.98 in Fig.9a and they are 8.1 and 7.83 in Fig.9b. So the dynamic load in mesh pair 1 is always bigger than that in mesh pair 2 no matter the system vibrates in linear or nonlinear status. The load sharing between u_1 and u_2 is worse in case I especially in linear vibration status.

Fig.9c and Fig.9d are dynamic factors in linear (550Hz) and nonlinear areas (1155Hz) in case II using the parameters as Fig.5. The dynamic factors are equal in tow mesh pairs 1 in both linear and nonlinear areas and the maximum dynamic factors in Fig.9c and Fig.9d are 1.32 and 8.2 respectively. This means load sharing in case II is better than that in case I and dynamic load in linear area in case II is smaller than that in case I. So the designers should choose case II other than case I to improve the load sharing and reduce the dynamic load on the teeth.



(a)

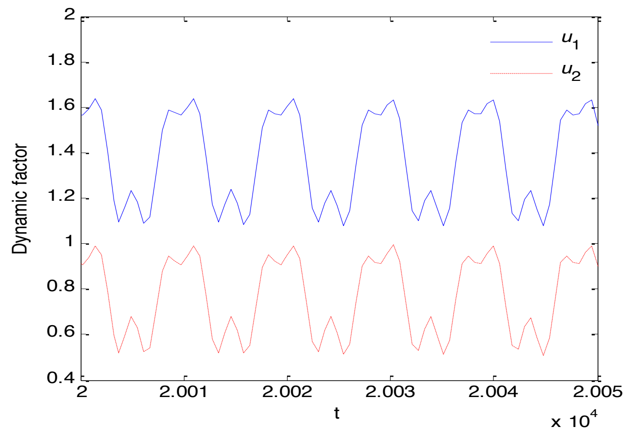


(b)

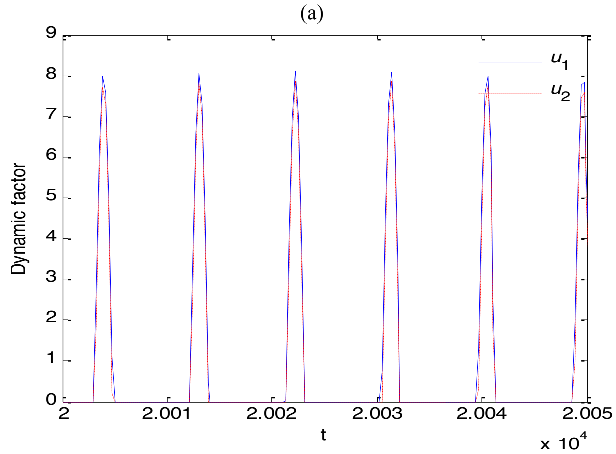
Fig.8 Influence of (a) TVS coefficient in w_1=w_2=0.08mm, T_1=T_2 =100Nm, (b) backlash in T_1=T_2=100Nm, κ_1=κ_2=0.2

5. Conclusion

A nonlinear dynamic model of double-mesh helical gear train was established. Simulation results showed that there are linear and nonlinear areas in the frequency responses of the system. In nonlinear area mesh pairs have tooth separation and impact and dynamic load is much larger than that in linear area. Axial displacements are reverse and vibrate in bi-directions. Big helical angle will increase nonlinear area and cause heavy nonlinear vibration in high frequency area; both axial stiffness and torsional stiffness could restrained the axial and torsional vibrations when they increase; big TVS coefficient could increase nonlinear area and vibration amplitudes greatly, but have no influence to linear area; larger backlash will cause heavier nonlinear vibrations. The dynamic factors in nonlinear status are much bigger than that in linear status. The load sharing and dynamic factors are much better in case II than that in case I.



(a)



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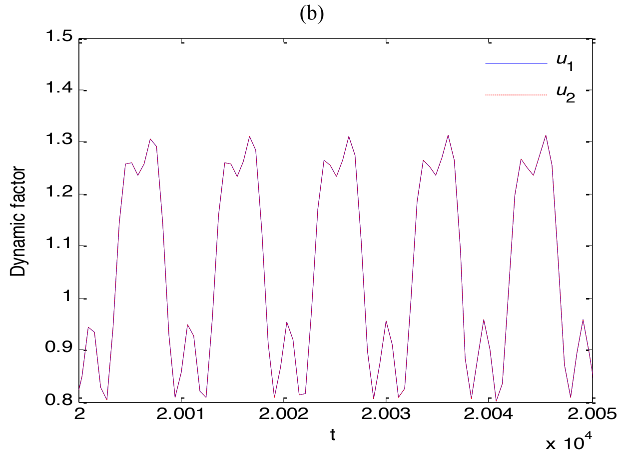
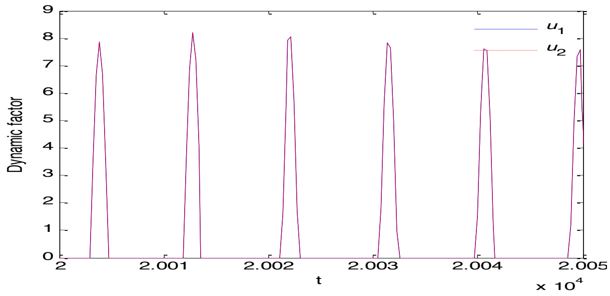


(c)



(d)

Fig. 9 Dynamic factor in mesh pairs (a)550Hz in case I, (b)1155Hz in case I, (c)550Hz in case II, (d)1155Hz in case II

6 Acknowledgements

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