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Channel Estimation Algorithm Based on Factor Graph for GFDM-UWA System

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Abstract

According to the characteristics of the wireless channel, channel estimation algorithms based on the factor graph are proposed for Generalized Frequency Division Multiplexing over Underwater Acoustics (GFDM-UWA) system to improve the precision of channel estimation and bandwidth efficiency, which include two-dimension joint channel estimation and one-dimensional channel estimation algorithm. Time-varying frequency-selective fading channels can be modelled as 1st ordered autoregressive processes and approximating messages as Gaussian distribution, a novel channel estimation and symbol detection method for GFDM-UWA system is deduced by applying sum-product algorithm on the factor graph. Simulation results show that the algorithm can achieve good performance with low computational complexity.

Keywords: Generalized Frequency Division Multiplexing over Underwater Acoustics (GFDM-UWA); channel estimation; symbol detection; factor graph; Sum-product Algorithm (SPA).

1. Introduction

At present, there are limited channel estimation methods for Generalized Frequency Division Multiplexing over Underwater Acoustic channel (GFDM-UWA) technique, which is generally divided into pilot-based channel estimation and blind channel estimation. Blind estimation is difficult to implement in most application environments due to its complexity. The pilot-based channel estimation algorithms mainly include Least Square (LS) algorithm, Minimum Mean Square Error (MMSE) algorithm, and Decision Feedback (DF) algorithm. These algorithms are simple, but the performance is not enough ideal [1]. The Maximum Likelihood (ML) estimator can obtain theoretical optimal estimation performance, but computational complexity increases exponentially with increase of channel freedom, which limits its application in practice. Joint iterative channel estimation is an effective way to solve above Problem. It uses the pilot and detected data to iteratively estimate channel based on traditional auxiliary pilot channel estimation [2]. In [3], joint decoder and channel estimation module performs iterative channel estimation on OFDM system to improve channel estimation accuracy. On this basis, the blind multi-generation receiver of OFDM system based on factor graph has also received attention [4], but amount of computation is large and convergence speed is slow. In view of this, in this paper a new joint channel estimation algorithm is proposed. According to ML criterion, the optimal channel estimation problem is regarded and the observation information is solved using channel parameters and likelihood function of transmitted symbols. With the help of factor graph model [5], the initial channel parameter information is exchanged through time-frequency 2-D transfer function node, and optimal channel estimation result is obtained by using sum-product algorithm (SPA) rule. The algorithm reduces pilot overhead while improving performance. However, due to two-dimensionality of pilot signal in GFDM-UWA system, there is a "ring" in the factor graph model of the algorithm. Therefore, in order to obtain better results, performance of the algorithm must be improved by multiple iterations, if factor graph model is constructed. If there is no loop, sum-product estimation algorithm can obtain accurate posterior estimation results, and the amount of calculation is also significantly reduced. In order to further reduce the computational complexity of the joint iterative algorithm, the characteristics of wireless channel is considered [6-7], in this paper 2-D joint channel estimation is decomposed in to cascaded two 1-D channel estimation algorithms.

2. System Model

The received frequency domain observation signal is expressed as:

$$Y[n,k] = X[n,k]H[n,k] + W[n,k] \quad (1)$$

Where $Y[n,k]$, $X[n,k]$ represent the received and transmitted signal on k^{th} subcarrier in the n^{th} GFDM-UWA symbol; $W[n,k]$ is a zero mean and σ^2 variance Gaussian white noise; $H[n,k]$ represents the k^{th} subcarrier channel frequency response (CFR) at n^{th} time.

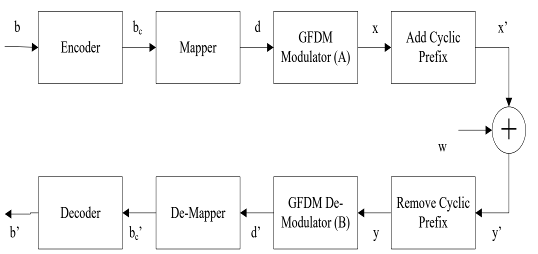

Fig. 1. Baseband Model for GFDM-UWA system.

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Consider GFDM-UWA system (fig. 1) for a frequency selective channel, the correlation between two-point channel response with time intervalΔn, frequency domain interval Δk can be expressed as:

$$r_H(\Delta n, \Delta k) = r_t(\Delta n) r_f(\Delta k) \quad (2)$$

Where r_t represents channel time domain correlation function, mainly depending on Doppler frequency shift caused by relative movement of transmitter and receiver; r_f represents frequency domain channel correlation function.

According to literature [9], transfer function $\Delta[\Delta n, \Delta k]$ is used to describe the information interchange between channel parameters, which reduces computational complexity of algorithm. In this paper, only first-order autoregressive (AR) model [10] is considered, i.e., $\Delta n + \Delta k = \pm 1$. According to characteristics of wireless channel parameters, the Gaussian distribution is approximated the transfer function i.e., $\Delta[\Delta n, \Delta k] \sim N(0, \sigma_\Delta^2[\Delta n, \Delta k])$. The variance for time domain transfer function Δt is:

$$\sigma_{\Delta t}^2 = 2 - 2J_0(2\pi f_d T_s) \quad (3)$$

Where τ_{max} is GFDM-UWA symbol interval; $f_d$is relative Doppler extension of transmitter and receiver; $J_0(\cdot)$ is 1^{st} order Bessel function. The variance for frequency domain transfer function Δf is expressed as:

$$\sigma_{\Delta f}^2 = 2 - sinc(\tau_{max} F) \quad (4)$$

Where τ_{max} is maximum transmission delay; F is subcarrier spacing.

According to above GFDM-UWA system model, the optimal detection of the transmitted symbol using the ML algorithm can be expressed as:

$$\hat{X} = arg \underbrace{max}_{X} p(Y|X) \quad (5)$$

Where X represents sequence of transmitted symbols; Y represents sequence of received symbols.

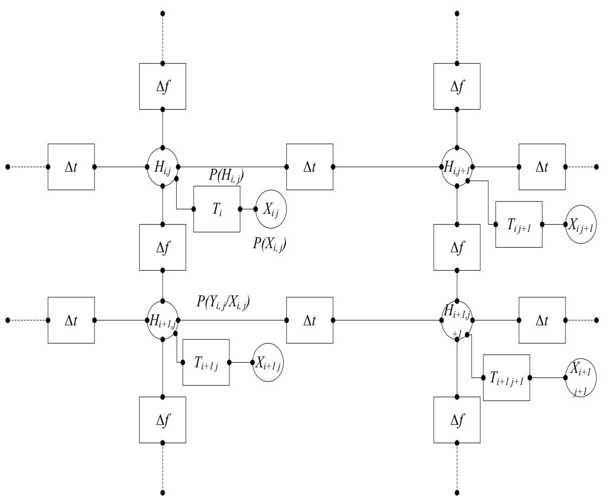

Fig. 2. Factor Graph based receiver model in GFDM-UWA system.

3. Joint Channel Estimation Based on Graph Factor

3.1. Two-Dimensional Joint Channel Estimation Algorithm Based on Factor Graph

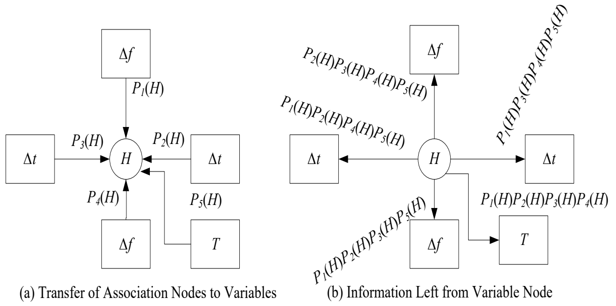

Fig. 3. Structure of Channel State Information Updater.

In fading environment, likelihood function in equation (5) is decomposed in to the correlation of symbol estimates at different time and frequency points:

$$p(Y|X) = \int_H p(Y|X,H) p(H) dH = \int_H \prod_{X \epsilon X_P} p(Y|X,H) \prod_{X \epsilon X_P} p(Y|H) p(H) dH \quad (6)$$

Where H is channel response matrix; X_P is training sequence of transmitted symbol.

According to characteristics of wireless channel, likelihood function is decomposed into product of local probability functions, and factor graph is used to represent relationship between each decomposed term and variable. A factor graph is a bidirectional graph consisting of function nodes, variable nodes, and graph edges. If function contains variable, edge of variable and function node need to be connected by the edge [4]. The factor graph model of the GFDM-UWA system receiver is shown in Figure 2. The squares in figure represent function nodes, the circles represent variable nodes, and straight lines represent edges of function nodes and variable nodes. $\Delta f, \Delta t$ represents frequency and time domain transfer function nodes between adjacent channel parameters$T_i \equiv p(Y_{i,j}|H_{i,j}, X_{i,j})$.

According to the above receiver model, a 2-D joint channel estimation algorithm is obtained by applying the sum product algorithm. From the factor graph model as shown in Fig. 2, each graph edge information is iteratively updated to each node until the algorithm converges. The factor graph based 2-D joint channel estimation algorithm in GFDM-UWA system is described as follows:

Step 1: Initialization. According to the received signal Y_0 and training sequenceX_p, the initial channel estimation value H_0 is obtained by conventional LS algorithm. According to factor graph model as shown in Fig. 2, channel information at pilot position is exchanged by transfer function node by means of sum-product algorithm rule, and mean and variance of each node are iteratively updated in sequence. Assuming that the initial transmitted symbol have has equal priori probability, i.e., $p(X_{i,j}) = 0.5, i = 1,2,..,K; j = 1,2, \dots N$, where K, N represents number of subcarriers and symbols in GFDM-UWA.

Step 2: Get the information of the variable node channel parameter based on symbol updated information and is given by:

$$p(H_{i,j}) = \sum_{X_{i,j}\in\{\pm 1\}} p(Y_{i,j}|H_{i,j}, X_{i,j})p(X_{i,j}) \quad (7)$$

Step 3: The channel parameters to gaussian channel model are updated, according to Fig. 3. Because it is assumed that $p(H)$ is subject to gaussian distributions, their products are still subject to Gaussian distributions

$$\prod_{j=1,j\neq m}^{N} p_n(H) \sim N(\mu_m, \sigma_m^2) \quad (8)$$

Where:

$$\mu_m = \frac{\sum_{n=1,n\neq m}^{N} \frac{\mu_n}{\sigma_n^2}}{\sum_{n=1,n\neq m}^{N} \frac{1}{\sigma_n^2}}, \sigma_m^2 = \frac{1}{\sum_{n=1,n\neq m}^{N} \frac{1}{\sigma_n^2}}$$

Step 4: The information passed from the observation function node to the symbol variable node based on the updated channel parameter information is gathered, and represented as:

$$p(Y_{i,j}|X_{i,j}) = \frac{1}{\pi(|X_{i,j}|^2 \sigma_{H_{i,j}}^2 + \sigma_n^2)} \times \exp\left(-\frac{|Y_{i,j} - \hat{H}_{i,j}X_{i,j}|^2}{|X_{i,j}|^2 \sigma_{H_{i,j}}^2 + \sigma_n^2}\right) \quad (9)$$

Repeat Step2~Step4 until the algorithm converges.

Step 5: The estimated value of the transmitted symbol from all the information related to the symbol is gathered:
Step 5 is evaluated by all information related to the symbol to send the symbol.

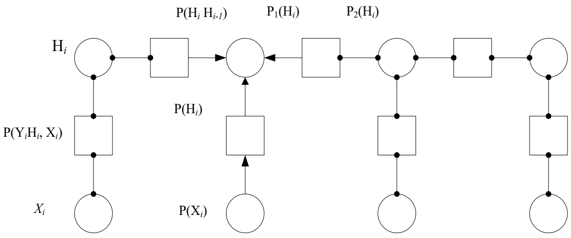

Fig. 4. 1-D Channel Estimation Factor Graph Model.

3.2. Two Cascaded 1-D Channel Estimation Algorithm

Since the factor graph of the above two-dimensional algorithm has a loop, multiple iteration operations are required, and the calculation amount is large, and the correlation of the wireless channel can be separated by the equation (2), so the two-dimensional joint channel estimation algorithm is passed through two cascade of one-dimensional channel estimation is implemented, that is, an appropriate factor graph model is established in the time domain and the frequency domain, respectively, when the channel parameter information is updated, only the transfer function in each direction is considered, and the other dimension is ignored. Correlation, the ring graph model no longer contains loops. Due to the incompleteness of channel parameter information transmission caused by algorithm approximation, the performance of the one-dimensional joint channel estimation algorithm is degraded compared to the performance of the two-dimensional joint channel estimation algorithm. The factor graph model of the one-dimensional channel estimation algorithm is shown in Figure 4. Consider the frequency domain channel estimation first in the time domain estimation.

Step 1: Time domain channel estimation

In this case, the factor graph model does not contain the symbol variable node and the observation node, so only the channel parameter information needs to be obtained. According to the channel parameter model, the channel parameter changes to a Markov process, and $p(H_i|H_{i-1})$ is the time domain transfer function and is represented by Δt in Fig.4. The output information of the channel can be calculated from (9), and then similar operations are performed in all subcarriers containing pilot symbols.

Step 2: Frequency domain channel estimation

The channel estimation value in the time domain is taken as known data, and then 1-D channel estimation is performed in the frequency domain, and the frequency domain transfer function Δf is used instead of $p(H_i|H_{i-1})$. Since each subchannel of the GFDM-UWA system is equivalent to a flat Rayleigh fading channel, the likelihood function $p(Y_{1:k}|X_{1:k})$ is decomposed into:

$$p(Y_{1:k}|X_{1:k}) = \int_{H_{1:k}} p(Y_{1:k}|X_{1:k}, H_{1:k})\, p(H_{1:k})dH_{1:k} = \int_{H_{1:k}} p(Y_i|X_i, H_i)p(H_0)p(H_i|H_{i-1})dH_i \quad (10)$$

According to the factor graph model as shown in Fig. 4, the specific steps of 1-D joint channel estimation algorithm is expressed as follows:

1. Assume that the initial transmitted symbol a priori is equal probability, i.e., $(X_i) = 0.5, i = 1,2,..,K$.
2. Obtain the channel parameter information, which is expressed as:

$$p(H_i) = \sum_{X_i\in\{\pm 1\}} p(Y_i|H_i, X_i)p(X_i) \quad (11)$$

3. Update the channel state information, because $p_1(H_i) \sim N(\mu_1, \sigma_1^2)$, $p_2(H_i) \sim N(\mu_2, \sigma_2^2)$, then $p_1(H_i) \cdot p_2(H_i) \sim N(\mu_H, \sigma_H^2)$.

$$\mu_H = \frac{\sigma_2^2\mu_1 + \sigma_1^2\mu_2}{(\sigma_2^2 + \sigma_1^2)}$$

$$\sigma_H^2 = \frac{\sigma_1^2\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)} \quad (12)$$

4. According to the updated channel state information, the information of the symbol variable node is:

$$p(Y_i|X_i) = \frac{1}{\pi(|X_i|^2\sigma_{H_i}^2 + \sigma_n^2)} \cdot \exp\left(-\frac{|Y_i - \hat{H}_iX_i|^2}{|X_i|^2\sigma_{H_i}^2 + \sigma_n^2}\right) \quad (13)$$

Repeat steps (2) to (4) until maximum iterations are met.According to (5), the symbol estimate using ML criterion is as follows:

$$\hat{X}_i = \arg\max_{X_i} p(Y_i|X_i) \quad (14)$$

4. Simulation Results

The simulation parameters of GFDM-UWA system is as follows: the number of subcarriers K= 64 , the number of GFDM-UWA symbols per frame is N = 13, using BPSK modulation; under Rayleigh fading multipath channel, the number of multipath is 3; Normalized Doppler frequency

shift is $f_d T_s = 0.0114$ and the maximum delay $\tau_{max} = 3000ns$.
Fig. 5 shows performance comparison between the algorithm and accurate channel estimation (Bit Error Rate, BER).

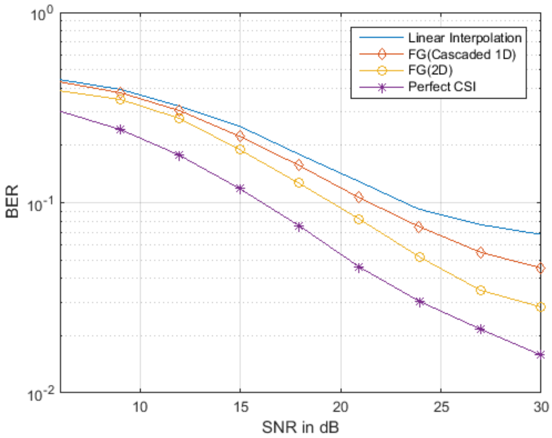

Fig. 5. BER Comparison of algorithm with the linear interpolation algorithm.

The linear interpolation algorithm represents the BER result of classical linear interpolation channel estimation algorithm, FG (cascaded 1D) and FG (2D) respectively represent two cascaded 1-D channel estimation and factor graph based 2-D channel estimation algorithm based on factor graph. The BER performance, perfect CSI, represents the BER performance of the ideal channel estimate. The simulation is based on the BER performance of the ideal channel estimation and the classical linear interpolation algorithm. It can be seen from the simulation results that the linear interpolation algorithm has the worst performance because the pilot interval cannot be tracked in time when the pilot interval is slightly larger, so the performance is not ideal. Compared with the linear interpolation algorithm, based on the factor graph, the joint channel estimation algorithm performs better, and the 1-D channel estimation algorithm takes the second place. The performance of the simplified 1-D channel estimation algorithm is lost because of the algorithm has an approximation in the channel estimation process, i.e., only the transfer function node in the 1-D direction is considered, resulting in loss of information when the channel parameter information is transmitted. On the other hand, since each function node is only connected to its associated variable node, the computational complexity of the simplified algorithm is significantly reduced. The two algorithms presented in this paper have a compromise between performance and complexity.

Fig.6 and Fig.7 show the BER performance comparison of 2-D and 1-D channel estimation algorithms for different pilot intervals in frequency domain.

In the simulation, the pilot intervals in frequency domain are $D_f = 6, D_f = 10$ and $D_f = 10$, respectively selected, and the time domain interval is 6. The simulation results show that pilot interval is also an important parameter affecting the performance of system. As pilot interval decreases, the channel estimation algorithm based on factor graph approximates the accurate channel state information (CSI). Therefore, pilot-assisted channel estimation must compromise spectral efficiency and estimation performance.

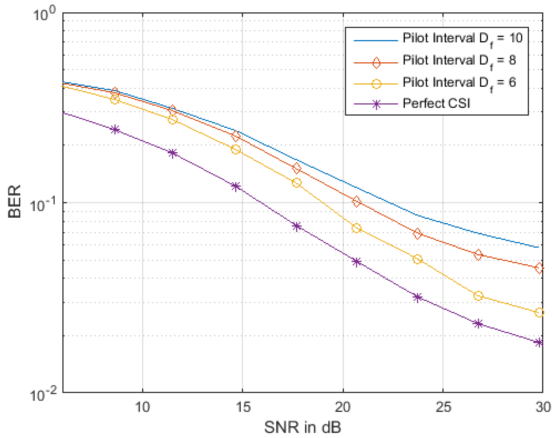

Fig. 6. BER comparisons of 2-D channel estimation algorithms at different pilot intervals.

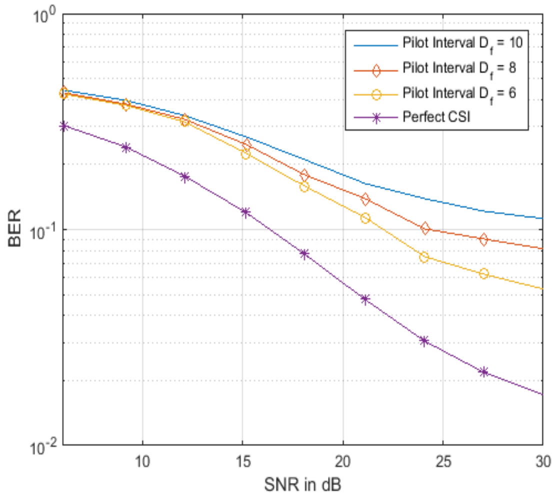

Fig. 7. Comparison of BER of cascaded 1-D channel estimation algorithm under different pilot intervals.

5. Conclusion

In this paper, channel estimation algorithm for GFDM-UWA system is proposed based on factor graphs. Firstly, the likelihood function that implements the optimal estimation of the transmitted symbols is decomposed to obtain a suitable factor graph model, and then the specific expression of the iterative estimation algorithm is derived according to the sum-product algorithm rules. The analysis and simulation results show that the proposed two-dimensional joint channel estimation algorithm can approximate the optimal estimation performance, and the simplified one-dimensional channel estimation algorithm can still achieve relatively good results under the condition of moderate computational complexity. Here the channel estimation is studied for SISO-GFDM-UWA system, and in future MIMO-GFDM-UWA channel estimation algorithms will be studied.

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