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Waterflood Optimization: Review on Gradient-Ensemble Based Optimizers and Data Driven Proxies

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Abstract

Waterflooding is the most common secondary recovery technique used in oil and gas industries today, owing to its cheap investment cost and easy implementation. However, major challenges are encountered in terms of oil sweep efficiency and breakthrough time which poses a risk to production and economic lifecycle of reservoirs. As a result, reservoir engineers are tasked with improvising optimal production strategies with the goal of maximizing profit. This review extensively describes some common optimization techniques reported in improving oil reservoir production. Also, their formulation, limitations and advantages with respect to production rates, oil well placement and control, inter-well connectivity and reservoir sweep efficiency were reviewed. While there are several optimization algorithms used in waterflooding, the emphasis in this work involves only the gradient and data driven optimizers since it is impossible to cover all optimization technique in a single review paper. Basically, no algorithm has been globally accepted as superior to the other since the sole aim is to improve productivity and economic profit, and each of these techniques has its unique practicability. However, when considering factors like design limitations, computational and economic cost, implementation timeframe, availability of data, some technique may suffice.

Keywords: Enhanced Oil Recovery, Waterflooding, Self-optimizing control, Net Present Value, Optimization

1. Introduction

Waterflooding recovery is an enhanced oil recovery technique that involves the injection of water into an oil reservoir thereby increasing the underground pressure [1–3]. This pressure increase causes oil to flow to the surface. Waterflooding is one of the most used enhanced oil recovery technique due to the fact that water is readily available and discounted to sustain [4]. Waterflooding enhanced oil recovery has shown to be predominant on the basis of [5]; water availability, injection simplicity, sweep efficiency and ability of water to displace oil. However, with the efficacy of implementing waterflooding recovery technique, about 35% of the original oil in place (OOIP) is produced [6]. In reality, conventional waterflooding schemes may not suffice in increasing the yield of produced oil due to a poor sweep efficiency [7]. So, to account for oil productivity, a system involving reservoir management lifecycle called Closed-Loop Reservoir Management (CLRM) is developed to tackle this production shortcoming [8]. CLRM basically involves the application of real time data and mathematical models to propagate the long and short term decision making strategies for new and existing oil reservoirs [8], [9]. CLRM primarily consist of two workflows; the first is history matching which relies on historical data assimilation; the second is the optimization of control inputs which relies on some optimization algorithms (Fig. 1). The aspect of optimization is the primary focus of this review.

2. Waterflood Optimization

Waterflooding problems are commonly formulated as optimization problems. The problem is usually formulated to optimize a key performance index by manipulating the optimal variables such as production and injection wells, bottom hole pressure [10], [11]. In reservoir engineering, the process of finding varied optimum reservoir parameters such as injection and production rates etc., is known as well control optimization [12]. The study on optimization techniques for waterflooding has over the years been considered a pathway for successful realization of new and existing oil reservoir production, and authors tend to be explicit on the choice of publishing organization due to field specific relevance and accessibility. Fig. 2 presents a word cloud on selected publishers used by authors for waterflood optimization problems.

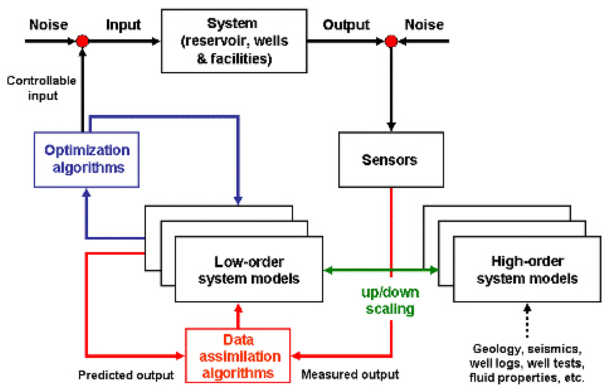

Fig. 1. Closed-Loop Reservoir Management (CLRM) Workflow [44].

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3. Gradient based Waterflood Optimization

Real reservoirs are inherently heterogeneous which makes it almost difficult to obtain actual solutions to production efficiency. So, in this case, to account for such scenarios, geological and economic uncertainties are introduced into the optimization sets. One of the methods that has found usability for this kind of problem is what is called ensemble Optimization(EnOpt) [13]–[18]. EnOpt has found keen interest by researchers due to its ease of approximating ensemble gradients rather than adjoint estimations [19]. The uncertainty are approximated by distributing the performance indicator into a finite number of possible outcome and then optimized over the production period of the reservoir [20]. Successful approaches have been reported for several problems including production optimization [21–25]. Upstream oil exploration are quite complex, hence utilization of conventional optimization strategy will not suffice because it only provide solutions of single uncertainty realization [26]. Real world optimization problems are faced with constitutive challenges such as data uncertainty, difficulty in implementation of generated optimal solutions, large scale problems even though global optimization may be practically applicable. Beyer and Sendhoff [27] described scenarios where singularities in global optimal design are experiential. They observed that global optimization formulation described previously can only be suitable for static systems. Real world problems of optimization are dynamic, which makes the optimality effectiveness unstable. General optimization technique is shown to be sensitive to minor changes. However, to deal with sophistication of design objective, the robustness of systems that are insensitive to uncertainties are identified. The idea of formulating robust designs in the presence of uncertainty is what is referred to as robust optimization [28]. The common uncertainty cases encountered in design process are: A) uncertainty in operating conditions. B) uncertainty in design parameters. C) uncertainty that is obvious in the performance of a system. Robust optimization is an optimization approach used to consider investigation of optimal parameter under system uncertainty [29]. The concept of robust optimization focuses on specific fields that exhibit probabilistic design theories, which is closely related to dynamical approach to system observation. The concept of robust optimization has gained more proximity to robust control techniques. In robust optimization, the investigated reservoir model is not usually stochastic but rather deterministic. One of the major parameters in robust optimization that is tractable is what is referred to as the uncertainty parameter sets [30]. They are values of parameter uncertainty that are considered in optimization and they are usually specified by the user. In robust optimization, parameters can be expressed either linearly or nonlinearly depending on the nature of uncertainty. Ben-tal and Nemirovski [31] described the mathematical illustration of robust designs applied to linear programming problems, were the robust counterpart;

$$\min_{t,x}\{t: t \geq c^T x,\ \ Ax \geq\ b \qquad \forall(c\ , A, B) \in U\} \qquad (1)$$

for an uncertain linear programming problem of the form;

$$\left\{\min_{x}\{c^T x\ :\ Ax\ \geq b\} | (c, A, b) \in U \subset R^n x R^{m\,x\,n}\ x\ R^m\right\} \qquad (2)$$

Is comparable to a very computational approachable case, provided the uncertainty set is computationally responsive.

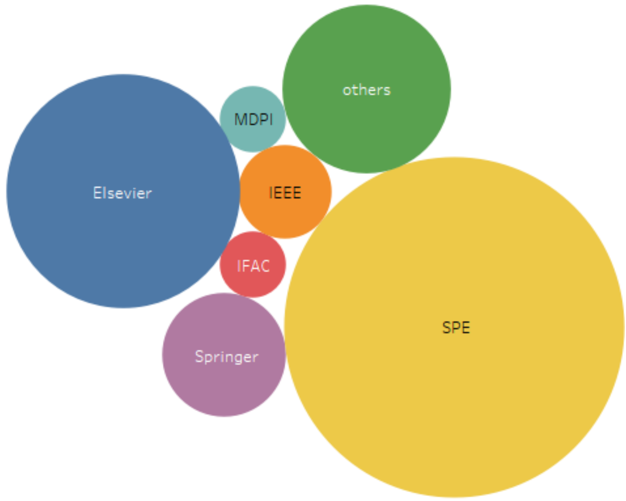

Fig. 2. Selected Publishers for Research In Waterflood Optimization.

Waterflood optimization problems are in most cases modelled-based, however there are inherent limitations that may arise from unknown reservoir implicit behaviour and varying economic conditions such as currency devaluation from market instability. One way to account for such scenarios is to consider optimization scenarios (commonly called scenario based optimization) that will leverage the reservoir data for better performance. Siraj et al [18] investigated the applicability of a scenario-based optimization to waterflooding robust optimization. The author described the possibilities of providing a robust performance for various geological uncertainties. Worst case optimization has been given for deterministic models where the uncertainty is designed as a variable say θ which takes values in a deterministic set Θ [32]. The approach of optimizing a waterflooded reservoir through scenario-based optimization is to distribute the dimension of the uncertainty and then to establish a worst-case scenario optimization basis.

Another aspect of robust optimization that involves system randomness is the stochastic based robust optimization. Stochastic optimization involves the use of nature-based algorithm to optimize a process in the presence of system randomness [33]. Over the past few years, this method of optimization has seen a significant increase in usage trend both for businesses, sciences and engineering. System randomness usually occur in two distinct ways. Either via objective function or process constraints. Moraes et al [34] presented a system that integrates stochastic gradient and multiscale forward simulation for robust optimal well control of waterflood reservoirs with geological uncertainties. Here, the authors considered well control parameters such as the pressure, rate of valve settings for different well configurations. The multiscale simulation was used to evaluate the response of the model, while stochastic simplex approximate gradient was used to compute the gradient of the objective function by implementing forward simulation reaction. Stochastic optimization based on evolutionary technology was also implemented by Ambia [35] to optimize the waterflooded performance index such as the NPV and recovery factor. A synthetic model was built to determine the optimum well pattern, spacing, production and injection scheme that will improve the NPV and recovery factor.

Capolie et al [36] investigated the efficacy of open and closed loop optimization applied to oil reservoir waterflooding reservoir to maximize the NPV and RF. In robust optimization, it is convenient to incorporate the standard deviation to actual optimization objective function. As a case study, Wellano et al [37] implemented a combination of mean and standard deviation of the objective function (NPV) as a single function, and a risk factor which recognizes a trade-off between the mean and standard deviation of NPV. Secondly, the NPV of the reservoir and its corresponding standard deviation were included as a set of constraints. However, this optimization strategy was applied both to single and multi-objective cases. For the multi objective case a formulation based on the so called pareto idea was used.

Risk management in robust optimization techniques have being shown to play a very vital role in establishing successive reservoir modelling decisions that are faced with uncertainties for diverse traditional optimization approaches [38]. Siraj et al [39] addresses the idea of risk management for a reservoir with deviation extrapolation and how the risk can be implemented to the objective function. In the literature, geological and economic uncertainties were considered. One of the risk measures considered is worst case optimal approach, and the conditional value at risk approach [40]. Since waterflooding optimization is a large non-linear optimization, gradient based optimization techniques could be used to obtain a base approach. Gradient optimization are used by solving a system of adjoint equations to obtain the gradients [41–43]. This will be discussed in detail in subsection 3.2 of this article. We've seen that most traditional robust optimization are carried out in an open loop(offline) fashion and as such the models are not usually validated. However, Siraj et al [44] had identified the robust optimal designs of waterflooded reservoirs in an online fashion using the so called residual analysis. The analogy was to find a way to reduce the model uncertainty in an online setting. As the reservoir models are nonlinear in nature, a deterministic metric such as the best fit ratio (BFR) is used in defining the invalidation sets. The residual is said to be the difference between measured output and the computed output [44]

$$\epsilon = y - \hat{y} \quad (3)$$

The best fit ratio (BFR) is given as [46]:

$$BFR = 100\% * \max\left(1 - \frac{\|\epsilon\|_2}{\|y - y\|_2}, 0\right) \quad (4)$$

The BFR is often used in system identification. A low BFR shows a poor fit to data while a high BFR shows a good fit.

In most waterflooded oil reservoirs, water injection rates are commonly used as decision parameter that affects the economic feasibility of the project. However, in some literatures, the compatibility of the injected water are being studied with respect to the reservoir type [45]. One of this compatibility is studied on low salinity waterflooded reservoirs. Although low water salinity has shown to be a better system in yielding optimal recovery, conventional recovery techniques are still common choices in terms of sweep efficiency. With the great impact of unconventional waterflooding, the scheme greatly depends on wettability conditions. Wettability are conditions were the tendencies of a fluid spreading over a rock surface are efficient. Wettability parameters are shown to affect the optimal recovery process over a production trajectory. Wettability are measured by considering the contact angle θ or through the interfacial force between two fluids that are immiscible when in contact with a solid. Dang [45] defines the contact angle used to illustrate the wettability, which is the tangent to the water-oil surface estimated through the water phase. Here, the author presented a well placement robust optimization strategy for low salinity waterflooding case. The optimization was presented on several geological uncertainty realizations. The results were asserted for optimal wettability alterations and sweep efficiency. This was done by locating optimal well placement positions. A contact angle of 0 shows a system that is highly water wet, while a contact angle of 180 indicates a system that is oil wet. Table 1 shows a relationship between angle of contact and wettability phase.

Table 1. Relationship Between Angle Of Contact And Wettability Phase [47]

Angle of contact	**Wettability**
0-30	Strong water wet
30-90	Considerable water wet
90	Neutral wettability
90-150	Considerable oil wet
150-180	Strongly oil wet

Yasari and Reza [46] investigated the effect of pareto-based optimization to variabilities in uncertainty realization in reservoir permeability. The idea of the pareto optimization is on the basis of multiple objective functions. The pareto optimality for two objective function is defined by [46]:

$$\forall: f_i(x_{-1}) \geq f_i(x_{-2}) \text{ and } \exists: f_i(x_{-1}) > f_i(x_{-2}),\ i = 1, \dots \dots N \quad (5)$$

The optimum injection policies gave a higher expected net present value and a lower variance. The study gave an efficacy of the pareto-based solutions for the injection wells under uncertainties in reservoir permeability.

System uncertainty in reservoir waterflooding has been described under conditions of high profit and low risk cases, a singularity in financial modelling referred to as 'portfolio selection' or sometimes called mean variance portfolio selection (MVO) formulated by Markowitz [47]. Portfolio selection explains the systematic trade-offs between profit investment in the presence of uncertainty [50], [51]. Mean variance has found tremendous applicability in portfolio analysis and selection due to its primary ability to consider uncertainty. Based on Markowitz's mean variance analysis, associated risks through variances in portfolio selection are identified by measuring the expected value of returns on investment. The returns in investment are maximized via pareto-optimality by setting up the portfolio's associated risks as upper or lower bound [50], [51].

Mean variance analysis (portfolio selection) considers several uncertainties involving monetary policies and product availability thereby maximizing actual(mean) returns and minimizing the variance(risks). This attribute gave it applicability in waterflood optimization problem. Capolei et al [17] applied mean variance selection in optimizing a waterflooded reservoir by considering geological uncertainty. This technique was further implemented by Siraj et al [26] for both geological and economic uncertainty. Economic risks in oil production are reduced by including the expected Net Present Value (NPV) and the risk associated with it in the ensemble of reservoir model. Just like RO, the idea behind portfolio selection basically involves risk reduction, hence risk management tools for such cases are also introduced. One of the commonly used risk management tool in mean variance optimization is the Value-at-risk (VaR) and Conditional Value-at-risk(CVaR) [52]. They are used in portfolio optimization. VaR measures the degree of losses in business or portfolio finances during a specific period of investment [53]. CVaR on the other hand is used to measure the degree of loss that occurs beyond a certain threshold of VaR in an investment [54]. CVaR has shown to be more robust in mitigating risks as compared to VaR [55], [56]. Hanssen *et al* [57] formulated a stochastic reservoir optimization problem based on CVaR to handle oil production constraints. It was further extended to consider multiple risk scenarios [58]. For large ensemble realizations, retrospective optimization was found to be an optional technique [59]. Table 2 gives a summary of performance for Gradient based algorithm.

3.1. Ensemble Kalman Filters (EnKF)

Ensemble based history matching models have found convenience in reducing system non-linearities. One of which is the Ensemble Kalman Filters (EnKF). The EnKF is a type of ensemble base approach that involves predicting and updating reservoir model parameters and states. It is a Monte Carlo approach for data adjustment. The EnKF approach requires no derivation of adjoint equation and backward integration in time [60]. Discrete model equations for Kalman filters in a simple linear system is given by the equation [61]:

$$y_n^f = Ay_{n-1}^f \tag{6}$$

$$C_{y_n^f} = AC_{y_{n-1}^a}A^T + C_\varepsilon \tag{7}$$

$$y_n^a = y_n^f + K_n\left(d_{obs,n} - Hy_n^f\right) \tag{8}$$

$$K_n = C_{y_n^f}H^T(HC_{y_n^f}H^T + C_{d_n})^{-1} \tag{9}$$

$$C_{y_n^a} = (1 - K_nH)C_{y_n^f} \tag{10}$$

y denotes the state vector that is projected. d_{obs} indicates the observed estimate. K_n indicates the Kalman gain parameter matrix at a time index n. C_{d_n} indicates the covariance matrix of the estimated error. $C_{y_n^f}$ indicates the covariance matrix. C_ε represents the model noise. A^T denotes the dynamics of the system at time T. K_n can be derived using several approaches. One way is by solving a least square problem through additional constraints such as the time independent evaluation of the estimated noise [62]. Another way is by implementing the Bayesian inference [63], [64]. EnKF was also extended for nonlinear systems having large degrees of measurement noise, changing Equation 6 with $y_{n+1}^f = F_n(y_n^a)$ such that F_n represents a differentiable function. However, for large scale problems involving the extended Kalman Filter, several alternatives have been introduced [65–68].

Production optimization using EnKF is based on low computational time in the prevailing condition of large reservoir non-linearities and geological uncertainties [69]. EnKF has been reported for several reservoir history matching scenarios [13], [36], [70–77]. Automatic history matching using EnKF was reported by Yaqing and Oliver [78]. EnKF was also used for three phase flow conditions in a waterflood optimization using a quarter five spot well arrangement [79]. Here, the authors investigated the dependence of covariance localization on the dynamics of flow. They used water and gas phase streamlines as a resource for covariance localization. The EnKF follows an ensemble realization vector y_k represented by model prediction vector d_k at time k, dynamic variable m_k^d and a static variable m_k^s while p indicates the prediction state [69];

$$y_k^p = \begin{bmatrix} d_k \\ m_k^d \\ m_k^s \end{bmatrix} \tag{11}$$

d_k could be the bottom hole pressure, well water cut and gas oil ration. m_k^d indicates pressure or phase saturation and m_k^s could be the relative permeability or rock porosity. EnKF was combined with a multi-layered capacitance resistance model (CRM) for waterflood prediction [80]. The EnKF was used to calculate the connectivity coefficients for each layer in the CRM.

Table 2. Pros and Cons Of Gradient Based Algorithms [49] – [61].

Pros	Cons
• Converges faster at a possible solution. • Effective when considering multiple injectors. • Efficient for single history matched solutions.	• Computationally expensive. • High possibility of converging at a local optimum. • Access to simulator source code is needed. • Requires a very good initial guess.

3.2. Optimal Control Theory

Optimal control problem is a gradient driven technique that allows the investigation of control parameters which will minimize or maximize an objective function or cost function through an adjoint (costate) equation [81]. Optimal control tends to adjust control parameters of a dynamic system in an open loop fashion [82]. For every optimal control designs, there are sets of element that must be inherent [83]: a control variable that is chosen from several control sets, the system to be controlled and a state equation that defines the relationship between the control variable. In an optimal control problem, the objective function J [84]:

$$\min_u J(u) = \varphi\left(x(T)\right) + \int_0^T L\left(x(t), u(t)\right)dt \tag{12}$$

Is minimized with respect to a dynamic system;

$$x(t) = f\left(x(t), u(t)\right) \tag{13}$$

Having a state variable $x(t)$ and control inputs $u(t)$, with an initial condition;

$$x(0) = \bar{x}_0 \tag{14}$$

And a constraint variable subject to path;

$$h\left(x(t), u(t)\right) \leq 0 \tag{15}$$

through a control constraint;

$$u(t) \in \mu, \forall_t \in [0.T] \tag{16}$$

Such that;

$$\mu = \{q \in R^m : u_{min} \leq q \leq u_{max}\} \tag{17}$$

In solving a typical optimal control problem for waterflood optimization, two approaches are used; direct and indirect method. Direct method involves computing the derivative of the objective function directly [43], [85–90]. Direct method was implemented in a closed loop reservoir optimal and then compared to the open loop case [91]. Indirect method on the other hand involves a calculus of variation (adjoint function) such that the derivative of Hamiltonian function $H(k)$ is obtained [9]. Equation 12 – 17 was modified for single equality constraint by Agus [92]. Lagrange multiplier was introduced in the constraints to Eq. 18 such that [93];

$$J(u) = \varphi(x(T)) + \int_0^T L(x(t), u(t)) + \lambda^T(t)[A(x(t))x(t) + B(x(t)u(t) - x(t)]dt \tag{18}$$

Were λ is the Lagrange multiplier and T the transpose symbol. Introducing the Hamiltonian function as [93];

$$H(x(t), u(t), \lambda(t) = L(x(t), u(t)) + \lambda^T(t)[A(x(t))x(t) + B(x(t)u(t)] \tag{19}$$

By putting Eq. 19 in Eq. 18;

$$J(u) = \varphi(x(T)) - \lambda^T(T)x(T) + \lambda^T(0)x(0) + \int_0^T \{H(x(t), u(t), \lambda(t)) + \lambda^T(t)x(t)\} dt \tag{20}$$

The first order partial variation of $J(u)$ can be computed for a small change in u [93];

$$\partial J(u) = \left[\frac{\partial\varphi(x(T))}{\partial x} - \lambda^T(T)\right]\delta x(T) + \lambda^T(0)\delta x(0) + \int_0^T \left\{\left[\frac{\partial H(x(T),u(t),\lambda(t))}{\partial x} + \lambda^T(t)\right]\delta x(T) + \frac{\partial H(x(t),\lambda(t))}{\partial u(t)} + \delta u(t)\right\} dt \tag{21}$$

Lagrange multiplier can be set such that;

$$\lambda^T(t) = -\frac{\partial H(x(t),u(t),\lambda(t))}{\partial x} \tag{22}$$

$$\lambda^T(T) = \frac{\partial\varphi(x(T))}{\partial x} \tag{23}$$

Eq. 22 and 23 represents the costate(adjoint) equation of any given system. For unconstrained u, it is optimized for a first order necessary condition of a constrained optimal control;

$$H(x(t), u_{opt}(t), \lambda(t) \leq H(x(t), u(t), \lambda(t) \tag{24}$$

Eq. 24 is called the Pontryagin Maximum Principle [94 - 95].

In oil reservoir waterflood optimal control problems, the calculation generally constitutes a forward integration of the reservoir dynamic system as well as the backward integration of the adjoint equations. The adjoint equations are used to compute the system gradient [96]. waterflood optimal control is made up of [81], [97]:

- Reservoir dynamic system of the form;

$$g(u^k, x^{k+1}, x^k, \varphi) = 0 \tag{25}$$

Where g is a nonlinear function, u is the input vector, k is the system timesteps, x^{k+1} and x^k is the reservoir state, φ is a vector of parameters.

- Initial conditions of the dynamic system [85];

$$x_0 = \bar{x}_0 \tag{26}$$

- A set of injection and production rates at timestep k and $k+1$.
- Adjoint (costate) equations [97];

$$\lambda(k)^T = \left[-\frac{\partial J(k)}{\partial x(k)} - \lambda(k+1)^T \frac{\partial g(k)}{\partial x(k)}\right]\left[\frac{\partial g(k-1)}{\partial x(k)}\right]^{-1} \tag{27}$$

$\frac{\partial J(k)}{\partial x(k)}$ is a vector of partial derivatives of the objective function J^k with respect to state variable x. $\frac{\partial g(k)}{\partial x(k)}$ and $\frac{\partial g(k-1)}{\partial x(k)}$ are the Jacobian of the reservoir dynamics. The objective function could be the NPV or Production profiles. J^k for the NPV is given in the form [98];

$$J^k = \sum_{n=1}^{T} \frac{\Delta t^k}{(1+b)^{\frac{t^k}{365}}} \left[\sum_{i=1}^{N_p}(P_0 q_{o,1}^n - P_{wp} q_{wp,i}^n) - \sum_{j=1}^{N_I}(P_{wI} q_{wI,j}^n)\right] \tag{28}$$

N_p stands for the Number of production wells, N_I is the Number of injection wells, b is the Discount factor, Δt^k is the Time step size, t^k is the evolution time, T is the time unit. The water injection rates are commonly used as the decision variables.

- Final conditions of the adjoint systems.

Taking the reservoir as an equality constraint problem, the objective function J^k is summed up using the Lagrange multipliers [99][101];

$$J = \sum_{k=0}^{k-1} J(k) + \lambda(k+1)^T g(k) = \sum_{k=0}^{k-1} H(k) \tag{29}$$

Production optimization was obtained using an augmented Lagrange method [99]–[101]. This method was compared with the straightforward adjoint equations in the absence of reservoir uncertainty. Whereas, uncertainty cases involving smart wells was also studied [81], [91], [96], [102–106]. Some authors have studied the augmented approaches used in optimal control theory. One of which is the Bang-Bang technique which is applicable when the objective function is linear and the upper and lower bound are the only control constraints [84], [107], [108]. Optimal control is considered efficient due to its fast approach to obtaining solutions, however, one of the major drawbacks is that the gradient of the objective function solely relies on the adjoint equations. And this Adjoint are computationally expensive and obtaining it solution requires knowledge of programming. Table 3 gives a summary of performance for optimal control theory (OCT).

Table 3. Pros and Cons of OCT [85], [86], [93] – [109].

Pros	Cons
• Converges faster at a possible solution. • Effective when considering multiple injectors. • Efficient for single history matched solutions.	• Requires an adjoint equation. • High possibility of converging at a local optimum. • Difficult to implement on complex nonlinear space. • Access to simulator source code is needed. • Requires a very good initial guess.

5. Data-Driven Optimization Approach

Availability of data has become a resource to system performance. With reservoir production data, it is convenient to obtain an optimal reservoir performance without recourse to analytical models. Data driven optimization is basically a black box approach because a prior knowledge of the reservoir geological information is not needed and its simulation time is brief. Data driven approaches does not require a derivative technique for estimating the objective function as opposed to the gradient system of optimization.

In reservoir optimization, several data driven approaches have been looked at with respect to well placement and production. The first discussed in this review is the Inter-Well Connectivity models (ICM). One of the earliest ICMs used in reservoir production optimization is the Capacitance Resistance Models (CRM). CRM is a correlation-based technique based on material balance law that pairs injection wells to production wells [109]. CRM is robust in handling dynamic boundary conditions when production and bottom hole pressure (BHP) data are available. CRM was established by Albertoni and Lake [110], and advanced by Yousef et al [111] for injectors and producers using space superposition. CRM involves two parameters for injection-producer. The first is the allocation factor (connectivity coefficient) and the second is the time constant. Allocation factors aids in estimating the inter-well connectivity between water injectors and producers by equating the total water injection rates that flows toward the production wells. Several research on CRM has been carried out with respect to waterflood optimization including the use of single and multi-layered reservoirs considering data from production logging, BHP and crossflows [80], [112], [113]. Cao et al [114] combined CRM with the Koval model to predict the waterflood production. The Koval model is used to address the characterization of viscous and heterogeneity effects using the so-called viscous fingering [115]. Wang et al [116] later implemented an improved CRM-Koval model coupled with aquifer support using the Karst reservoir as a case study. While the Koval model may be a good choice for production prediction from carbonate reservoirs, it is not sufficient to describe production from mature fields. A major setback with CRM is that the allocation factor remains constant during the span of production, whereas it changes as the multiphase flow of the system is dynamic. Another limitation lies on the fact that the multiphase flow system requires empirical models in estimating the fractional flow. For this reason, an approach called Inter-well Numerical Simulation Model (INSIM) was developed [116], [117].

INSIM is a physics-based data driven model which is able to predict the rates of production in well pairs by using an augmented Buckley-Leverett theory. INSIM has being derived from the principal of mass conservation and Darcy's law with compressibility of fluid and rock consideration. It was assumed that for two phase isothermal flow of oil and water, constant viscosities and negligible gravity and capillary force, the total volume balance for the i^{th} well is written as [116]:

$$\sum_{j=l}^{n_w} T_{i,j}^{n}(t)(\left(p_j(t)-p_i(t)\right)+q_i(t)=c_{t,i}(t)V_{p.i}(t)\frac{dp_i(t)}{dt} \quad (30)$$

$c_{t,i}$ stands for the total compressibility of well $V_{p,i}$, q_i is the subsurface fluid rate for injection and production well. p_i stands for the average pressure of i^{th} well at time t. $V_{p.i}(t)$ stands for half the summation of control volume pore volume of connective units to the i^{th} well. INSIM is able to estimate inter-well connectivity as well as monitor water cut [117].

INSIM was first employed by Zhao et al [117], [118] for one-dimensional two-phase flow reservoir. An improved approach called INSIM Flow Tracking (INSIM-FT) was studied by Guo et al [119], [120] for 2-dimensional flow and later extended for 3-dimensional flow to history match the production history of multi-layered reservoirs [121] and reservoir wells with gravitational effect [122]. Recently, INSIM-FT was extended to include a Flow Path Tracking (INSIM-FPT) for production optimization, history matching and inter-well connectivity [123].

Another data driven approach used in oil reservoir waterflooding is the Reduced Order Model (ROM). They are large models that are discretized from set of Partial Differential Equations (PDEs). The two mostly used technique in generating ROM for waterflood optimization problem is the Proper Orthogonal Distribution [97], [124–126], and Trajectory Piecewise Linearization (TPWL) [127]. Data driven based optimal control algorithms have found tremendous applicability in waterflood optimization. With the availability of production measurement, it's quite convenient to implement a control-based optimization framework. Of the many control-based optimization, one of the commonly used techniques is the Model Predictive Control (MPC). MPC is a numerical optimization technique that corresponds to a finite horizon optimal control [128], a new optimization strategy called Receding Horizon approach since the system's state is updated at every given sampling period [129]. The complexity of MPC solely depends on the complexity of the model. For example, a linear problem will settle for a linear predictive control. Considering the complexities and nonlinearities of reservoir model, a Nonlinear MPC is better suited. Several approaches in waterflood optimization using NMPC has being studied for conventional wells [129–132] and nonconventional well production [133], including the Receding Horizon approach [129], [134], [135]. Based on the data reviewed in this work, data driven algorithms are the second most used optimization techniques in the oil and gas fields for more than two decades, with an average implementation of about 33% after gradient-based techniques which has a record of about 40% usage (Fig. 3).

Attention has been given to a state of optimal control strategy in waterflooding optimization, where the feedback closed loop control strategy is improved by investigating control variables that are less sensitive to reservoir uncertainties. The control variables are maintained at a constant set point in the presence of uncertainty to make the process near optimal. This concept has been described by Halvorsen et al [136] as self-optimizing control (SOC).

Self-optimizing control describes scenarios of optimality with acceptable loss and in turn, the need to reoptimize in the presence of disturbance will be minimized. Halvorsen et al [140] described the proximate relation of SOC approach to self-regulating control, a scenario were by controlled activity is minimized as the dynamic performance of the process becomes acceptable. Skogestad [141] gave an outline of the variables to control to attain a self-optimizing control:

- Controlled variables
- Manipulated variables
- Measurements selection
- Structure of the controller configuration
- Selection of the controller strategy (PID, decoupler, fuzzy etc)

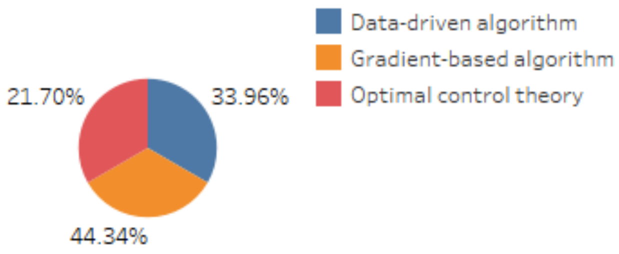

Fig. 3. Algorithms Used for Waterflood Optimization.

Table 4. Pros and Cons Of Data Driven Algorithm [130] – [140]

Pros	Cons
• The predictable value is a good quantity. • Uncertain parameters are the only needed measure. • It is easy to implement with availability of data.	• In some cases, a very large amount of data will be required to improve the performance. • Discrepancies in data quality will eventually produce inaccurate results.

Process system are basically controlled in diverse manners, whereby control strategies are implemented in the local and plantwide layers. Plantwide control layers are generally responsible for the entire controllability of the process in order to maintain the local controlled variables [138], [139]. However, it is possible to link the layers to form a single control unit through the concept of SOC. According to Cao [140], control scheme of a self-optimizing system is chosen accordingly:

- Stabilization control
- Constrained control
- Self-optimizing control

MVs are control values that must be adjusted in order to achieve a specific output, while the CVs are quantities that have being controlled. On the other hand, the uncontrolled variable is called the disturbance [141]. CVs are selected via two basic ways, locally or globally. The local method has been described as the linear approximation of a nonlinear model around a nominal point. By doing this, the solution is said to be local [142]. Ye et al [143] described the linearized model between independent variables and measured outputs around a nominal point as:

$$y = G^y u + G_d^y W_d d + W_n n \tag{32}$$

G^y and G_d^y are steady state gain matrices for inputs and disturbance. W_d and W_n are the diagonal matrices that are used to normalize d and n. d and e denote the disturbance and errors of the control system respectively. The selection of subsets as alternatives for CV or their combination is considered as combinatorial optimization [143]. Solution for this kind of problem has been proposed by an approach called branch and bound methods [144–146]. The global method involves the direct use of gradient functions as CVs so that global optimum could be achieved[147]. This proposed method could be seen in the works of [138], [140], [148].

Self-optimizing control for waterflooded reservoir optimization has being reported by Grema et al [149], [150]. Grema and Cao [32] applied a data driven self-optimizing control. This method involves investigating controlled variable that are applied to oil reservoirs with uncertainties. The CVs were investigated from measured production data in an offline manner and it was then implemented online in a closed loop feedback approach, and then compared to the open loop control (OC) scheme. This same approach was applied to multivariable waterflooding optimal control by Grema et al [150]. Recently, SOC was extended for smart well problems [151]. Table 4 gives a summary of performance for data driven algorithms while table 5 shows the summary of the entire review.

Recently, machine learning models have been implemented to oil reservoir optimization problems. Machine learning as a branch of artificial intelligence is a data driven technique used for predictive analysis. It comprises of several algorithms with specific purposes such as regression, classification and clustering problems. The ability of this algorithm to self-learn from available data makes it a choice of preference for waterflooding optimization [152]. A typical framework for machine implementation is presented in Fig 4. Machine learning is classified based on the nature of learning. This are supervised learning, unsupervised learning and reinforcement learning. In supervised learning, the data is made up of input and output variables, while unsupervised learning is comprised of input variables where the developed machine learning is expected to obtain patterns and produce appropriate output. Reinforcement learning is concerned with knowledge based derivation from the model [153]. Deep learning models are the most commonly used machine learning algorithms in waterflood optimization. Deep learning algorithms are made of complex interconnecting units called neurons divided into layers. Some of them include the deep feedforward neural network (DFFNN), nonlinear autoregressive and external inputs (NARx), support vector machines (SVM), convolutional neural networks (CNN), recurrent neural network (RNN) etc.

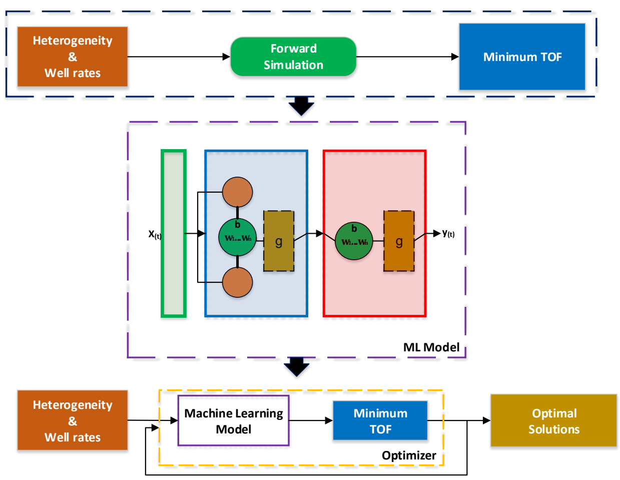

Fig. 4. Workflow For Machine Learning Waterflood Optimization.

Effective machine learning has been applied to problems that includes geophysical exploration, logging curve construction, drilling and completion methods, surface facility engineering and well logging [153]. Machine learning have been applied for well control optimization [154 – 157], production optimization [158 – 159], reservoir estimation based on reinforcement learning [160]. Machine learning algorithms are of great importance due to their robust data driven capabilities and strengths.

Table 5. Review Summary.

Optimization Algorithm		**Reference**
Data driven proxies	Self-Optimizing Control	[33], [146], [151], [153] – [155]
	Correlation based models	[82], [112] – [126]
	Reduced order models	[100], [127] – [130]
	Model Predictive control	[132] – [139]
	Machine learning	[157] – [165]
Gradient based Algorithm	Conditional Value at Risk	[57] – [61]
	Mean Variance optimization	[18], [27]
	Robust optimization, Sequential Quadratic programming (SQP)	[14] – [32], [35] – [39]
	Optimal Control theory	[10], [85]–[93], [94] – [111]
	Ensemble Kalman Filter	[14], [38], [71] – [82]

5. Conclusion

Research on waterflood optimization has been one of the most predominant topics covered in the oil and gas industry. Typically, with more emphasis on well placement pattern, well control, oil and gas production rates and how these optimization algorithms are implemented. Traditional model-based algorithm has found great use for optimization cases however, it may still be full of flaws as to implementation and obtainable solutions. As more data become available, researchers tend to focus more on leveraging them in obtaining possible and efficient solutions thereby establishing more techniques that may suffice against the existing ones. New ways in making oil recovery efficient and profitable has gained traction over the years. Such kind of technology like the use of machine learning and deep learning has become predominant in the oil and gas industry. Computational limitations have become one major challenge in identifying reservoirs, especially those of complex geological properties. Investigating efficient production or Net Present Value might be rigorous and time taken. Monumental oil well data, rock and fluid properties will require extensive study for cases of modelling. This key limitation will require advanced computational technique like the deep machine learning models. Several optimization algorithms used in waterflood optimizations where discussed, the pros and cons of these algorithms were also studied with respect to formulation and implementation. It is however important to note that, no algorithm suffices against the other. Hence, the need to improve on the existing technique becomes imminent.

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