

Optimization of Drone Delivery Routes for E-commerce in Urban Areas

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Abstract

Drone delivery is expected to improve operations by enhancing flexibility and reducing congestion effects induced by last-mile deliveries. With rising digitalization and urbanization, however, flight routing of drones is constantly grappling with the challenge of uncertain demand. In this study, an opportunity-constrained model that considered both the endurance and payload capacity of drones and stochastic customer demand was developed to solve the drone delivery planning problem for E-commerce in urban areas. Considering the challenging solvability of the model, historical customer demand data were leveraged to linearly transform opportunity constraints with unknown true distribution. In addition, a constructive heuristic algorithm was used to solve the established opportunity-constrained model, and an effective strategy was developed to generate the quality of initial flight paths. The experimental results based on various scales of classic cases demonstrate that an effective drone delivery route scheme can be generated by the proposed heuristic algorithm compared to traditional savings algorithm, allowing drones to complete tasks in shorter flight distances at the same payload rate, thus reducing overall costs. Compared to deterministic models, more robust flight path schemes for addressing continuously changing customer demands can be obtained. The model and insights in this study could be used as a reference for the application of drone delivery in urban areas to promote logistics efficiency.

Keywords: Drone delivery routes; E-commerce; Mixed integer program; Heuristic algorithm; Demand uncertainty

1. Introduction

With the rapid development of electronic commerce (E-commerce), urban logistics distribution is facing unprecedented challenges and opportunities. Traditional distribution methods are gradually exposing their limitations in terms of efficiency, cost, and timeliness, and innovative solutions in the field of urban logistics distribution are being provided through the constantly advancing development of drone technologies. As a novel distribution method, drone distribution effectively circumvents the restrictions of ground transportation, significantly improves distribution efficiency, reduces labor costs, and gradually becomes a strong candidate for solving the last-mile distribution problem in urban areas.

Recently, researchers have conducted extensive research into the drone routing problem across various application scenarios. These studies mainly explore the reasonable assignment of distribution tasks to drones and optimizing their flight paths simultaneously, such as the collaborative assignment problem of drones in joint operation tasks and the logistic optimization problem of drones in the distribution of medical supplies [1-2]. Most of these studies have been based on optimization theories, constructing route optimization models for drone distribution under different application scenarios, and then developing accurate or intelligent optimization algorithms for the solutions, which provide strong theoretical references for the practical applications of drones. Restricted by technical difficulties such as limited

battery and load capacity, existing studies have mainly focused on the ground-air cooperative distribution mode based on drone technology. However, in real scenarios, urban areas, which are important carriers of economic activities, are characterized by dense buildings, narrow roads, and heavy foot traffic. For this type of closed, specific, and relatively complex distribution environment, a stand-alone drone-delivery mode is the optimal distribution solution. Additionally, in an E-commerce environment, logistics distribution demand in urban areas is characterized by diversification and high frequency. Route optimization of drone distribution is required not only to possess a high degree of flexibility and real-time capability but also to adequately consider factors such as the urgency of the distribution task, the density of the customer distribution, and the heterogeneity of the goods. The route optimization of drone distribution presents new challenges due to the uncertainty of distribution demand. Therefore, under the realistic constraints of limited drone load capacity and battery capacity, planning drone distribution route schemes to efficiently solve the problem of the uncertainty of customer demand in urban areas and realize a reduction in drone distribution costs are of important significance to engineering.

This study investigated a drone distribution route optimization problem under the uncertainty of customer demand, considering limited battery and load capacities. A constructive heuristic algorithm was also designed. Example tests were conducted using the drone models used by China Post, JD.com, and SF Express. Finally, a reliable drone distribution route path scheme was obtained, which provides

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a reference for research on the optimization of drone distribution.

2. State of the art

A considerable number of research achievements in drone distribution route optimization have been achieved by researchers and industry experts. These studies primarily focused on the algorithm design for distribution route optimization and minimization strategies for cost or energy consumption. Arafat et al. [3] considered the factors of charging station layout and drone flight range, established a joint routing and charging strategy (JRCS) model, and developed a mixed integer linear programming (MILP)-based method for optimized solution of distribution routes, to address the problem that logistics drones are limited by battery life and flight function when undertaking long-distance distribution tasks. In addition, in urban logistics distribution, Shao et al. [4] considered the factors of service benefit and risk cost, established a cost-benefit assessment model and a global heuristic path search rule, and proposed a path planning method based on risk mitigation and customer service to address the problem, which effectively reduced the risk cost of the paths while guaranteeing service quality. These studies provide valuable references for the optimization of drone distribution routes in different scenarios.

Aiming at the short endurance and limited distribution range of drones, several researchers have proposed a new distribution pattern for truck-drone cooperative distribution: the truck-drone routing problem (TDRP) [5]. Schermer et al. [6] developed a mixed-integer programming model for drone path planning and derived multiple sets of valid inequalities to improve solver performance. Kuo et al. [7] considered the joint truck-drone distribution problem with a time window and designed a variable neighborhood search algorithm to solve it. Zhou et al. [8] proposed a two-echelon vehicle that allows drones to make multiple round trips from customer nodes where the vehicles are parked and used an accurate branch-and-bound algorithm to solve it. Tamke and Buscher [9] considered the effect of drone speed on power consumption. Subsequently, Montemanni, Amico, and Corsini [10] studied the problem of parallel distribution with multiple trucks and drones and developed an energy-consumption model for drones during distribution. Stodola and Kutěj [11] proposed a multi-warehouse drone vehicle distribution problem and designed an improved ant colony algorithm. In addition, Kim, Ko, and Moon [12] considered the factors of cooperative work of drones and trucks as well as constraints on time windows. They developed a model for the vehicle routing problem with time windows and drones (VRPTW-D) and developed a three-stage savings-based heuristic (TSH) method to solve the problem. Their approach enhances the efficiency of the distribution system and reduces operating costs. These studies provide theoretical references for practical applications of the truck-drone cooperative distribution patterns.

Drone distribution is affected by factors such as uncertainty in distribution time and customer demands. Recently, researchers have paid more attention to the impact of these uncertainties on drone delivery to obtain more reliable and robust solutions. Han, Liu, and Li [13] considered time windows and dynamic customer demands and proposed a two-stage optimization model based on different demand response strategies, which provided a useful reference for

handling dynamically changing customer demands in the actual distribution process. Ghiasvand et al. [14] adopted a data-driven robust optimization approach to effectively deal with the uncertainty in a multi-trip truck-drone distribution problem by developing a two-phase clustering algorithm and a dimensional separation technique to construct the uncertainty set. This approach reduces the conservatism and computational complexity of the model and provides a new solution for drone distribution route optimization under uncertainty. Gu, Liu, and Poon [15] proposed a framework combining a Markov decision process and a heuristic algorithm to address the uncertainty of the pick-to-order problem in a dynamic truck-drone routing problem to improve the total profits of the logistics system and the acceptance rate of customer requests. Yin et al. [16] considered the uncertainty of customer demand and travel time, studied a class of collaborative transportation problems between drones and trucks in humanitarian logistics, proposed a framework combining robust optimization and a branch-and-price algorithm to solve the problem, and verified the superiority and practicability of this method through a real case analysis. Zhao et al. [17] proposed a robust multi-drone traveling salesman problem (TSP) to address parcel delivery. Their approach involves trucks coordinating a heterogeneous fleet of drones in uncertain navigation environments. They constructed a temporal-based robust model to minimize the desired travel time and limit the standard deviation to balance the delivery efficiency and synchronization risk. Pugliese, Guerriero, and Scutellà [18] investigated the last-mile delivery problem using trucks and drones in urban areas. They considered the uncertainty in drone energy consumption and managed drone energy consumption through a multi-commodity flow model and time segmentation. To prevent energy disruptions in the worst case, they applied robust optimization and used the Benders decomposition method to obtain a solution. Faiz, Vogiatzis, and Noor-E-Alam [19] utilized drones to address the problem of uncertainty in post-disaster humanitarian logistics, particularly in situations where infrastructure damage has resulted in unclear demand information. They effectively handled demand uncertainty by first deploying communication drones to provide communication coverage, followed by supply distribution. Zhang et al. [20] proposed a new variant of the drone arc routing problem in the context of humanitarian logistics. Their approach addressed the challenge of rapid assessment of ground transportation networks after a disaster by considering the uncertainty in assessment time. The aim is to maximize the arc informative profits collected within a predefined time limit. They achieve this by adopting a graph transformation technique and a robust optimization method. Tadić, Krstić, and Radovanović [21] proposed a novel hybrid fuzzy multi-criteria decision-making model. This model effectively analyzed and evaluated the barriers in the application of drones in last-mile logistics by combining fuzzy Delphi-based fuzzy factor relationship (Fuzzy D-FARE) and fuzzy comprehensive distance-based ranking (Fuzzy COBRA) methods. Additionally, they proposed corresponding strategies to overcome these barriers, especially in dealing with the uncertainty associated with the conflicting goals of multiple stakeholders. These studies provide insights into solutions to uncertainty problems.

Based on the review of the studies related to drone delivery problems, the following findings are presented.

(1) Existing studies have conducted extensive and in-depth research on drone logistics scheduling problems; however, most studies have focused on deterministic drone

distribution scheduling problems, that is, it was assumed that the distribution time and customer demand were known and fixed. Moreover, there were very few studies that considered uncertainty in customer demand or differences in the distribution time.

(2) For technical problems, such as the short endurance of drones and limited distribution range, most of the existing studies considered the ground–air collaborative distribution pattern. However, with the rapid development of E-commerce, the speed of ground (i.e., truck) distribution falls behind the distribution demands of logistics parks, especially when faced

with an increasingly congested traffic environment, and complete drone distribution is often used to solve this problem.

(3) In existing studies that consider the uncertainty of customer demand, robust optimization models are usually built to deal with uncertainty by constructing uncertainty sets of distribution of customer demand. However, with this modeling approach, the complexity is relatively high or there would be difficult to solve. Table 1 compares this study with existing related studies to demonstrate the differences between this study and the existing drone distribution scheduling studies more intuitively.

Table 1. Review of the relevant papers in routing optimization of drone delivery problem

Literature	Uncertainty in demand	Research object		Constraint		Model
		Drone distribution	Truck-drone distribution	Load capacity	Battery capacity	
[7] Kuo et al.			√	√	√	Mixed integer programming model
[8] Zhou et al.			√	√	√	Mixed integer linear programming model
[9] Tamke and Buscher			√	√	√	Mixed integer linear programming model
[10] Montemanni, Amico, and Corsini			√	√	√	Chance-constrained model
[11] Stodola and Kutěj			√	√	√	Mixed integer linear programming model
[12] Kim, Ko, and Moon			√	√	√	Three-stage optimization model
[13] Han, Liu, and Li	√		√	√		Two-stage optimization model
[14] Ghiasvand et al.	√		√	√		Robust optimization model
[15] Gu, Liu, and Poon	√		√	√		Markov decision process model
[16] Yin et al.	√		√	√	√	Robust optimization model
[17] Zhao et al.	√	√		√		Robust optimization model
[18] Pugliese, Guerriero, and Scutellá			√	√	√	Robust optimization model
[19] Faiz, Vogiatzis, and Noor-E-Alam	√	√		√		Two-stage optimization model
[20] Zhang et al.	√		√	√		Robust optimization model
[21] Tadić, Krstić, and Radovanović	√	√			√	Hybrid fuzzy multi-criteria model
The present study	√	√		√	√	Chance-constrained model

Therefore, in this study, a drone route planning problem with uncertain customer demand was investigated to minimize the total mileage of the drone and reduce the cost when considering both the capacity and range of the drone. For this purpose, an integer programming model was developed. Based on this deterministic model, the model was further linearly transformed by considering uncertain customer demands and introducing chance constraints. Finally, a constructive heuristic mileage-saving algorithm based on load capacity and battery capacity constraints was designed to solve the model, resulting in an effective logistics drone distribution route planning scheme.

The remainder of this study is organized as follows. In Section 3, the characteristics of drone logistics distribution scheduling problem are described, a mathematical model is established, and the chance constraints are transformed for linearization. In Section 4, the effectiveness of the proposed model and algorithm is verified using classical examples of different scales. In final section, the study is summarized, along with prospects for future research.

3. Methodology

3.1 Problem description

In E-commerce, the logistics delivery network within urban parks is represented by a directed graph G consisting of a goods distribution center O and N . The set $N = \{1, 2, \dots, n\}$

represents the customer points with demands. The demands of customers are independent, and their distribution is unknown. The total demand at any customer point does not exceed the maximum payload of one drone, and the total demands at any customer point are satisfied by one drone in a single delivery. The set of drones, denoted by K , comprises drones of identical models, payload, and battery capacities. All drones depart from the distribution center, complete delivery tasks, and return to the distribution center. During delivery, drones only unload goods and do not load new ones. The power consumption coefficient of drones is constant e , and the power consumption of drones is directly proportional to the flight mileage. The question is how to plan the number of drones and their routes to minimize the total flight mileage of all drones for completing all delivery tasks within constraints on drone payload and battery capacity.

To clearly express the research question, this study defines the symbols and variables shown in Table 2.

Table 2. Definitions of symbols and variables

Symbols	
O	Goods distribution center;
N	Set of customer demand points, $\forall i, j \in N$;
K	Set of drones, $\forall k \in K$;
e	Power consumption per kilometer for drones;
Q	Battery capacity of drones;
L	Maximum flight mileage for one drone in a single delivery;
q	Maximum effective payload weight of drones;

l_{ij}	Distance between node i and node j ;
M	A sufficiently large positive integer;
Variables	
d_i	Demand at customer demand point i , a random variable;
p_{ik}^1	Remaining battery level of drone k upon arrival at node i ;
p_{ik}^2	Remaining battery level of drone k upon departure from node i ;
Decision Variables	
x_{ijk}	If drone k traverses arc (i, j) , it takes 1; otherwise, 0.
y_{ik}	if drone k visits node i , it takes 1; otherwise, 0.

To ensure the rigor of the problem formulation, basic assumptions were made based on previous studies.

(1) The influences of the drone flight resistance on the power consumption were neglected, and the drone power consumption was proportional to the flight mileage.

(2) All drones departed from the same center or warehouse, and the starting point was the same.

(3) All the drones maintained a fixed and stable flight speed during delivery.

(4) All the drones were assumed to avoid obstacles and hazardous areas.

(5) The demand at any demand point is less than the payload capacity of one drone, considering only the weight of the goods and not their volume.

(6) The positions of the demand points are known. The demands at each demand point are served by only one drone, and the demands are indivisible.

3.2 Model establishment

First, a drone delivery route optimization model was established under demand uncertainty as follows:

$$\min z = \sum_{k=1}^K \sum_{i=0}^N \sum_{j=0}^N c_{ij} x_{ijk} \quad (1)$$

$$s.t. \quad \sum_{k=1}^K y_{ik} = 1, \forall i \in N \quad (2)$$

$$\sum_{i=1}^N x_{ijk} = y_{ik}, \forall j \in N, \forall k \in K \quad (3)$$

$$\sum_{j=1}^N x_{ijk} = y_{ik}, \forall i \in N, \forall k \in K \quad (4)$$

$$\sum_{i,j \in S \times S} x_{ijk} \leq |S| - 1, S \in \{1, 2, \dots, N\}, \forall k \in K \quad (5)$$

$$P \left[\sum_{i \in N} \tilde{d}_i y_{ik} \leq q \right] \geq 1 - \alpha, \forall k \in K \quad (6)$$

$$0 < p_{0k}^1 \leq Q, \forall k \in K \quad (7)$$

$$p_{ik}^1 = p_{ik}^2, \forall i \in N, \forall k \in K \quad (8)$$

$$p_{jk}^1 \leq p_{ik}^2 - x_{ijk} l_{ij} e + M(1 - x_{ijk}), \forall i \in N, \forall j \in N, \forall k \in K \quad (9)$$

$$p_{ik}^1 \geq 0, \forall i \in N, \forall k \in K \quad (10)$$

$$x_{ijk} \in \{0, 1\}, \forall i \in N, \forall j \in N, \forall k \in K \quad (11)$$

$$y_{ik} \in \{0, 1\}, \forall i \in N, \forall k \in K \quad (12)$$

Equation (1) is the objective function, which represents the minimum sum of the flight distances of all drones involved in the delivery. Equation (2) indicates that each demand point is visited once by a single drone. Equations (3) and (4) ensure that each arc is traversed only once and that all drones return to the distribution center after visiting the last node. Equation (5) eliminates the sub-circuit constraints. Equation (6) imposes an opportunity constraint on drone payload capacity, where $P[*]$ is the feasibility of the event $[\ast]$, that is, the probability that the event of a drone's payload not exceeding its maximum payload capacity is not less than the confidence level given by the decision-maker $1 - \alpha, \alpha \in [0, 1]$. Equation (7) defines the initial battery-level constraint for the drones, which should not exceed the maximum battery capacity. Equations (8), (9), and (10) represent the battery-level constraints for drones when each demand node is reached. Equation (8) indicates that the drone's battery level remains unchanged before and after visiting node i . Equation (9) shows that the battery level decreases when a drone passes from nodes i to j . Because the maximum battery capacity of the drones is Q and $M \leq Q$, replacing M with Q can significantly reduce the solution space. Equation (10) requires that the drones have battery power upon reaching any node. Equations (11) and (12) specify the range of decision variable values.

3.3 Deterministic equivalent treatment of opportunity constraints

Customer demands are independent and their distribution is unknown. For any customer $\forall i \in N$, the demand \tilde{d}_i is a set of independent random variables with a mean μ_i and standard deviation σ_i . The mean and standard deviation of the total customer demand for the drone k 's flight path is given by:

$$M_k = \sum_{i \in N} \mu_i y_{ik}, \forall k \in K \quad (13)$$

$$S_k = \sqrt{\sum_{i \in N} \sigma_i^2 y_{ik}^2}, \forall k \in K \quad (14)$$

If there exists a constant τ such that:

$$P \left[\left(\sum_{i \in N} \tilde{d}_i y_{ik} - M_k \right) / S_k \leq \tau \right] = 1 - \alpha \quad (15)$$

Constraint (6) can be represented by the following deterministic constraint:

$$M_k + \tau S_k \leq q, \forall k \in K \quad (16)$$

Expanding (16) yields:

$$\sum_{i \in N} u_i y_{ik} + \tau \sqrt{\sum_{i \in N} \sigma_i^2 y_{ik}^2} \leq q, \forall k \in K \tag{17}$$

Theorem 1. If the demand satisfies (1) the probability distribution of the demand \tilde{d}_i is independent; (2) for $\forall i \in N$, $\sum_i (\tilde{d}_i y_{ik} - M_k) / S_k$ and $(\tilde{d}_i - u_i) / \sigma_i$ have the same distribution; (3) σ_i^2 is a constant multiple of u_i (i.e., $\sigma_i^2 = \pi u_i$, π : a constant multiple), then there exists a constant \bar{q} in which the linear constraint (18) and the opportunity constraint (6) are equivalent.

$$\sum_{i \in N} u_i y_{ik} \leq \bar{q}, \forall k \in K \tag{18}$$

Proof: The decision variable y_{ik} in this study is a 0-1 decision variable. Thus, $(y_{ik})^2 = y_{ik}$. Therefore:

$$S_k = \sqrt{\sum_{i \in N} \sigma_i^2 y_{ik}^2} = \sqrt{\sum_{i \in N} \pi u_i y_{ik}^2} = \sqrt{\pi \sum_{i \in N} u_i y_{ik}^2} = \sqrt{\pi M_k} \tag{19}$$

Substituting Eq. (19) into Eq. (16) yields:

$$M_k + \tau \sqrt{\pi M_k} \leq q \tag{20}$$

This yields Eq. (21):

$$M_k \leq \frac{2q + \tau^2 \pi - \sqrt{\tau^4 \pi^2 + 4q\tau^2 \pi}}{2} = \bar{q} \tag{21}$$

3.4 Design of constructive heuristic algorithm

The problem considered in this study was NP-hard. It cannot be effectively solved using traditional approaches, such as genetic and savings algorithms. Therefore, we proposed an improvement strategy, namely, prioritizing the insertion of arcs with larger savings values, to obtain high-quality initial solutions. A heuristic savings algorithm that integrates the problem characteristics was constructed.

3.4.1 The classic savings algorithm

The savings algorithm proposed by Clarke and Wright [22] was used to solve the traveling salesman problem (TSP), where a traveler starts from a city, visits n cities only once, and returns to the starting point, to plan the route with the lowest travel cost or the shortest distance. When there are multiple travelers and each city can only be visited by one traveler, the problem becomes a vehicle routing problem (VRP). Therefore, the saving algorithm can also be employed to solve the VRP. The principle of the traditional heuristic saving algorithm is to consider n visit locations as nodes, with one of the nodes selected as the depot (starting point). For example, taking node 1 as the depot, connecting each node to the depot forms round-trip routes $1 \rightarrow j \rightarrow 1$ ($j(j = 2, 3, \dots, n)$), an initial path scheme comprising $n - 1$ routes were obtained. The total distance traveled by the traveler is expressed as:

$$z = 2 \sum_{j=2}^n c_{1j}$$

where c_{1j} is the distance of the route from the depot $j(j = 2, 3, \dots, n)$ and we assumed that $c_{ij} = c_{ji}, \forall j$. When connecting nodes i and j , if the traveler no longer passes through arcs $(i, 1)$ and $(1, j)$ on the arc (i, j) , the savings value $s(i, j)$ of the route segment caused by this can be calculated as follows.

$$s(i, j) = 2c_{1i} + 2c_{1j} - (c_{1i} + c_{1j} + c_{ij}) = c_{1i} + c_{1j} - c_{ij}$$

3.4.2 Improved savings algorithm

Based on the principle of the traditional savings algorithm, for different point i and point j , if the value of $s(i, j)$ is greater, the distance saved by the drone when passing through the arc (i, j) will be greater. Therefore, to minimize the drone's flight mileage, the arc (i, j) corresponding to the larger $s(i, j)$ values should be prioritized in the route, allowing the drone to pass through these arcs as much as possible. The traditional savings algorithm was improved by this study through the addition of insertion conditions for the arc (i, j) , to generate a higher-quality initial solution. Subsequently, the arc in the initial route were evaluated to determine whether each route's distance and payload were within the drone's maximum distance and payload constraints. If not, the arc was removed. Finally, it was verified whether the routes within the maximum distance and payload constraints of the drone could meet all demands. Thus, a feasible set of routes was obtained.

This improved heuristic approach not only generated a better initial solution but also enhanced the efficiency and quality of problem-solving. By prioritizing routes with greater distance savings, the algorithm is more likely to find optimal route plans during the initial solution generation phase, reducing the search space in the subsequent optimization processes and accelerating algorithm convergence. Moreover, the proposed algorithm avoids generating solutions that are not in line with the actual flight capabilities of the drone, thereby improving the solution quality and feasibility. Thus, ensuring solution quality, the improved savings algorithm proposed in this study minimizes the search space and enhances the algorithm efficiency, providing an effective method for solving drone delivery scheduling problems.

The algorithm flowchart of the constructive heuristic savings algorithm is shown in Fig. 1 to provide a more intuitive presentation of the algorithm designed in this study.

4. Result Analysis and Discussion

To validate its feasibility and effectiveness, the proposed drone delivery route model considering customer-demand uncertainty was tested using randomly generated simulation examples. The solution results and scheduling schemes were compared with those of a deterministic drone delivery scheduling model. In addition, to assess the effectiveness of the constructive heuristic savings algorithm designed in this study, multiple classical examples of varying scales were tested, and the results were compared with those of traditional savings algorithms. The algorithm was developed in MATLAB 2018b. All test cases were independently run 10 times on a computer equipped with a Core i9 3.50 GHz processor and 31.7 GB RAM.

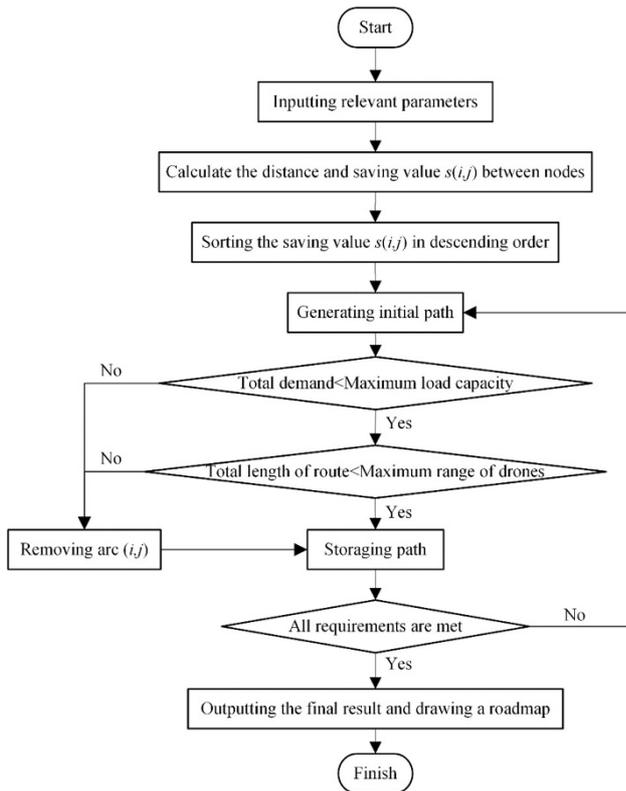


Fig. 1. Algorithm flowchart of the constructive heuristic algorithm

4.1 Parameter settings and case generation

Classical examples of various scales were tested. Table 3 lists the relevant information for these examples. In these examples, the numbers of customer demand points for the small-, medium-, and large-scale cases were 22, 45, and 78, respectively. Node 1 served as the distribution center in all cases, with coordinates (145, 215), (61, 99), and (46, 12), respectively.

In addition, to obtain more applicable drone delivery route schemes, three locations of logistics distribution centers were tested: at the center of the demand points, away from the center of the demand points, and at the edges of the demand points. Fig. 2 illustrates the location maps.

Currently, there are five main types of drones used for logistics delivery. These cases test utilized drone models employed by three major Chinese logistics companies, including China Post, JD.com, and SF Express, as listed in Table 4. Considering the constraints of drone quantity and cost, the China Post Jie Yan TR5 drone model, which has a relatively lower payload capacity, was employed for the small-scale case with 22 demand points. For the medium and large-scale cases, the JD Y-3 and SF Express XAIRWAY drone models were utilized. Additionally, the unit flight cost of the drones was set at 1 CNY per kilometer, and the energy consumption coefficient was set at 1 kWh/km. In all cases, the demand at each point was considered as a set of independent random variables: $\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_n$, and \tilde{d}_i , following a normal distribution with a mean u_i and a variance of $\sigma_i^2 = \pi u_i$. The risk level α of drone overload was set to 0.1.

Table 3. Data for different scale examples

	Node No.	X	Y	Demand Weight (g)	Node No.	X	Y	Demand Weight (g)
Small Scale	1	145	215	0	12	128	231	1200
	2	151	264	1100	13	156	217	1300
	3	159	261	700	14	129	214	1300
	4	130	254	800	15	146	208	300
	5	128	252	1400	16	164	208	900
	6	163	247	2100	17	141	206	2100
	7	146	246	400	18	147	193	1000
	8	161	242	800	19	164	193	900
	9	142	239	100	20	129	189	2500
	10	163	236	500	21	139	182	1800
	11	148	232	600	22	139	182	700
Medium Scale	1	61	99	0	24	93	33	1700
	2	95	7	1400	25	39	45	2200
	3	45	87	100	26	89	33	800
	4	15	47	1600	27	47	77	1600
	5	39	75	2300	28	29	19	2000
	6	55	23	1200	29	13	65	1200
	7	29	71	600	30	33	9	2200
	8	87	79	500	31	63	9	2000
	9	75	63	100	32	41	13	1200
	10	65	61	1300	33	67	75	1400
	11	73	35	2000	34	41	27	2500
	12	17	35	1400	35	49	77	1700
	13	39	99	1800	36	57	81	1900
	14	75	77	700	37	45	5	2000
	15	49	37	800	38	83	7	1500
	16	85	31	2100	39	81	61	200
	17	89	71	800	40	57	81	900
	18	89	43	2400	41	93	89	1000
	19	79	81	2000	42	17	13	600
	20	45	5	1900	43	89	27	1100
	21	93	69	1300	44	7	25	2100
	22	49	69	300	45	35	35	2400
	23	63	25	2600				
Large Scale	1	46	12	0	40	23	86	2600
	2	51	4	1400	41	8	18	500
	3	52	30	1700	42	0	74	200

4	80	70	1700	43	20	44	1400
5	18	90	1600	44	56	7	1100
6	59	39	1900	45	14	10	2100
7	23	59	1700	46	88	40	2000
8	77	48	500	47	96	38	1800
9	82	30	1200	48	59	31	200
10	18	82	400	49	22	87	1900
11	11	41	200	50	59	36	1200
12	7	9	200	51	24	83	2200
13	88	33	2600	52	83	37	1400
14	23	88	200	53	53	5	2300
15	0	76	700	54	0	37	2500
16	85	34	1800	55	84	78	800
17	17	46	600	56	27	93	300
18	52	10	600	57	61	12	900
19	13	45	1800	58	69	43	2100
20	19	85	200	59	54	9	300
21	86	77	1400	60	20	98	2200
22	54	6	500	61	18	50	600
23	83	32	900	62	25	84	2200
24	15	10	400	63	31	69	2000
25	53	5	300	64	58	36	2000
26	14	42	1500	65	0	11	500
27	13	10	400	66	61	36	1300
28	57	32	2300	67	18	49	600
29	20	85	700	68	57	8	1400
30	65	46	2100	69	0	49	1600
31	61	42	400	70	56	8	1200
32	87	52	100	71	62	45	2300
33	79	51	600	72	83	32	500
34	25	91	1600	73	53	10	1200
35	89	34	400	74	82	53	1500
36	26	100	2000	75	21	85	2100
37	0	88	500	76	64	41	400
38	63	43	1400	77	80	50	2300
39	55	10	1400	78	16	10	1900

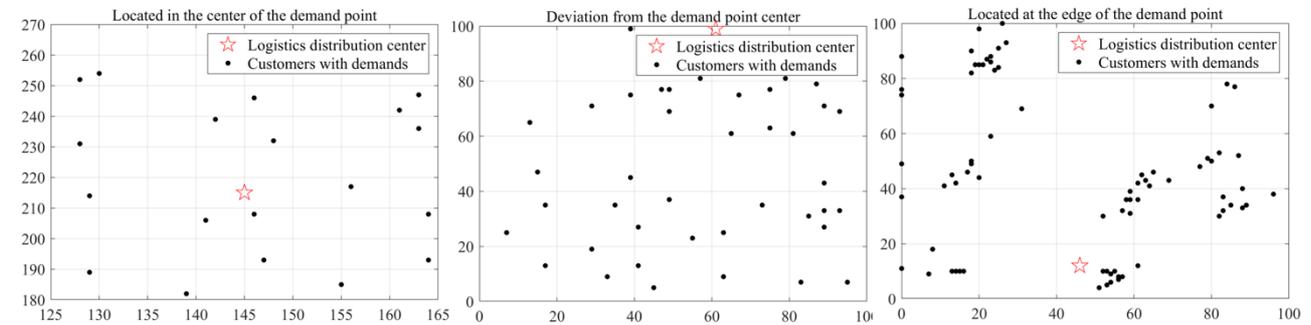


Fig. 2. Locations of logistics distribution centers

Table 4. Specifications of the drones

Drone Model	Payload (kg)	Endurance (kWh)
China Post Jie Yan TR5	6	20
JD Y-3	10	20
SF Express XAIRWAY	10	20

4.2 Algorithm performance testing

To assess the performance of the heuristic algorithm proposed in this study, it was compared with the traditional savings algorithm. Actual cases of different scales were used for testing, and for π set to 0.5, the results obtained by the two methods are presented in Table 5. Additionally, the average payload rate was introduced as a metric to indicate the capacity of the drones to perform delivery tasks. The average payload rate L_r is given.

$$L_r = \frac{\sum q_k}{|K| \times q}$$

where $q_k (\forall k \in K)$ is the actual payload of drone k that is performing the delivery tasks. $|K|$ represents the total number of drones in the fleet. To better illustrate the differences in solution quality between the two methods, the objective function value was compared with the average payload rate, as shown in Fig. 3.

As shown in Table 5 and Fig. 3:

- (1) The average payload rates of the drone delivery routes obtained using the two methods are the same. This means that the total payload capacity of the drones required to fulfill all customer delivery demands was also the same.
- (2) In all tested cases, the heuristic algorithm produced better drone delivery routes, fulfilling demands with shorter distances and cutting costs, despite the same payload rate.
- (3) Flight routes obtained through the two methods differed. The flight routes obtained by the two methods were distinct for different case scales and positions of the logistics distribution centers, as illustrated in Fig. 4.

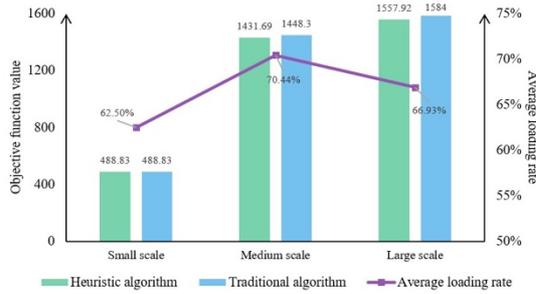


Fig.3. Comparison of solution quality between traditional savings algorithm and the heuristic method

Table 5. Comparison of results between traditional savings algorithm and the heuristic method

Case	Drone Quantity	Method	Objective Function Value	Payload
Small Scale	6	Heuristic Algorithm	488.83	[4.1, 3.4, 3.9, 3.9, 4.0, 3.2]
		Traditional Algorithm	488.83	[3.9, 3.9, 4.1, 3.2, 4.0, 3.4]

Medium Scale	9	Heuristic Algorithm	1431.69	[7.4, 7.3, 7.0, 7.3, 7.1, 7.2, 7.1, 6.1, 6.9]
		Traditional Algorithm	1448.3	[7.3, 7.3, 7.5, 7.2, 7.5, 7.4, 6.3, 7.4, 5.5]
Large Scale	14	Heuristic Algorithm	1557.92	[7.4, 6.0, 7.4, 6.3, 7.1, 6.0, 7.5, 7.5, 6.1, 7.2, 6.5, 7.0, 7.2, 4.5]
		Traditional Algorithm	1584.0	[7.4, 6.5, 7.5, 7.0, 6.8, 7.1, 4.4, 6.9, 7.4, 7.1, 7.0, 6.0, 7.4, 5.2]

This study compared and analyzed the optimal drone delivery schedules generated by deterministic model and opportunity-constrained model considering demand uncertainty. Table 6 and Fig. 5 present the experimental results and the drone flight routes generated using the deterministic model. Table 7 lists the comparison results of the deterministic and opportunity-constrained models.

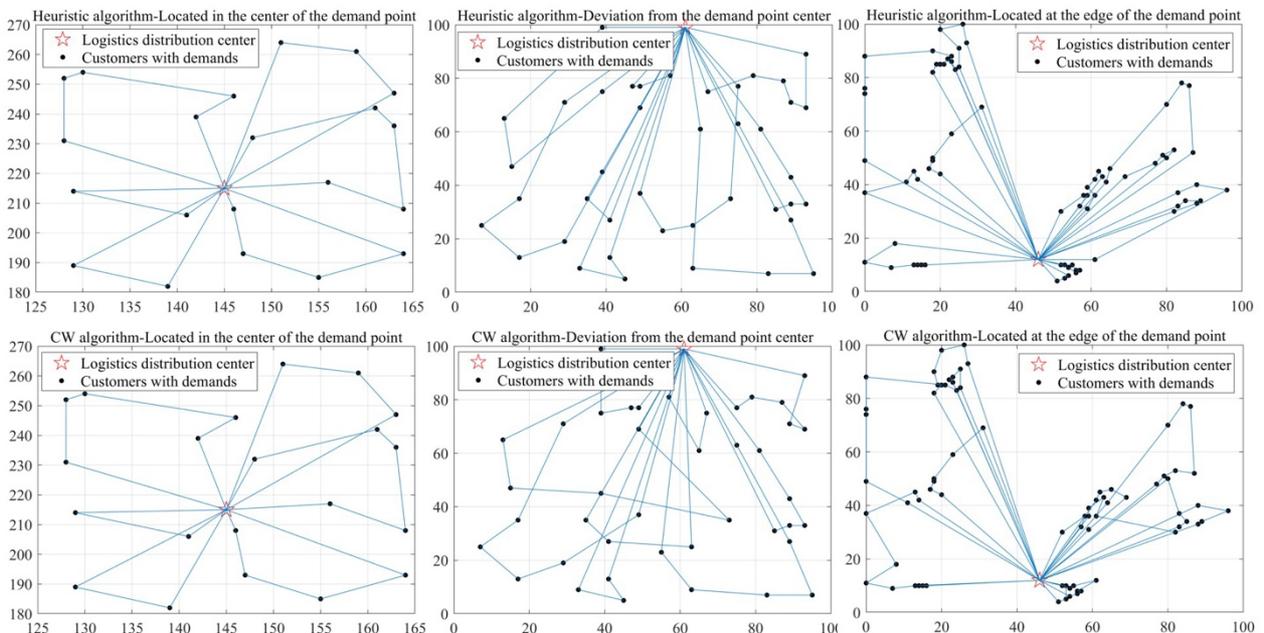


Fig. 4. Flight routes generated by the traditional savings algorithm and heuristic method

Table 6. Experimental results of the deterministic model

Case	Objective Value	Drone Quantity	Payload
Small Scale	488.83	4	[5.7, 5.3, 5.9, 5.6]
Medium Scale	1431.69	7	[9.9, 9.4, 9.9, 9.7, 9.8, 8.5, 6.2]
Large Scale	1557.92	10	[9.8, 9.8, 7.9, 9.5, 9.4, 9.4, 9.9, 9.2, 9.6, 9.2]

Medium Scale	Deterministic Model	7	1431.69	90.57%
	Chance-constrained Model	9	1448.3	70.44%
Large Scale	Deterministic Model	10	1557.92	93.7%
	Chance-constrained Model	14	1584.0	66.93%

Table 7. Comparison of results of deterministic and opportunity-constrained models

Case	Model	Drone Quantity	Objective Function Value	Average Payload Rate
Small Scale	Deterministic Model	4	488.83	95.75%
	Chance-constrained Model	6	488.83	62.5%

From Tables 6 and 7, for the three different case scales with varying demand points, the payload rate of the drones was above 90%, with an average payload rate of up to 93.34%. Compared with the results obtained by opportunity-constrained model considering uncertain customer demands, the payload rates of the deterministic model were higher by 33.25%, 20.13%, and 26.77%, with an average increase of 26.72%. Although the deterministic model's drone flight routes more efficiently use the drone capacity, they could not

accommodate sudden increases in customer demand due to the absence of spare payload space. Therefore, the drone deployment schemes of deterministic model may not be suitable for continuously changing customer demands in E-commerce.

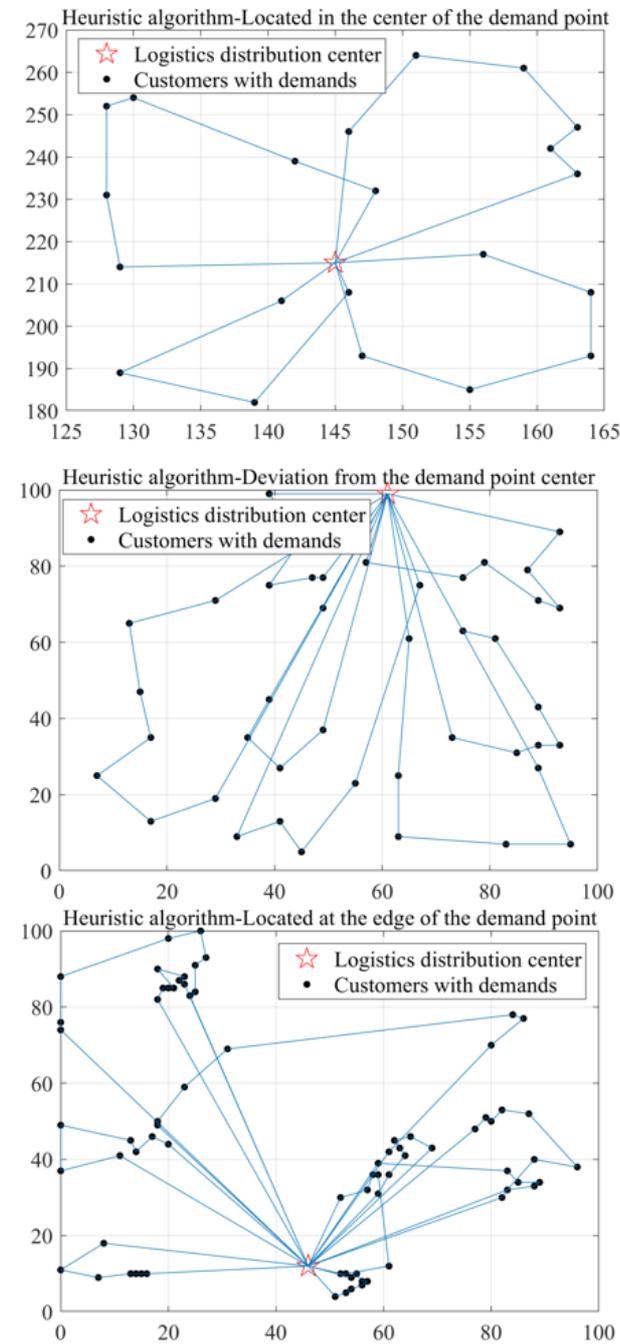


Fig. 5. Schematic diagram of drone flight paths under examples of different scales

4.3 Parameter sensitivity analysis

In previous studies, customer demands are fixed and assumed to be the mean u_i . By contrast, our opportunity-constrained model considers random customer demands \tilde{d}_i , with known mean u_i and covariance σ_i . The difference lies in the maximum payload constraint of the drones. Given that the probability of the drone payload not exceeding the capacity limit is not less than $1-\alpha$, the value of \bar{q} is determined by parameter π , Therefore, different values of π led to

different solutions produced by the opportunity-constrained model, as shown in Table 8 and Fig. 6.

Table 8. Results for different π values

Case	π	\bar{q}	Obj	Load(kg)
1	0.0	6.00	375.28	[5.4, 5.6, 5.9, 5.6]
	0.1	5.09	427.40	[4.9, 4.8, 3.4, 4.5, 4.9]
	0.2	4.75	449.52	[4.5, 4.0, 4.6, 3.6, 4.5, 1.3]
	0.3	4.51	457.75	[4.5, 4.5, 4.0, 3.8, 3.6, 2.1]
	0.4	4.31	469.29	[3.2, 3.4, 3.5, 4.3, 4.2, 3.9]
2	0.0	10.0	1169.40	[9.8, 9.9, 7.9, 9.9, 9.3, 9.1, 7.5]
	0.1	8.80	1294.82	[7.6, 8.7, 7.7, 8.4, 8.4, 8.6, 7.9, 6.1]
	0.2	8.34	1312.36	[7.6, 7.9, 8.3, 7.7, 8.3, 7.3, 8.3, 8.0]
	0.3	8.01	1364.99	[7.7, 8.0, 8.0, 7.3, 7.5, 8.0, 7.2, 7.9, 1.8]
	0.4	7.74	1396.69	[7.7, 6.0, 7.3, 7.7, 7.4, 7.3, 7.6, 6.3, 6.1]
3	0.0	10.0	1251.81	[9.7, 9.9, 9.6, 9.4, 9.8, 9.7, 9.9, 9.8, 9.9, 6]
	0.1	8.80	1369.79	[8.4, 8.0, 8.7, 8.5, 6.7, 8.4, 7.2, 8.7, 8.7, 8.2, 3.5]
	0.2	8.34	1447.74	[8.2, 8.0, 8.3, 8.2, 8.0, 8.0, 8.3, 7.8, 8.2, 7.9, 7.6, 5.2]
	0.3	8.01	1495.67	[8.0, 7.3, 7.1, 7.8, 7.9, 7.9, 7.9, 6.7, 8.9, 6.0, 6.6, 7.4, 5.1]
	0.4	7.74	1547.51	[6.8, 7.2, 7.3, 7.1, 7.7, 6.4, 7.6, 7.6, 7.7, 6.4, 7.1, 7.7, 7.1]
	0.5	7.52	1557.92	[7.4, 6.0, 7.4, 6.3, 7.1, 6.0, 7.5, 7.5, 6.1, 7.2, 6.5, 7.0, 7.2, 4.5]

Fig. 6 illustrates the following:

(1) The drone’s flight distance gradually increased as π increased, implying a continuous increase in the total cost. This could be because the increase in π corresponds to a larger covariance of customer demands $\sigma_i^2 = \pi u_i$ that is, greater volatility of random customer demands. To satisfy all customer demands, more drone resources and longer flight distances are required, resulting in higher transportation costs.

(2) When $\pi = 0$, the problem is transformed into a deterministic problem, where customer demands are known to be u_i . Consequently, the opportunity-constrained model is aligned with the deterministic model, resulting in identical drone flight costs and route schemes.

(3) The payload rate of drones decreased as π increased. This was because as the volatility of random customer demands increased, reducing the actual payload of drones could increase the spare payload to meet all uncertain customer demands, improving the robustness of the scheme.

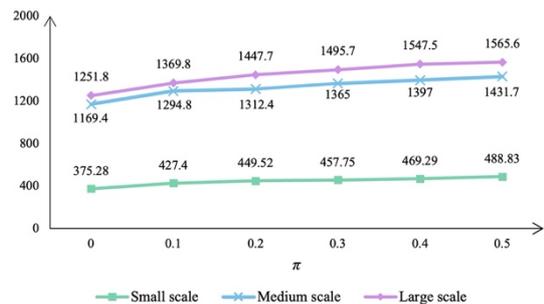


Fig. 6. Variation in the objective function values for different π values

5. Conclusion

An integrated drone delivery route optimization model was carefully crafted to tackle the complexities and uncertainties inherent in urban E-commerce logistics, particularly those posed by fluctuating customer demands. Recognizing the need for efficiency and flexibility in delivery routes, this model aims to optimize drone deliveries, ensuring faster, more reliable service. To rigorously evaluate its effectiveness, the model's performance underwent extensive testing through a series of experiments on classical cases of varying scales. These cases encompassed a wide range of scenarios, from small-scale residential deliveries to large-scale commercial shipments. The results were promising, highlighting the model's versatility and potential for widespread application. The key findings from these experiments are summarized below:

(1) The proposed model, incorporating deterministic integer linear programming and opportunity constraints, effectively handles demand uncertainty. It maximizes drone payload and minimizes total flight mileage, optimizing resource allocation under limited drone endurance and payload constraints.

(2) The effectiveness of the drone delivery route optimization model was validated through experiments on classical cases. Compared to traditional savings algorithms, our approach achieved better route schemes, demonstrating its superiority in addressing uncertain customer demands.

(3) From a managerial perspective, this model provides practical insights for businesses to make wiser decisions in resource allocation and utilization, enhancing service quality, customer satisfaction, and gaining a competitive edge in the market.

This study leveraged historical customer demand data to linearize opportunity constraints, facilitating the solution process. A heuristic algorithm tailored to the problem's characteristics was designed, along with a strategy for generating high-quality initial solutions. Future research directions include exploring alternative methods for transforming opportunity constraints, considering uncertain delivery times, and developing precise algorithms for more accurate drone delivery route schemes.

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