An Effective Simulation Model to Predict and Optimize the Performance of Single and Double Glaze Flat-Plate Solar Collector Designs

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Abstract

This paper outlines and formulates a compact and effective simulation model, which predicts the performance of single and double glaze flat-plate collector. The model uses an elaborated iterative simulation algorithm and provides the collector top losses, the glass covers temperatures, the collector absorber temperature, the collector fluid outlet temperature, the system efficiency, and the thermal gain for any operational and environmental conditions. It is a numerical approach based on simultaneous guesses for the three temperatures, \( T_p \), plate collector temperature and the temperatures of the two glass covers \( T_{g1} \), \( T_{g2} \). A set of energy balance equations is developed which allows for structured iteration modes whose results converge very fast and provide the values of any quantity which concerns the steady state performance profile of any flat-plate collector design. Comparison of the results obtained by this model for flat-plate collectors, single or double glaze, with those obtained by using the Klein formula, as well as the results provided by other researchers, is presented.

Keywords: solar collector simulation, double glaze, losses coefficient, performance prediction, optimization

Nomenclature

- \( A_c \) collector area (m\(^2\))
- \( C_b \) bond conductance
- \( D_i \) tube inner diameter (m)
- \( D_o \) tube outer diameter (m)
- \( F \) fin efficiency for straight fins with rectangular or tubular profile
- \( F' \) collector efficiency factor
- \( F_R \) collector heat removal factor
- \( I_T \) global solar radiation intensity on the collector (W/m\(^2\))
- \( Nu \) Nusselt number (dimensionless)
- \( Pr \) Prandtl number of the air inside the collector (dimensionless)
- \( Q_o' \) heat (rate) gain of the collector panel normalized to its surface (W/m\(^2\))
- \( Ra \) Raleigh number (dimensionless)
- \( Ra' \) Raleigh number, generalized for tilted layers; \( Ra' = Ra \cos(\beta) \) (dimensionless)
- \( T_a \) ambient temperature (K)
- \( U_L \) total energy losses coefficient (W/m\(^2\)K)
- \( U_t \) top heat losses coefficient (W/m\(^2\)K)
- \( T_{f,i} \) fluid inlet temperature (K)
- \( T_{f,o} \) fluid outlet temperature (K)
- \( T_g \) glass cover temperature (K)
- \( T_p \) absorber plate temperature (K)
- \( T_{p,m} \) absorber plate mean temperature (K)
- \( T_s \) equivalent black body sky temperature (K)
- \( W \) distance between the fluid tubes (m)
- \( c_p \) specific heat capacity of the fluid in the collector tubes, J/kgK
- \( h_{f,j} \) fluid heat transfer coefficient inside the tube (W/m\(^2\)K)
- \( h_{P-G1} \) convective heat transfer coefficient between absorber plate and glass cover (W/m\(^2\)K)
- \( h_{g1-g2} \) convective heat transfer coefficient between the inner and outer glass cover (W/m\(^2\)K)

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1. Introduction

Flat-plate solar collector modular designs for Domestic Hot Water (DHW) and innovative solar collector type designs for space heating in buildings or other applications are techno-economically attractive and effective for energy savings and green buildings, as argued in the scientific articles in [1]. The performance of solar collectors, especially the flat-plate ones, has been investigated so far by many researchers, starting with the classical works [2-6], then proceeding to further investigations on various parameters in order to improve the solar collector design and performance, [7-11] and finally reaching to more detailed experimental and theoretical investigations with sophisticated simulation models [12-18]. The efforts to increase the solar collector gain and its efficiency are continuous [19-23] and focus on the reduction of the \( U_1 \) coefficient and the increase of the ratio \( \alpha / \varepsilon \), which in turn affects \( U_1 \), too, as one moves from conventional absorbing surfaces to selective ones [12]. There is, also, a concern for double glaze flat plate collectors, especially, when the annual meteorological conditions in a site are in favor, such as high wind velocity and low ambient temperatures. In the past, attempts to get to a simple expression of \( Q_1'' \) as a function of \( \alpha / \varepsilon \) considered an absorbing surface, but not a real solar collector as described in [12], while other attempts described in [13] present a smart set of formulae to predict \( T_{ap} \) and \( T_{gc} \) for any value of \( \alpha \) and \( \varepsilon \), but for single glass flat-plate collectors. On the other hand, the thermal analysis of solar collector designs with innovative features, such as vee corrugated collectors [24], double glazing systems, is complicated [25]. It is necessary to incorporate into the model a whole set of proper mathematical expressions which describe such collector systems and deal with the most probable to happen physical phenomena with all the possible modes of heat transfer and radiation taken into consideration. Conclusively, an accurate prediction of the thermal performance of any flat-plate solar collector design is a challenge. In the optical analysis concerning the \( I_1 \) through a solar collector glass cover, an improvement was described in [26]. The energy analysis, with heat transfer and IR radiation taken into account may be handled by various models which are structured on energy balance equations. Such fundamental approaches are outlined in [2,27], while in [5,6,9] some improved versions are presented. Two families of such approaches are distinguished, in general. The empirical one, with representative formulæ for the determination of \( U_1 \) coefficient as in [4,13] and the numerical one as in [18-20,24,25,28]. Within this scope, the numerical techniques showed some errors which resulted from regrouping of the heat convection and IR radiative terms. Improved expressions were elaborated to determine the absorber plate and glass cover temperatures for single and double glaze collectors [25,29,30] with a good accuracy, where the glass cover solar radiation absorption is also taken into account. However, these papers are concerned only with the thermal analysis of the top losses and the glass cover temperatures for various environmental conditions and do not develop a complete and compact model analysis for flat-plate collectors with their fluid properties studied as a whole system. A detailed analysis to answer when a double glaze collector is preferable vs a single glazed one is a must, especially when optimization factors do challenge for the most cost-effective solutions. The latter is of high importance as solar collector type designs embedded into building structures require cost effective innovations to gain competitiveness. A complete and friendly solar collector simulation model to determine the heat losses coefficients, the absorber and glass covers temperatures, the heat gain and the efficiency, taking into account the glass covers radiation absorption, is presented in this study.

2. The solar collector simulation model: details and iterative techniques

The numerical simulation model developed for either single or double flat-plate collectors considers:

a. The geometrical characteristics of a double glaze flat plate collector: collector area \( (A_c) \), the collector plate thickness \( (\delta) \) the tube diameters \( (D_{out}, D_{in}) \), the spacing between the fluid tubes \( (W) \), the absorber plate to cover-1 and cover-1 to cover-2 distances, \( l_{g1} \) and \( l_{g1-g2} \), respectively, the glass cover thicknesses \( l_{g1} \), \( l_{g1} \) and \( l_{g2} \) air gap spacing between glass cover-1 and glass cover-2 \( (m) \)

\[ m \quad \text{fluid mass flow rate (kg/s)} \]

\[ a \quad \text{absorber plate absorption coefficient} \]

\[ \beta \quad \text{collector tilt with respect to horizontal (°)} \]

\[ \delta \quad \text{absorber plate thickness (m)} \]

\[ \varepsilon_c \quad \text{absorber plate emissivity coefficient} \]

\[ \varepsilon_g \quad \text{glass cover emissivity coefficient} \]

\[ \eta \quad \text{solar collector instant efficiency} \]

\[ \nu \quad \text{air kinematic viscosity (m²/s)} \]

\[ \sigma \quad \text{Stefan-Boltzman constant (W/ m² K⁴)} \]

\[ \tau \quad \text{glass cover transmittance} \]

\[ h_{r,g1} \quad \text{radiative heat transfer coefficient between absorber plate and glass cover (W/m²K)} \]

\[ h_{r,g2} \quad \text{radiative heat transfer coefficient between the outer glass cover and the environment (W/m²K)} \]

\[ h_w \quad \text{heat transfer coefficient from the outer glass cover (W/m²K)} \]

\[ k \quad \text{thermal conductivity of the collector material (W/mK)} \]

\[ k_a \quad \text{thermal conductivity of air (W/mK)} \]

\[ k_{g1}, k_{g2} \quad \text{thermal conductivity of the glass covers-1 and -2 respectively (W/mK)} \]

\[ l_{g1}, l_{g2} \quad \text{thickness of the glass cover-1 and glass cover-2 (m)} \]

\[ l_{g1-g2} \quad \text{air gap between absorber plate and glass cover-1 (m)} \]
l_p, the tilt of the collector with reference to the horizontal surface β, etc.

b. The thermo-physical characteristics of the material used in the collector such as: the thermal conductivity k of the collector material, as well as the glass covers conductivity k_g1, k_g2, the collector’s fluid specific heat capacity c_p, the Prandtl number, Pr, the kinematic viscosity ν, the air conductivity coefficient k_a. Their dependence on temperature was also taken into account. The corresponding values of the above quantities Pr, ν, k_a at the boundary layers of the collector two sections, i.e. the absorber plate to cover-1 zone and cover-1 to cover-2 zone, are fitted to a double exponential for k_a and Pr, and to a quadratic expression for the kinematic viscosity ν, for the range of 250-1000 K. Therefore, the program uses a simple self-adaptive data retrieval mode, useful in the various iterations of this algorithm for increased accuracy.

c. The ambient temperature T_a and the fluid inlet temperature T_i, the fluid mass flow rate m, the collector outer surface heat transfer coefficient h_o, the values of the absorption coefficient α and the emissivity coefficient ε_g of the absorbing surface, the glass cover(s) emissivity coefficient ε_g, the fluid heat transfer coefficient inside the tubes h_f, calculated according to the formulae in Appendix 1.

d. the solar irradiance on the collector I_{rs} and the effective product of the coefficients r and α, known as (αr), estimated by a set of formulae provided in Appendix 2. The simulation model developed estimates the following quantities: U_i, T_p, T_g1, T_g2, T_o, Q_H, Q_W and η vs T_f with parameters T_f, α, ε_g and h_a. The numerical approach uses the following set of equations to take into account heat convection between absorber plate and glass cover-1, glass cover-1 and glass cover-2 and outer glass to the environment; also, the IR radiation exchanges between the various zones of the collector, plus the one between the collector surface and the environment and finally, the thermal conductivity in the glass covers. The overall energy transfer coefficient from the absorber to the environment is given by the expression:

\[
U_i = \left( \frac{1}{h_{p-g}} + \frac{1}{h_{g-a}} + \frac{1}{h_{w-a}} \right) \left( \frac{l_g}{k_g} + \frac{l_2}{k_2} \right) \]

(1)

This formula is easily developed by using the electric equivalent circuit for the heat transfer through the collector where:

\[
h_{p-g} = \alpha \left( T_p^2 + T_g^2 \right) \left( T_p + T_g \right)
\]

and

\[
h_{g-a} = \alpha \left( T_g^2 + T_a^2 \right) \left( T_g + T_a \right)
\]

and

\[
h_{w-a} = \varepsilon_g \cdot \varepsilon_a \left( T_p^2 + T_a^2 \right) \left( T_p + T_a \right)
\]

(2)

T_i is the sky temperature given by:

\[
T_i = 0.0552(T_a)^{1.5}
\]

(4)

T_p and T_a are both in Kelvin, as discussed in [27].

Factors:

\[
R_{k1} = \frac{l_g}{k_g} \quad \text{and} \quad R_{k2} = \frac{l_2}{k_2}
\]

(5)

are defined as the thermal resistances which represent heat conduction through the glass cover-1 and cover-2, respectively. Finally, the convection heat transfer coefficient h_o for the outer glass cover is determined with respect to the wind velocity v_w, on the collector surface by the following formulae, in cases the forced flow convection prevails:

\[
h_o = 5.7 + 3.8v_w \quad [29]
\]

(6a)

\[
h_o = 2.8 + 3.0v_w \quad [30]
\]

(6b)

\[
h_o = 4.5 + 2.9v_w \quad [31]
\]

(6c)

For cases of natural heat convection, h_o may be replaced by h_o, to be determined by a set of equations provided in [33].

The values of T_p, T_g1 and T_g2 are generally unknown, while T_a and T_i may be measured experimentally along with the solar irradiance on the collector I_{rs}. Hence, they are given as input guess values or as predefined initial parameters in this algorithm. Recurrence formulae which relate T_p, T_g1 and T_g2 can be easily constructed assuming that the energy flow from the collector’s absorber plate to cover-1 is the same with the energy flow from cover-1 to cover-2 and finally to the environment. Conclusively, steady state conditions and no side losses are considered, while the temperature difference due to heat conduction in the glass covers is taken into account, along with the convection and the radiative losses in the various zones the collector consists of.

An exercise on the continuity principle for the energy flow which crosses the solar collector leads to the following recurrence formulae for the collector glass cover-1 and glass cover-2 temperatures.

\[
T_{g1} = T_p - \frac{U_i \left( T_g - T_p \right)}{\left[ h_{p-g} + h_{g-a} \right] + R_{k1}}
\]

and

\[
T_{g2} = T_i - \frac{U_i \left( T_f - T_p \right)}{\left[ h_{w-a} + h_{a-f} \right] + R_{k1}}
\]

(7)

However, as T_g1 and T_g2 are not known, U_i cannot be accurately calculated, and hence eq.(7) cannot be effectively handled. The iteration starts with a guess for T_g1 and T_g2. T_g1 and T_g2 simultaneously. A fast converging set of various modes of iterations is developed, where the convection heat transfer coefficients h_{p-g1} and h_{p-g2} are calculated from the Nu number according to the following expressions.
\[ Nu_1 = \frac{h_{p-g1} T_1^*}{k} = \frac{h_{p-g1} T_p - T_{g1}}{k}, \]
\[ Nu_2 = \frac{h_{p-g2} T_2^*}{k} = \frac{h_{p-g2} T_p - T_{g2}}{k}, \]  
(8)

\( l^* \) is the characteristic length which in this case is the absorber plate to glass cover-1 distance \( l_{p-g1} \), while \( l_{p-g2} \) is the cover-1 to cover-2 distance. The \( Nu \) numbers \( Nu_1 \) and \( Nu_2 \) for those two zones are calculated from the equations below:

\[ Nu = \frac{h l^*}{k} \left( 1 + 1.446 \left( 1 - \frac{1708}{Ra} \right) \right) \text{, for } 1708 < Ra < 5900 \text{ or} \]
\[ Nu = 0.220 (Ra)^{1/2} \text{ for } 9.23 \cdot 10^4 > Ra > 5900 \text{ or} \]
\[ Nu = 0.157 (Ra)^{1/2} \text{ for } 9.23 \cdot 10^4 < Ra < 10^5, \]  
(9a)
(9b)
(9c)

where, to cater for the collector inclination \( \beta \), \( Ra' \) is defined by:

\[ Ra' = Ra \cos \beta \]  
(9d)

Detailed discussion on \( Ra \) for various angles of inclination is found in [27,33]. Introducing the corresponding value of \( l^* \) for the zone investigated, the associated \( Ra \) (Rayleigh) numbers may be estimated by the expression:

\[ Ra_{1} = \frac{g \cdot \rho \cdot \Delta T \cdot l_{p-g1}^2 \cdot \rho_{t}}{v_i^2}, \]
\[ Ra_{2} = \frac{g \cdot \rho \cdot \Delta T \cdot l_{p-g2}^2 \cdot \rho_{t}}{v_i^2}, \]  
(10)

where, the values of \( k_{_{air}}, v, Pr \) for the air space within the collector can be determined from Tables at the corresponding boundary layer temperature \( T_{bl} \). Generally the values of the above thermo-physical quantities for air and water may be taken from relevant Tables [33] and especially in the algorithm developed are estimated by the fitted exponentials or polynomial expressions mentioned above. The program determines automatically the thermo-physical properties of air and water, along with the other calculations in each loop. For gasses, \( \beta' \) and \( T_{bl} \) are interrelated:

\[ \beta'_{1} = \frac{1}{T_{bl}} = \frac{1}{(T_p + T_{g1})/2} \]

and

\[ \beta'_{2} = \frac{1}{T_{bl}} = \frac{1}{(T_p + T_{g2})/2} \]  
(11)

\( T_{bl1} \) and \( T_{bl2} \) are the temperatures, in K, at the boundary layers of plate to cover-1 and cover-1 to cover-2 zones, respectively. Then, the temperature differences are defined by:

\[ \Delta T_{1} = T_p - T_{g1} \]
\[ \Delta T_{2} = T_p - T_{g2} \]  
(12)

These relationships provide a constraint in the guess process.

The, \( T_p \) guessed value should be different than the guessed \( T_{g1} \) and \( T_{g2} \) as \( \Delta T \) obtained from eq.(12) must be \( \neq 0 \) for the numerical process to take place without problems. In case \( \Delta T = 0 \), then \( Ra = 0 \), which is not true as thermal losses do exist.

3. Iterative Process: Steps and Iterations

A comprehensive iterative procedure is shown in Fig.1 and the steps are briefly outlined below:

1. \( T_p \) and \( T_{g1}, T_{g2} \) are guessed with the constraint as required above. An approximate value of \( U_i \) is estimated using eqs.(1-6) and eqs.(8-12).
2. The program starts a number of loops to determine a better value of \( T_{g1} \) for the guessed values of \( T_p \) and \( T_{g2} \) using eq.(7) and eqs.(1-6).
3. The program re-evaluates \( U_i \) from eqs.(1-6) for the new \( T_{g1} \) value, keeping the previously guessed values \( T_p, T_{g2} \) for a subsequent correction.
4. Then, starts another loop to determine a better value of \( T_{g2} \) for the guessed value of \( T_p \) and the value of \( T_{g1} \) as estimated before in Step 2, using eq.(7) and eqs.(1-6).
5. Taking into consideration the values of \( T_p, T_{g1}, T_{g2} \) as resulted from the iteration procedure, i.e. Steps 2 – 4, and the initially guessed value of \( T_p \), the program re-evaluates \( U_i \) from eqs.(1-6).
6. The program sets \( U_{i(1)} = U_i \) as the purpose is not to calculate \( U_i \) but, as said above, to investigate top losses for various environmental and operational conditions. In fact, \( U_i \) approaches \( U_{i(1)} \) since an effective back and side insulation is placed in the collector frame. It is obvious that the side and backwards direction losses could be easily calculated, as discussed in [34]. For this set of values \( T_p, T_{g1} \) and \( T_{g2} \), the program calculates \( \Delta T_1, \Delta T_2 \), \( T_{bl1}, T_{bl2} \), i.e. \( \beta'_{1}, \beta'_{2} \) and \( R_{bl1}, R_{bl2} \). Then, it calculates the heat removal coefficient, \( F_k \) given by the related formulae below, as fully presented in [27].

\[ F_{k} = \frac{m \cdot c_{p} \cdot [1 - \exp(-A \cdot U_{i(1)} / m \cdot c_{p})]}{A \cdot U_{i(1)}}, \]  
(13)

\[ F' = \frac{U_{i(1)}}{W \cdot [U_{i(1)} \cdot (D_i + (W - D_i) \cdot F') + \pi \cdot D_i \cdot h_i]] + C_{v}}, \]
(14a)

\( F' \) is the collector efficiency factor, which provides for the ratio of the heat transfer resistance from the absorber plate to the environment over the heat transfer resistance from the fluid to the environment, and

\[ F = \tan \theta \frac{m \cdot [(W - D_i)/2]}{m \cdot (W - D_i/2)}, \]  
(14b)

\( m = \frac{U_{i(1)}}{k \cdot \delta} \)
bond conductance of the fluid tubes attached on the absorber. Cs typical values must be > 30 W/m²K.

7. Having estimated \( U_t \) in Step 6, the program calculates \( Q^\prime \), i.e. the heat rate gain, by the energy balance formula of Hottel – Whillier – Bliss and the calorimetry equation as it regards the fluid flow in the collector tubes:

\[
Q^\prime = F_e \left[ \left( \frac{\alpha \alpha}{U_s} \right) - U_t \left( T_{g2} - T_{g1} \right) \right] = \left( m c_p \right) \left( T_{g2} - T_{g1} \right)
\]

where, \((\alpha \alpha)\) is the effective product of the solar radiation transmission loss coefficient of the glass cover(s) system and the absorption coefficient of the collector plate. It is estimated according to the set of formulae provided by Appendix 2.

8. The program calculates the value of the \( T_{f,o} \) from the second part of eq.(15) and determines a better value of \( T_p \) from a formula which provides the absorber plate mean temperature \( T_{p,m} \), where we assume \( T_p = T_{p,m} \), i.e.

\[
T_p = T_{p,m} = T_{g2} + \frac{Q^\prime}{U_s} \left( 1 - F_e \right)
\]

Another equivalent expression to calculate \( T_p \) is given by:

\[
T_p = F_e T_{g2} + T_{g2} \left( 1 - F_e \right) - \frac{L_e \left( \alpha \alpha \right) \left( 1 - F_e \right)}{U_s}
\]

which is derived from the obvious identity:

\[
Q^\prime = F_e \left[ \left( \frac{\alpha \alpha}{U_s} \right) - U_t \left( T_{g2} - T_{g1} \right) \right] = \left( m c_p \right) \left( T_{g2} - T_{g1} \right)
\]

9. Substituting \( T_{p,m} \) for \( T_p \), the program repeats once again the first set of loops, see Step 2, to determine a better value for \( T_{g1} \) and then for \( T_{g2} \), as described before.

10. Then, the program repeats all the above steps as before until the new \( T_{p,m} \) differs from the previous one by a preset value according to the accuracy required.

An improved version of this set of iterations was tested, where the separate iterations associated with the determination of each one of the three parameters \( T_{p,m} \), \( T_{g1} \), \( T_{g2} \), are handled in parallel within the same loop, see flowchart in Fig.1. This procedure is much faster and the results converge within 9-13 iterations, regardless of the initial guess values given to \( T_{p,m} \), \( T_{g1} \), \( T_{g2} \). The convergence is set to the 3rd decimal point, so that the iteration ends when the three temperatures \( T_p \), \( T_{g1} \), \( T_{g2} \) do not differ more than 0.001°C from their corresponding values in the previous iteration.

The above iterations give as outputs the required \( T_p \) and \( T_{g1} \), \( T_{g2} \) values, necessary for the calculations of the performance indicators for the solar collectors. It, also, provides \( T_{g1} \), \( Q^\prime \) and finally, the solar collector efficiency, \( \eta \), which is determined by:

\[
\eta = \frac{Q^\prime}{I_T}
\]

The results of this simulation technique for \( U_t \) with the guessed temperatures \( T_p \) and \( T_{g1}, T_{g2} \) are compared with the ones obtained by using the Akhtar & Mullick model [25] and the generalized formula proposed by Klein [4]:

\[
U_t = \left[ \frac{N}{C} \left( \frac{\left( U_p + U_{g1} \right) + \left( U_p + U_{g2} \right)}{2} \right) \right]^{1/ \gamma} + \frac{\sigma \left( \frac{U_p + U_{g1} + U_{g2}}{2} \right)}{\left( \frac{U_p + U_{g1} + U_{g2}}{2} \right)}
\]

where, \( N \) is the number of the glass covers; in this case, \( N = 2 \).

\[
f = (1 + 0.089h_{u} - 0.1166 h_{u} a)(1 + 0.7866 N)
\]

\[
C = 520(1 - 0.00051 \beta^2)
\]

while, for \( 70^\circ \beta < 90^\circ \) the eq.(20(b) is used with \( \beta = 70^\circ \)

\[
e = 0.43(1 - 100/T_{p,m})
\]

According to eq.(20), only the \( T_p \) has to be given when the Klein empirical formula is chosen for the estimation of \( U_t \). This is done with a little cost in the accuracy, as to be discussed below. In this approach \( R_t \) is estimated directly using eqs.(9a-12), while \( T_{g1} \) and \( T_{g2} \) may be determined in this mode using the Klein formula for \( U_t \) via eq. (7), provided the \( U_t \) value by the Klein formula is introduced into the iteration procedure, just described before. However, in this approach \( T_p \) is arbitrarily given whereas it is straightforwardly associated to the \( I_T \) to \( T_{g1} \), plus the parameters \( \alpha \), \( \epsilon \), \( h_{u} \). Using the Hottel-Whillier-Bliss equation and the calorimetry one, see eq.(18), the program estimates a new \( T_{p,m} \) and \( T_{g2} \) for a given \( I_T \). The iteration is repeated until \( T_{p,m} \) does not change significantly. This is the version of the program which provides results based on the Klein formula.

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**Fig.1 Flow chart of the proposed simulation model**

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4. Results

The proposed model with its sets of formulae, as outlined in this paper was developed in MATLAB. Functions were also developed to elaborate results based on both the Akhtar & Mullick model and the Klein empirical formula.

This model was executed systematically to provide results for $\eta$, $T_{fi}$, $Q_{in}$, $U_i$, $T_{p}$, $T_{fi}$, $T_o$ for any environmental conditions, as it regards the ambient temperature $T_a$ and the wind velocity $v_w$ for various values of $I_T$, $T_{fi}$ and the coefficients $\alpha$, $\epsilon_p$ and $h_w$. The purpose was first to investigate the accuracy of its results compared with the ones obtained by the Klein formula [4] and also by the Akhtar & Mullick model [25]. Both of these models require as inputs the value of $T_p$ which is $I_T$ dependent, whereas the model proposed in this paper is complete and compact only taking as input data the $I_T$, $T_a$, $T_{fi}$ which make up a realistic set of solar collector operation variables.

The results provided by the above three models for $U_i$ vs $I_T$, are shown in Figs.2-5 for the case of double glaze solar collector for various sets of parameter values $T_{fi}$, $\alpha$, $\epsilon_p$ and $h_w$. It is obvious from those figures that this model provides $U_i$ results which are very close to the ones by the Akhtar & Mullick approach. They differ less than 1% in all cases. It should be noted that the results shown for the Akhtar & Mullick’s approach are based on their formula using $T_{fi}$ instead of $T_a$ otherwise the results obtained for their model deviated significantly from normal especially for low $I_T$ values. On the other hand, in the proposed model $T_s$ was calculated by eq.(4). As it concerns the Klein formula results, they differ to this model by about 2-3 % for low $I_T$ values. This difference is lower than the uncertainty in $U_i$ obtained by the Klein formula, as it is discussed by Duffie & Beckman [27]. As $I_T$ increases, this difference becomes much lower and Klein model results get close to this model. Generally, the Klein formula underestimates $U_i$ for low $\epsilon_p$ values. However, as $\epsilon_p$ increases Klein formula provides an overestimation in the $U_i$ results and for high $h_w$ and $T_{fi}$ values this overestimation reaches around 3-4% compared to the ones from this model and the model in [25].

Fig.2. Heat losses coefficient $U_i$, vs $I_T$ for a double glaze flat-plate collector as estimated by the proposed model and compared to Klein model [4] and model [25], for $T_a=20^\circ$C, $\epsilon_p=0.1$, $h_w=5$ W/m$^2$K, with $T_{fi}=20^\circ$C and $T_{io}=50^\circ$C.

Fig.3. Heat losses coefficient $U_i$, vs $I_T$ for a double glaze flat-plate collector as estimated by the proposed model and compared to Klein model [4] and model [25], for $T_a=20^\circ$C, $\epsilon_p=0.95$, $h_w=5$ W/m$^2$K, with $T_{fi}=20^\circ$C and $T_{io}=50^\circ$C.

Fig.4. Heat losses coefficient $U_i$, vs $I_T$ for a double glaze flat-plate collector as estimated by the proposed model and compared to Klein model [4] and model [25], for $T_a=20^\circ$C, $\epsilon_p=0.1$, $h_w=20$ W/m$^2$K, with $T_{fi}=20^\circ$C and $T_{io}=50^\circ$C.

Fig.5. Heat losses coefficient $U_i$, vs $I_T$ for a double glaze flat-plate collector as estimated by the proposed model and compared to Klein model [4] and model [25], for $T_a=20^\circ$C, $\epsilon_p=0.95$, $h_w=20$ W/m$^2$K, with $T_{fi}=20^\circ$C and $T_{io}=50^\circ$C.

Fig.6 shows the $U_i$ results provided by the 3 approaches for single glaze flat-plate collectors. In this type of collector, the Klein formula provides results which are systematically higher than the ones provided by the other two approaches. However, this difference is not higher than 5-8% for the worst cases, that is, for high $h_w$ and high $\epsilon_p$ values displayed in Fig.6. It is evident that the last two coefficients affect greatly the $U_i$ profile.
As said, the model provides the values of the $T_p$, $T_{g1}$ and $T_{g2}$ temperatures vs the solar irradiation on the collector $I_T$ and these results are compared with those obtained by the formulae described in [25]. It obvious from Figs.7-10 that, for all environmental conditions this model was tested, the results compared to those by the model [25] almost coincided; the difference was less than 0.2 °C in a scale of 100 °C. To get results from [25] with coincidence to this model the $T_o$ value was set equal to $T_{so}$. The efficiency of the flat-plate collectors, $\eta$ vs $(T_{f,i} - T_o)/I_T$ or $\eta$ vs $(T_p - T_o)/I_T$ for either single or double glaze collectors was investigated for various $h_w$, $\varepsilon$ and $\alpha_w$ values, as shown in Figs.11-12. In this case, the potential of the proposed model is clear as it provides answers about the collector efficiency for any environmental conditions. The effect of high $I_T$, where high $T_p$ temperatures are reached, is evident from the change of the $\eta$ curve. The efficiency curve from its initial linear behaviour, when the convection losses in the collector prevail, takes a curved profile of 2nd degree, as in this case the IR radiation losses prevail. The curved shape in the efficiency curve is prominent when the efficiency is given vs $(T_p - T_o)/I_T$. This is more realistic diagram, as it is the absorber plate which is heated by the $I_T$ absorbed and this affects greatly the whole energy performance of the collector. In Fig. 12 the effect of the parameters $\alpha$, $\varepsilon_w$ and $h_w$ is obvious. Also, Fig.12 shows a comparison between a single flat-plate solar collector efficiency against the double glaze one. The region of the environmental conditions the one type of collector is preferable over the other is clear. Figs. 13 and 14 provide the $\eta$ values vs $I_T$ keeping as parameter the fluid inlet temperature $T_{f,i}$. As expected, $\eta$ decreases as $T_{f,i}$ increases, while the efficiency increases with $I_T$ for constant $T_{f,i}$. For higher $T_{f,i}$ values $\eta$ reaches to its saturation value for higher $I_T$ values, around 500 W/m², compared to the cases of low $T_{f,i}$. The effect of the $\varepsilon_w$ in the collector efficiency is obvious by comparing the results in Figs. 13 and 14.

$\varepsilon_w=0.1$, $h_w=5 \text{ W/m}^2\text{K}$. The predicted values of $T_{g1}$ and $T_{g2}$ by this model and by [25] almost coincide.
The results obtained by this model, also, by the model outlined in [25] and the Klein formula [4] for a large set of environmental conditions show a consistency to each other. The predicted \( T_{gl} \) and \( T_{g2} \) by this model and the one of [25] almost coincide, as the difference between them is of the order of 0.1-0.2 °C at a range of 80°C – 130°C. On the other hand, this model provides the \( T_u \) values, too, which are not determined by [4, 25]. The three models provide the \( U_t \), coefficient. The results obtained by this model and the one of [25] for the case of double glaze flat-plate collectors show a difference of 1-2%, while the empirical Klein formula provides for \( U_t \) results which differ by 3-4% for the cases of high \( h_o \) and \( T_{gl} \) values. This difference gets smaller for low \( h_o \) and \( T_{gl} \) values. However, this difference for single glaze collectors reaches up to 5-8% for the worst conditions, that is for high \( h_o \) and \( \varepsilon_o \) values. Generally speaking the Klein formula gives an understimation in \( U_t \) for low \( \varepsilon_o \) values. However, as \( \varepsilon_o \) increases \( U_t \) is overestimated; see Figs. 3 and 5. Finally, the model provides the efficiency \( \eta \) of the flat-plate solar collectors. The results provided vs \((T_{gl} - T_{g2})/ I_f \) or \( \eta \) vs \((T_p - T_{gl})/ I_f \) for either single or double glaze collectors were investigated for various \( h_o \), \( \varepsilon_o \) and \( \varepsilon_p \) values, as shown in Figs. 11-13. At high \( I_f \) values which imply higher \( T_p \) values, than for low \( I_f \), the efficiency curve, \( \eta \) vs \((T_p - T_{gl})/ I_f \) deviates from linearity as the IR radiation exchanged between the collector and the environment increases fast with \((T_p^4 - T_{gl}^4)\). This model may serve as a dynamic tool to provide the energy performance, of a solar flat-plate collector, identified by \( \eta \), \( Q_s^a \), \( U_t \), \( T_{gl} \), \( T_{g2} \), and \( T_{gl} \). These results help for decisions to be taken over the environmental and operational conditions that the double glaze flat-plate collector is preferable against a single glaze one for the same conditions. Finally, this model requires 3 guessed values \( T_{gl}, T_{g2} \) and results are provided very fast converging to the 3rd decimal point. This model may be executed on any platform. The equations used, accept any combination of input data for the estimation of double glaze solar flat plate collector’s efficiency and heat gain. The initial guess values of \( T_{gl}, T_{g2} \) and \( T_{gl} \) should differ in order to let the program run. It is important to point out that whatever guessed values for \( T_{gl}, T_{g2} \) and \( T_{gl} \) are taken and with any set of \( T_{gl}, T_{g2} \) values, the program runs efficiently and converges fast, as said above.

5. Discussion

The results obtained by this model, also, by the model outlined in [25] and the Klein formula [4] for a large set of environmental conditions show a consistency to each other. The predicted \( T_{gl} \) and \( T_{g2} \) by this model and the one of [25] almost coincide, as the difference between them is of the order of 0.1-0.2 °C at a range of 80°C – 130°C. On the other hand, this model provides the \( T_u \) values, too, which are not determined by [4, 25]. The three models provide the \( U_t \), coefficient. The results obtained by this model and the one of [25] for the case of double glaze flat-plate collectors show a difference of 1-2%, while the empirical Klein formula provides for \( U_t \) results which differ by 3-4% for the cases of high \( h_o \) and \( T_{gl} \) values. This difference gets smaller for low \( h_o \) and \( T_{gl} \) values. However, this difference for single glaze collectors reaches up to 5-8% for the worst conditions, that is for high \( h_o \) and \( \varepsilon_o \) values. Generally speaking the Klein formula gives an understimation in \( U_t \) for low \( \varepsilon_o \) values. However, as \( \varepsilon_o \) increases \( U_t \) is overestimated; see Figs. 3 and 5. Finally, the model provides the efficiency \( \eta \) of the flat-plate solar collectors. The results provided vs \((T_{gl} - T_{g2})/ I_f \) or \( \eta \) vs \((T_p - T_{gl})/ I_f \) for either single or double glaze collectors were investigated for various \( h_o \), \( \varepsilon_o \) and \( \varepsilon_p \) values, as shown in Figs. 11-13. At high \( I_f \) values which imply higher \( T_p \) values, than for low \( I_f \), the efficiency curve, \( \eta \) vs \((T_p - T_{gl})/ I_f \) deviates from linearity as the IR radiation exchanged between the collector and the environment increases fast with \((T_p^4 - T_{gl}^4)\). This model may serve as a dynamic tool to provide the energy performance, of a solar flat-plate collector, identified by \( \eta \), \( Q_s^a \), \( U_t \), \( T_{gl} \), \( T_{g2} \), and \( T_{gl} \). These results help for decisions to be taken over the environmental and operational conditions that the double glaze flat-plate collector is preferable against a single glaze one for the same conditions. Finally, this model requires 3 guessed values \( T_{gl}, T_{g2} \) and results are provided very fast converging to the 3rd decimal point. This model may be executed on any platform. The equations used, accept any combination of input data for the estimation of double glaze solar flat plate collector’s efficiency and heat gain. The initial guess values of \( T_{gl}, T_{g2} \) and \( T_{gl} \) should differ in order to let the program run. It is important to point out that whatever guessed values for \( T_{gl}, T_{g2} \) and \( T_{gl} \) are taken and with any set of \( T_{gl}, T_{g2} \) values, the program runs efficiently and converges fast, as said above.
6. Conclusions

The simulation model developed is friendly to the user, parameterized and equipped with functions to determine (τao) and the hD values, as provided in the Appendices. The software built provides values of any quantity related to the flat-plate collector either of single or double glazing, given the operation characteristics, the material used, the fluid flow mass flow rate and its inlet temperature, along with the H values. Finally, the solar radiation absorption by the glass covers, may be easily tackled as a second small heat source using the superposition principle. This contribution of this effect seems to be insignificant as the term q*=(1/2k) (A1.4) which determines the max increase in Tg due to the radiation absorption by the glass cover, provides an increase of less than 1°C. Note that, q* is the heat rate generated per volume (W/m³) in the glass cover due to solar radiation absorption. As it is clearly shown from the results in Fig. 12 the decision to use a double glaze flat plate collector depends on the region of (Tg - T0)/Tf in which it will operate, and also on the ε and hD values, which prevail in the region.

The simulation program developed is easy to implement on any platform, while the results converge in 9-13 iterations with an accuracy to the 3rd decimal point, regardless of the initial values given to Tw, Twl and Tg. The proposed algorithm was developed in MATLAB, while additional functions for the model outlined in [25] and the Klein formula were also incorporated for comparison reasons. The software developed may also determine the Tw, Twl and Tg values for any Tw, Twl and Tg using the Uf value as obtained by the Klein formula. The model considers all possible modes of heat and energy transfer, such as conduction in the glass covers, convection in the air in the collector zones and in the fluid within the tubes, for laminar or turbulent flow, solar radiation absorption by the glass covers and the possible modes of energy transfer from the outer glass to the environment.

Appendix 1

Determination of fluid flow convection heat transfer coefficient hD inside a tube. The convection heat transfer coefficient hD for the fluid flow inside the absorber tubes is obtained by:

\[ Nu = hD D/L k = 1.86(Re Pr D/L)^{1/3} (\mu_b/\mu_0)^{0.14} \]  \hspace{1cm} (A1.1)

for laminar flow pattern which is developed when,

\[ Re < 2100 \text{ and } 100 > Re Pr D/L > 10, \]

where \( D \) is the hydraulic diameter of the tube and \( L \) is the length of the tube. \( \mu_b \) is the fluid dynamic viscosity coefficient at the bulk temperature \( Tw \). \( \mu_0 \) is the fluid dynamic viscosity coefficient at the boundary layer temperature \( Twl \), where,

\[ Twl = \left( Tw + Tw \right)/2 = \left( T_{bl} + T_{bl} \right)/2 + T_{bl}/2 \]  \hspace{1cm} (A1.2)

For the case when \( Re > 6000 \) then, flow is turbulent and holds,

\[ Nu = hD D/L k = 0.023 Re^{0.8} Pr^{1/3} \]  \hspace{1cm} (A1.3)

with all the thermo-physical properties estimated at the mean bulk temperature \( (T_{bl} + T_{bl})/2 \), unless it is specified to be estimated at the boundary layer temperature \( Twl \). For cases where, \( 1 < Gr/Re^2 < 10 \) and \( L/D_b > 50 \), where the fluid undergoes a transition phase from laminar to turbulent, then as discussed in [35]:

\[ Nu = hD D/L k = 1.75(\mu_b/\mu_0)^{0.14} \left((Re Pr D/L)^{1/3} \right. \left.+ 0.012(Gr)^{1/3} \right) \]  \hspace{1cm} (A1.4)

Appendix 2

Determination of (τao) for single and double glaze solar collectors. Let us consider the normal incidence of solar radiation on the collector plane as the calculations are elaborated with \( I_1 \) which implies the normal component on the glass cover. Let a glass cover with refractive index \( n=1.53 \) and thickness 3 mm. Let, also, the extinction coefficient \( K=4m^2 \). According to Snell Law the reflectance \( r \) is determined by,

\[ r = \left( n-1/(n+1) \right)^2 \]  \hspace{1cm} (A2.1)

Substitution of the values of the parameters to formula (A2.1) above gives, \( r=0.0439 \)

The transmittance of the solar light intensity, \( I_1 \) through \( N \) covers is provided by,

\[ \tau = \frac{1-r_2}{1+(2N-1)r_2} + \frac{1-r_1}{1+(2N-1)r_1} \]  \hspace{1cm} (A2.2)

where \( r_1 \) and \( r_2 \) represent the beam components polarised at normal and parallel to the medium of propagation. Assuming that the polarized beam components, normal and parallel to the glass, are of equal strength, then, \( r_1 = r_2 \).

For \( N=2 \), that is for double glaze collector the above formula becomes,

\[ \tau = (1-r)/(1+(3N-1)r) \]  \hspace{1cm} (A2.3)

In this case, eq.(A2.3) gives, \( \tau = 0.8448 \).

The transmittance which takes into account the absorption \( \tau_a \), for normal incidence of the solar radiation, is given by,

\[ \tau_a(0)= \exp(-K l_{g1}) \]  \hspace{1cm} (A2.4)

Generally, the transmittance \( \tau \) equals to \( \tau = \tau_a \). For a double glaze collector for normal incidence \( \tau(0) \) may be determined by:

\[ \tau(0)=I/I_0 = \exp(-K(l_{g1}+l_{g2})) \]  \hspace{1cm} (A2.5)

Finally, the total transmittance, \( \tau = \tau(0) \tau_a \).

When the above results are substituted into the equation above, it gives,

\[ \tau = \tau(0) \tau_a = \exp(-4x0.006)x0.8448=0.8248 \]
The effective product of $\tau$ and $\alpha$ that is the $(\tau\alpha)$ is given by,

$$(\tau\alpha) = \tau\alpha/(1-(1-\alpha)\rho_d) \quad (A2.6)$$

where, $\rho_d$ represents the reflectance of the diffuse radiation and it is estimated by:

$$\rho_d = \tau_\alpha = \tau_r \quad (A2.7)$$

In this case, $\rho_d = \tau_\alpha = 0.97629 – 0.8448 = 0.1315$. Usually, $\rho_d$ takes values in the region (0.13-0.20).

References

22. Chung-Fei Jeffrey Kuo et al “Using the Taguchi method and grey relational analysis to optimize the flat-plate collector process with multiple quality characteristics in solar energy collector manufacturing” Energy 36 (2011) 3554-3562