Jestr

Journal of Engineering Science and Technology Review 14 (1) (2021) 25 - 37

Research Article

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

A Machine Learning based Method for Managing Multiple Impulse Purchase Products: An Inventory Management Approach

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Received 3 November 2020; Accepted 2 April 2021

Abstract

Recently, some studies have begun to explore the potential that inventory management combined with machine learning algorithms could provide as a means of producing efficient and flexible inventory management methods. In this way, although there are some methods to carry out this practice, none are set up for impulse purchase products. This article illustrates this perspective within the context of an impulse purchase product provisioning problem and shows how group policies based on a clustering process can result in better (lower cost) groupings. To solve this problem, a method is proposed for finding a near-optimal inventory grouping solution. The key innovation in this solution is the idea to form groups for the items that have similar demand or ordering and cost characteristics. Subsequently, once the clusters have been formed, it was necessary to look at aggregating impulse purchase SKUs, and then two grouping techniques or heuristics that both consider common characteristics and develop some ordering decision rules are presented. The results show that the proposed method can be used to cluster impulse purchase products more effectively and the grouping techniques applied were efficient in terms of solution quality. The aim of the proposed unsupervised clustering-based method was not only to provide a classification of SKUs free of subjectivity processes but also to provide an approach to apply more efficient inventory policies for impulse purchase products.

Keywords: Cluster analysis, Impulse purchase products, Inventory management, Supply chain management, Industrial engineering

1. Introduction

In the highly competitive global landscape of today, organizations need to develop a strategic advantage by distinguishing their products or services, be it by cost, time, quality, and flexibility. In many cases, a company's ability to offer a differentiated strategy is related to its inventory and supply chain management operations and processes [1]. Thus, to achieve these goals effectively, it is key for organizations to be capable of easily integrating diverse methods and applications [2]. In the context of inventory management, machine learning methods could be employed to analyze inventory and categorize products in stock. In combination, these features could provide effective methods that could guide and support management decisions. Although among a large number of machine learning applications, the inventory management approach is still relatively scarce; Recently, some studies have begun to explore the potential that inventory management combined with machine learning algorithms could provide as a means of producing efficient and flexible inventory management method [1].

In this way, although there are some methods to carry out this practice, none are set up for impulse purchase products. This is a disadvantage because with the opening of new markets and the proliferation of consumer culture the economic importance of buying products on impulse always remains relevant [3–5]. This kind of merchandise can be defined as those products that a consumer acquires suddenly and immediately without a plan prior to purchase [6]. Impulse buying behavior has been described as a novelty or escape purchase that breaks the normal buying pattern [7]. Generally, these items are strategically displayed in hot spots (areas with a large circulation of people), such as near checkouts in retail stores. Along these lines, previous research that has been carried out does not consider the particularities of these products, leading to arbitrary or generalized models that are used for the management of the collaborative inventory of these goods. Making everything independent makes inventory management complicated and there are some problems with doing so: one is that if all the individual SKUs are done by themselves, and there are thousands of them, there will be totally uncoordinated orders. Also, by doing them individually, no common constraints can be considered; think of a budget, for example, firms have limited capital to hold as much inventory as they require to meet a certain level of service. Then there is the notion that if they are looked at independently, some consolidation opportunities will be missed because if products are ordered scattershot, instead of being put together and consolidated, perhaps when they are consolidated, some costs can be shared or economies of scale exploited. And the last, and possibly the most critical, is a waste of management time: If inventory management had to be done independently, a lot of time would be spent managing these items independently, and management time is one of the most scarce resources.

*E-mail address: davidgarciabarrios@gmail.com
ISSN: 1791-2377
doi:10.25103/jestr.141.02

This paper illustrates this perspective within the context of an impulse purchase product provisioning problem and shows how group policies based on a clustering process can result in better (lower cost) groupings. To solve this problem, a method is proposed for finding a near-optimal inventory grouping solution. The key innovation in this solution is the idea to form groups for the items that have similar demand or ordering and cost characteristics. In machine learning, classification is usually defined as supervised learning because the class label information is provided, which means that the learning algorithm is supervised in the sense that it indicates the class affiliation of each training tuple. Clustering is referred to as unsupervised learning since the class label information is not available [8]. In particular, there are some reasons to use cluster analysis: Clustering is a wide set of techniques to identify specific subgroups of observations in a given dataset. When observations are clustered, the intention is that observations in the same group will be similar and that observations in different groups will be different. Since there is no response variable, this is an unsupervised method, implying that it attempts to find relationships among the various observations without being trained by a response variable. Clustering makes it possible to identify which observations are similar and to potentially classify them [9].
Subsequently, once the clusters have been formed, it was necessary to look at aggregating impulse purchase SKUs and then two techniques that both consider common characteristics and develop some ordering decision rules are presented. More information on these heuristics, along with the algorithmic steps for their implementation, is given in section 3. It is important to mention that full aggregation may result in elevated costs if the specific order cost for low-demand products is high. In these circumstances, it may be more appropriate to order the low-demand products less frequently than the high-demand ones, an aspect that correctly achieves this method using these two grouping techniques mentioned. This approach means a reduction in the product-specific order cost related to the low-demand product. At last, in the validation stage, which is defined as a test stage where the method is executed to evaluate its response with respect to a real scenario, a case study was carried out with data from a retail company, in order to evaluate the results for a group of impulse purchase products in a period of two years. the proposed method can be a useful tool for managers to perform extensive what-if analysis and apply more efficient inventory control and monitoring.

This research contributes to the literature on cluster analysis and supply chain management of impulse purchasing products. This article is organized as follows: The related literature is reviewed in Section 2; Section 3 describes the proposed method; Section 4 presents the validation stage for this method; Section 5 concludes this paper and give some recommendations for future works.

2. Related work

From the review of recent literature, there were no studies related to machine learning methods focused on the management of impulse purchase product inventories policies. However, as for the study of retailing and consumer services related to impulse buying, Mittal, Chawla, and Sondhi (2016) [10] recognized and described three clear market niches of impulse buying consumers: proactive-impulsive, hesitant-hedonistic, and pragmatic-rationalist; offering producers and vendors a helpful strategic marketing tool to target multiple consumer groups. Bellini, Cardinali, and Grandi (2017) [6] examined the drivers of impulse buying in the context of further procurement planning and preparation. They employed a structural equation modeling approach and found that pre-purchase readiness influences impulse buying. Also, Hübner and Schaal (2017) [11] designed a model to maximize a retailer's profits by selecting the amount of facing and their location on the shelves with limited space. According to these authors, the more demand is impulse-driven, the more it is space-dependent. Page, Trinh, and Bogomolova (2019) [12] tested the effectiveness of a store layout with a central aisle that divides all other aisles, as compared to a conventional layout without a central aisle. According to them, these are standard store metrics, which can change due to the increased possibilities for impulse purchases. García-Barrios, Palencia, Solano, and Mendoza (2020) (García-Barrios, Palencia, Solano, & Mendoza, 2020) formulated a Vendor Managed Inventory model based on the direct participation of a vendor and a buyer (two-level supply chain) to agree on the procurement operations of a portfolio of impulse purchase products.

Furthermore, it should be mentioned that cluster analysis has played an important role in generating new research proposals for retailing and impulse buying in recent years. Holý, Sokol, and Černý (2017) [14] developed a model to group retail products using information from the market basket. Their model is presented as an optimization problem that is solved through a genetic algorithm. Besides, Balakrishnan, Cheng, Wong, and Woo (2018) [15] applied an intuitive clustering algorithm to identify useful trends in a matrix that matches customers with items they have previously purchased. In accordance with this work, the algorithm allows the retailer to maximize the customer's impulse purchases on their way to purchase. Similarly, Wang, Zhang, Xue, Lu, and Na (2020) [16] built a product recommendation system that was based on learning clustering representation. In summary, the results showed the effectiveness of this proposed system. Other authors develop analysis and clustering to investigate shopping behaviors, Wu and Yu (2020) [17] conducted a sequential search pattern analysis and clustering to study the search pattern of consumers during the entire purchase process. These authors suggest that this search pattern may be helpful in encouraging customers to buy more impulsively.

In addition, it is worth mentioning some approaches concerning inventory classification methods that are not strictly related to impulse purchase products but that correspond to novel research proposals in the recent literature. Balugani, Lolli, Gamberini, Rimini, and Regattieri (2018) [18] employed the k-means and the Ward method to aggregate items into homogeneous clusters to manage them with common inventory policies. In their research, these methods were a viable option for inventory control system simulation. Also, Zowid, Babai, Douissa, and Ducq (2019) [19] provided a Gaussian Mixture Model (GMM) to address the problem of multi-criteria inventory classification. The findings revealed that this model can perform well in terms of cost-service inventory efficiency. Sheikh-Zadeh, Rossetti, and Scott (2020) [20] designed a performance-based inventory sorting method that calculates a grouping solution for a multi-item, multi-tier inventory system. Empirical evidence showed that there is a minor gap between the application of the method and the optimal solution. In the same way, Sheikh-Zadeh, Farhangi, and Rossetti (2020) [21] developed a heuristic model which was based on a greedy approach, using the idea that items that have similar inventory policies should be aggregated. Their results suggested that this model provides a near optimal solution and significantly outperforms classification and clustering techniques.

As for the study of impulse buying, one of the most prevalent research approaches corresponds to examine elements of social psychology in consumer behavior. Pornpitakpan, Yuan, and Han (2017) [22] explored the effects of sellers' retail support quality and consumer perceptions of impulse buying. Their results showed that consumers who receive better service from sellers have stronger impulse buying intentions. Ferreira, Brandão, and Bizarrias (2017) [23] studied the outcome of the strategies adopted to tackle the negative emotions derived from the perception of crowding and the consumer's behavior calculated by impulse buying. According to these authors, there is a good response to human density in the retail environment. Bossuyt, Vermeir, Slabbinck, De Bock, and Van Kenhove (2017) [24] examined the relationship between impulse buying and misconduct. In summary, they noted that participants who bought impulse products were more likely to cheat to obtain an expensive product than those who bought ordinary items. Other authors study online impulse buying through the shopping process: Peña-García, Gil-Saura, Rodríguez-Orejuela, and Siqueira-Junior (2020) [25] investigated relevant factors in the implementation of e-commerce based on aspects of social psychology, particularly non-traditional elements such as impulse buying in online stores and comparing relationships in a multicultural context. The results revealed a clear inconsistency between intention and actual behavior. I.-L.Wu, Chiu, and Chen (2020) [26] designed a model to explore the factors that influence online impulse buying. Their research revealed a critical connection between perceived risk for online purchases, online store design and emotional responses. Additionally, Y. Wu, Xin, Li, Yu, and Guo (2020) [27] formulated a model to describe the impact of shortages on impulse buying. The findings provided evidence that the shortage increased the consumer's perception of excitement, which then resulted in an impulse purchase.

Finally, it is essential to mention the studies related to social networks and mobile commerce, which add enormous value to understanding impulse buying. C.-C. Chen and Yao (2018) [28] developed a mobile auction platform to analyze in what way situational factors influence impulse buying behavior. The results showed that personality-related factors, normative assessment, and positive affect were key elements of impulse buying. Zheng, Men, Yang, and Gong (2019) [29] focused on situational aspects and response factors in mobile commerce to examine impulse buying. In short, the findings showed that hedonic and utilitarian browsing were key drivers of impulse buying. Liu, He, and Li (2019) [30] studied the relationship between social networks websites and impulse buying. The results revealed that upward comparisons on social networks can induce young adults to increase impulse purchases. Last but not the least, Y. Chen, Lu, Wang, and Pan (2019) [31] designed a model to examine how recommendations for products on social media impact a consumer's need to buy impulsively. Their results indicated that the urge to buy impulsively is caused by affective trust in the recommender and affection for the recommended product.

3. Proposed Method

The main motivation for this method is to propose a useful solution for managing multiple impulse purchase SKUs. In short, this grouping problem was approached from a viewpoint of grouping SKUs based on the assumption that items within clusters should have similar inventory policies. This approach recognizes that if products have common policies, they are likely to have similar demand or ordering and cost characteristics, which are relevant to inventory management. Cluster analysis or clustering provides information about the data by dividing objects into clusters so that objects in a cluster are actually more related to each other than to objects in other clusters. Since it does not use external information, cluster analysis is often referred to as unsupervised learning in some areas such as machine learning [9]. Thus, this proposed method is built on K-means clustering, one of the most commonly applied algorithms in machine learning. K-means offers some different benefits over other clustering algorithms: K-means is remarkably straightforward and robust, as well as highly efficient and suitable for a broad range of data types. In this way, K-means is an algorithm that tries to identify K clusters that do not overlap. These groups are defined by their centroids (a group centroid is usually the average of the points in that cluster). Therefore, it is advantageous to go deeper into the computations involved in K-means:

The main concept behind k-means clustering is to identify clusters in a way that minimizes the total intra-cluster variation. The standard k-means algorithm establishes the total variation within the cluster (within-cluster variation) as the sum of the Euclidean squared distances between the items and the respective centroid, where x_i is a data point that belongs to cluster C_k, and μ_k is the mean of the points assigned to cluster C_k:

$$W(C_k) = \sum_{x_i \in C_k} (x_i - \mu_k)^2 \qquad (1)$$

Each observation x_i is allocated to a certain cluster so that the sum of squares distance of the observation to their designated cluster centers μ_k is minimized. Then, the total within-cluster variation can be defined as follows,

$$\sum_{k=1}^{k} W(C_k) = \sum_{k=1}^{k} \sum_{x_i \in C_k} (x_i - \mu_k)^2 \qquad (2)$$

Therefore, defining these clusters is a computationally intensive task that is quite complex. [32–34]. A typical k-means algorithm would operate by following iterative procedure: (i). Indicate the number of clusters (k) to be formed; (ii). Randomly select objects from the data set as the starting cluster centers or means; (iii). Assign each observation to their nearest centroid, in accordance with the Euclidean distance between the object and the centroid; (iv). For each one of the k clusters, the centroid is updated by computing new mean values for all data points in the cluster. The centroid of a kth cluster is a vector that contains the means of all variables for the observations in that kth cluster; and (v). Minimize in an iterative way the total within the sum of the square. In other words, repeat steps 3 and 4 until the cluster assignments do not change anymore or the maximum number of iterations is reached.

Once the clusters have been formed, it was necessary to look at aggregating impulse purchase SKUs and then two grouping techniques that both consider common characteristics and develop some ordering decision rules are presented. These grouping techniques were based on the work of F. Chen, Federgruen, and Zheng (2001) [35], Federgruen, Queyranne, and Zheng (1992) [36], Federgruen and Zheng (1993) [37], Lu and Qiu (1994) [38], and Roundy (1985) [39]. The first technique, which can be called “Grouping technique 1”, is one way to group similar SKUs using some ordering decision rules. The basic idea of this technique is to replenish SKUs according to their value: the higher value items are, the faster they will be replenished, where a “higher value” is defined as the annual demand (D_i) times the cost per item (C_i). In other words, SKUs are grouped by D_iC_i. Also, it is important to mention that this grouping technique should be used for a group of items with similar demand or ordering characteristics, which makes it necessary to execute the cluster analysis beforehand. Thus, the technique consists in choosing a base period of time p_0 (a week, a month, etc), and a series of ordering periods candidates that are multiples of that base period. Then, it is necessary to find those annual values where the total relevant cost (δ) ordering at p_j is equal to the total relevant cost of ordering at that next highest interval period p_{j+1}, as shown in Eq.3, where $\delta(p_j)$ and $\delta(p_{j+1})$ are the total cost as a function of Q (or total relevant cost). That is, $\delta(p_j) = C_rD_i/Q_{ij} + C_ihQ_{ij}/2$, and $\delta(p_{j+1}) = C_rD_i/Q_{ij+1} + C_ihQ_{ij+1}/2$. Those items that fall in the common sets will be grouped using some ordering decision rules, and they will be managed with similar inventory policies.

$$\delta(p_j) = \delta(p_{j+1}) \tag{3}$$

Supposing a value p_0 was chosen (one week, for example), and the time interval of supply (number of weeks, for example) to order for each item needs to be found. Therefore, the quantity ordered (Q_{ij}) for an item i at time period j will be equal to the annual demand (D_i) times p_j (as shown in Eq.4), whatever that candidate time period that was chosen, over a conversion factor ω (52, for example, considering that the base period was a week).

$$Q_{ij} = D_i\left(\frac{p_j}{\omega}\right) \tag{4}$$

In this way, what is being done is to set the total relevant cost (δ) for whether the product i is ordered at frequency p_j (left side of Eq.3), and to note what it is equal to if that product is ordering at frequency p_{j+1} (right side of Eq.3). Hence, it is possible to calculate what D_iC_i value is right at that breakpoint or "ordering decision rule". This is carried out and write it down in terms of C_r (average ordering cost), h (holding charge), and p_jp_{j+1}, as shown in the following equations:

$$\delta(p_j) = \delta(p_{j+1}) = \frac{C_rD_i}{Q_{ij}} + \frac{C_ihQ_{ij}}{2} = \frac{C_rD_i}{Q_{ij+1}} + \frac{C_ihQ_{ij+1}}{2} \tag{5}$$

$$C_rD_i\left(\frac{\omega}{D_ip_j}\right) + \frac{C_ih}{2}\left(\frac{D_ip_j}{\omega}\right) = C_rD_i\left(\frac{\omega}{D_ip_{j+1}}\right) + \frac{C_ih}{2}\left(\frac{D_ip_{j+1}}{\omega}\right) \tag{6}$$

$$\frac{\omega C_r}{p_j} + \frac{C_ihD_ip_j}{2\omega} = \frac{\omega C_r}{p_{j+1}} + \frac{C_ihD_ip_{j+1}}{2\omega} \tag{7}$$

$$\frac{C_ihD_i}{2\omega}\left(p_j - p_{j+1}\right) = \omega C_r\left(\frac{1}{p_{j+1}} - \frac{1}{p_j}\right) \tag{8}$$

$$D_iC_i = \frac{2\omega^2C_r}{h(p_j - p_{j+1})}\left(\frac{1}{p_{j+1}} - \frac{1}{p_j}\right) = \frac{2\omega^2C_r}{hp_jp_{j+1}} \tag{9}$$

Thus, a series of ordering decision rules is obtained, as shown in (10), where, if the annual value of D_i times C_i, is greater than or equal to $2\omega^2$ times C_r over h times p_jp_{j+1}, then the frequency p_j is selected. If it is less than that but greater than this value $2\omega^2C_r/hp_jp_{j+1}$ in the Eq.9, then the frequency p_{j+1} is selected, and so forth. In summary, it is possible to reduce management time and increase efficiency by consolidating similar items in the inventory system based on some common constraints and the calculated ordering decision rules assign SKUs according to the order frequency given to them. In practice, since there are hundreds of items and each item has its own optimal frequency, it is not feasible to manage the items independently.

$$Ordering\ decision\ rule_{[condition]} = \begin{cases} If\ D_iC_i \geq 2\omega^2C_r/hp_jp_{j+1}\ then\ select\ p_j \\ Else: if\ D_iC_i \geq 2\omega^2C_r/hp_{j+1}p_{j+2}\ then\ select\ p_{j+1} \\ Else: if\ D_iC_i \geq 2\omega^2C_r/hp_{j+2}p_{j+3}\ then\ select\ p_{j+2} \\ Else: if\ D_iC_i \geq 2\omega^2C_r/hp_{j+3}p_{j+4}\ then\ select\ p_{j+3} \end{cases} \tag{10}$$

The second way to group impulse purchase SKUs, which can be called “Grouping technique 2”, considers order a specific ordering frequency p instead of using an optimal ordering frequency p^* , where p^* is simply the optimal economic order quantity over the annual demand. This grouping technique is useful to ensure that a defined inventory policy can actually be implemented because in some cases is more critical to come with something that actually gets executed than something that could be theoretically optimal. Using the fact that $\delta(Q) = C_rD/Q + ChQ/2$ and $\delta(Q^*) = C_rD/Q^* + ChQ^*/2 = \sqrt{2C_rChD}$, where the optimal economic order quantity $Q^* = \sqrt{2C_rD/Ch}$; C_r = ordering cost per order; h = holding charge, D = the annual demand; and C = cost per item; $\delta(Q)/\delta(Q^*)$ can be readily obtained as follows,

$$\frac{\delta(Q)}{\delta(Q^*)} = \frac{\frac{C_rD}{Q} + \frac{ChQ}{2}}{\sqrt{2C_rChD}} = \frac{\sqrt{C_rD}}{Q\sqrt{2Ch}} + \frac{Q\sqrt{Ch}}{2\sqrt{2C_rD}} = \frac{\sqrt{2C_rD}}{2Q\sqrt{Ch}} + \frac{Q\sqrt{Ch}}{2\sqrt{2C_rD}} \tag{11}$$

Remember, δ the total relevant cost as a function of Q, the two terms that matter the most are this first term, which are calling the order costs (C_rD/Q), and then the second term, which is going to be the holding cost $(ChQ/2)$. These are the two critical values or cost components. The mathematical procedure for calculating the $\delta(Q)$ and $\delta(Q^*)$ is well-known, hence it can be entirely omitted. Then, according to Eq.11 and after rearranging terms, $\delta(Q)/\delta(Q^*)$ would be:

$$\frac{\delta(Q)}{\delta(Q^*)} = \frac{1}{2}\left(\frac{\sqrt{2C_rD}}{Q\sqrt{Ch}} + \frac{Q\sqrt{Ch}}{\sqrt{2C_rD}}\right) = \frac{1}{2}\left(\frac{Q^*}{Q} + \frac{Q}{Q^*}\right) \tag{12}$$

Based on the Eq.12, it possible to find what the change in the total relevant costs will be if a different p is picked compared to the optimal p^*, where $p^* = Q^*/D$, and therefore $Q^* = Dp^*$:

$$\frac{\delta(p)}{\delta(p^*)} = \frac{1}{2}\left(\frac{Q^*}{Q} + \frac{Q}{Q^*}\right) = \frac{1}{2}\left(\frac{Dp^*}{Dp} + \frac{Dp}{Dp^*}\right) = \frac{1}{2}\left(\frac{p^*}{p} + \frac{p}{p^*}\right) \tag{13}$$

Now, the basis for using this technique is that a base time period needs to be chosen (a week, a month, etc), and products could be ordered in powers of two of that $(2^0, 2^1, 2^2, ...)$, which guarantees that the total relevant cost will be within 6% of optimal, as will be shown later. In this way, whatever p^* is, a value α is picked so that p^* falls between 2^α and $2^{\alpha+1}$. For instance, suppose a base period of one week, then $2^0 = 1$ would mean ordering once a week; $2^1 = 2$ means ordering every two weeks; $2^2 = 4$ every four weeks, and so forth; Then, it is necessary to pick a value α where p^* falls in between those:

$$2^{\alpha} \leq p^* \leq 2^{\alpha+1} \tag{14}$$

Therefore, according to Eq.14, p^* could fall somewhere within a time period $p \leq p^* \leq 2p$, and the worst possible case where it could fall is right in the middle: the middle would be defined as where the error from the side p is equal to the error coming from the side $2p$. This concept of "worst possible case" implies to a hypothetical case: if $2^{\alpha} \leq p^* \leq 2^{\alpha+1}$, it could be possible to choose 2^{α} or $2^{\alpha+1}$ as p, or what p^* would be to maximize $\delta(p)/\delta(p^*)$. Hence, the worst means the maximum δ penalty using a power of two-time interval contrasted to the optimal cycle length. The whole idea is to make a practical policy using multiples of two of the time unit.

Thus, the point where those two are equal to each other is the worst possible error, because if p^* is close to this 2^{α} or close to this $2^{\alpha+1}$. Now, what is important to realize is that the point where those errors are the same is not the middle between 2^{α} and $2^{\alpha+1}$, the average of that would be $(2^{\alpha} + 2^{\alpha+1})/2$, when in fact, for those errors to be equal to each other is that it is actually where $2^{\alpha+1/2}$. Then, it is not that is in between the 2^{α} and the $2^{\alpha+1}$, it is where the errors are the same. This can be figured out by establishing some equations:

Substituting into Eq.13 the values p and $2p$, the point where those errors are the same would be,

$$\frac{\delta(p)}{\delta(p^*)} = \frac{1}{2}\left(\frac{p^*}{p} + \frac{p}{p^*}\right) = \frac{1}{2}\left(\frac{p^*}{2p} + \frac{2p}{p^*}\right) \tag{15}$$

$$\frac{p^*}{p} - \frac{p^*}{2p} = \frac{2p}{p^*} - \frac{p}{p^*} \tag{16}$$

$$\frac{2p^* - p^*}{2p} = \frac{2p - p}{p^*} \tag{17}$$

$$\frac{p^*}{2p} = \frac{p}{p^*} \tag{18}$$

According to Eq.18 and after rearranging terms,

$$\frac{p^*}{p} = \sqrt{2} \;\; and \;\; \frac{p}{p^*} = \sqrt{\frac{1}{2}} \tag{19}$$

Taking those terms in Eq.19 and put them right back into Eq.13, it is possible to note that turns out to be 6%. The notion here is if the power of two intervals is applied, it is assumed to be within 6% of the δ compared to the optimal interval.

$$\frac{\delta(p)}{\delta(p^*)} \leq \frac{1}{2}\left(\sqrt{2} + \sqrt{\frac{1}{2}}\right) \sim 1.060 \tag{20}$$

Therefore, the steps to use this grouping technique in practice involve to find p^*, choose a base time period p, and then find the lowest value of α that satisfies:

$$\frac{p^*}{\sqrt{2}} \leq 2^{\alpha} \leq \sqrt{2}p^* \tag{21}$$

Solving for α out of Eq.21 just by taking natural logs, the bounded terms for determining what that α is going to be can be found,

$$\frac{\ln\left(\frac{p^*}{\sqrt{2}}\right)}{\ln(2)} \leq \alpha \leq \frac{\ln(p^*\sqrt{2})}{\ln(2)} \tag{22}$$

Finally, a value $p_{practical}$, which is the practical cycle length that takes the lower bound, is calculated for each SKU as follows (ceiling brackets at the top means round up to the closest integer). The flowchart of the whole proposed method is shown in Fig. 1.

$$p_{practical} = 2^{\alpha} = 2^{\left\lceil \frac{\ln\left(\frac{p_i^*}{\sqrt{2}}\right)}{\ln(2)} \right\rceil} \tag{23}$$

4. Validation

To validate the method, a case study was conducted with the real data of a retail company. Data from the last 24 months of the company was considered in order to have real updated information. In addition, it is important to mention that the product catalog included 97 impulse purchase products that were marketed during this time interval, and that were supplied by national and international suppliers. On the other hand, some products from the company's catalog were not included in the study, as certain criteria were examined to debug products with atypical information:

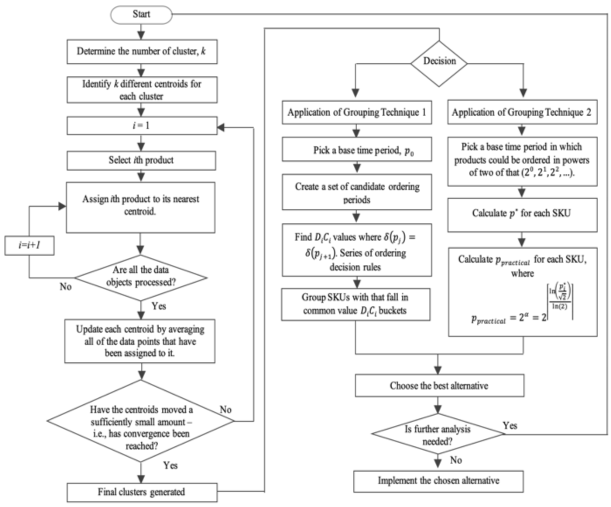

Fig. 1. Simplified flowchart of the proposed method.

- Products that were withdrawn from the points of sale for presenting low sales volume and, therefore, did not have enough historical data for their analysis.
- New products or products that were recently released (less than 23 months on display), did not have sufficient historical data for analysis. This is based on the need to work with sufficient representative information in the case study.
- Products that due to commercial agreements with the supplier were obliged to be exhibited but did not represent any significant behavior in the sales of the studied period.
- Products discontinued by the supplier in the period studied.

4.1. Cluster analysis

To perform the cluster analysis, the data was prepared as presented in Fig. 2. Rows were observations (impulse purchase products) and columns were variables (monthly demand). Also, any missing value in the data was removed. Thus, a dataset was used, which contained statistics in demand per 97 impulse buying products (P_i) in each of the 24 months (m_j). In other words, these considerations make it possible to analyze through a grouping into homogeneous sets, all those products that presented a similar behavior in the demand of the last two years.

$$
\begin{array}{c|ccccc|}
 & m_1 & m_2 & m_3 & \cdots & m_{24} \\
P_1 & P_1m_1 & P_1m_2 & P_1m_3 & \cdots & P_1m_{24} \\
P_2 & P_2m_1 & P_2m_2 & P_2m_3 & \cdots & P_2m_{24} \\
P_3 & P_3m_1 & P_3m_2 & P_3m_3 & \cdots & P_3m_{24} \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
P_{97} & P_{97}m_1 & P_{97}m_2 & P_{97}m_3 & \cdots & P_{97}m_{24}
\end{array}
$$

Fig. 2. Observations and variables.

The initial and most important step in using k-means is to specify the number of clusters (k) that will be generated in the final solution. Since k must be determined before starting the algorithm, it is usually beneficial to use different values of k and to look at differences in the results. In this way, the process was executed for 2, 3, 4, and 5 clusters, and the results are shown in Fig. 3, where observations are presented as points on the plot using principal components and an oval is formed around each cluster. However, although this visual examination provides information on where the true boundaries between the clusters occur, it does not indicate the optimal number of clusters. The k-means algorithm initiates by selecting in a random way k objects from the entire dataset to be used as initial centers or centroids for the clusters. Next, the remaining objects are assigned to the nearest centroid, where closest is defined by the Euclidean distance between the object and the cluster mean; After the assignment step, the algorithm calculates a new mean for each cluster. Each observation is checked again to determine if it could be closer to a different group.

In this sense, the analyst indicates the number of clusters to be used. To assist the analyst, the three most popular methods for determining optimal clusters are explained below. Firstly, the Elbow method by Thorndike (1953) [40] was employed to define how many clusters are appropriate. The key concept behind this method is to identify clusters such that the total within-cluster sum of square is minimized, as follows: $min\{\sum_{k=1}^{k} W(C_k)\}$, where C_k is the kth cluster and $W(C_k)$ is the within-cluster variation. The total within-cluster sum of squares measures the variability within each cluster. and must be as small as practicable. Hence, the following steps to define the optimal clusters can be applied: *(i)* Run k-means clustering process for several values of k; *(ii)* For each k, estimate the total within-cluster sum of square; *(iii)* Calculate the curve of within-cluster sum of squares according to k; and *(iv)* the position of a bend or elbow in the plot is interpreted as a guide of the optimal number of clusters. In this way, this method was implemented in RStudio, and Fig. 4 illustrated the variance within the clusters and suggests the optimal number of clusters somewhere between 2 and 4. As shown, there was an obvious inflection point when $k = 2$.

As a second, it was possible to calculate the average Silhouette width and use this value to judge the optimal number of clusters. In brief, the method calculates the average silhouette from objects for several k values. The average silhouette determines the quality of a cluster. The highest value of the index is used to calculate the optimal number of clusters [41, 42]. The optimal number of clusters maximizes the average silhouette in a given range of possible values for k. From Fig. 5, the results indicate that 2 clusters maximize the average silhouette values.

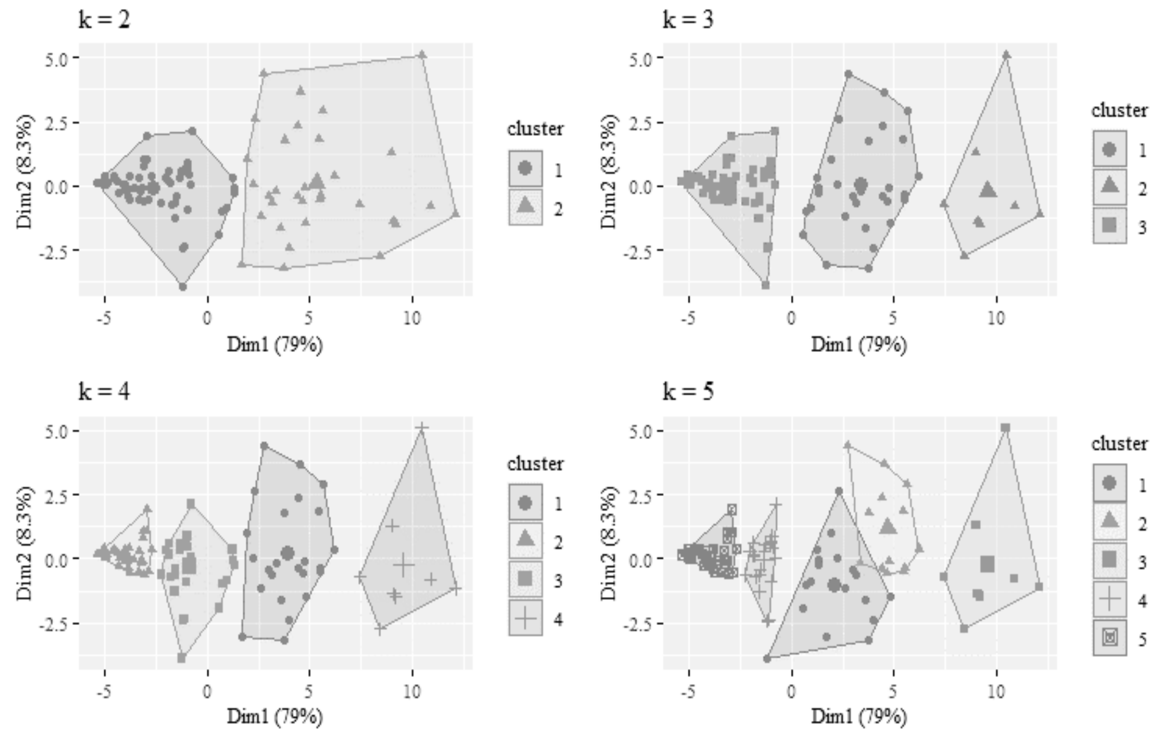

Fig. 3. Number of cluster.

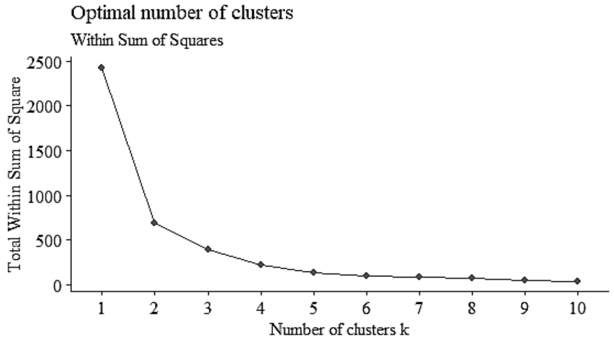

Fig. 4. Optimal number of cluster: Elbow method.

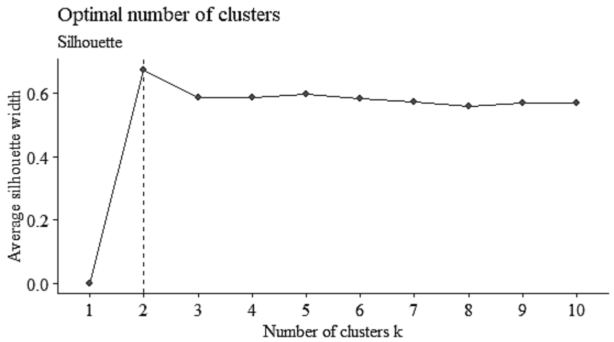

Fig. 5. Optimal number of cluster: Average Silhouette method.

In addition, the Gap Statistic examines the total intracluster variation for different k-values with their expected values [43]. The reference dataset is generated using Monte Carlo simulations of the sampling process. That is, for each variable (x_i) in the data set, the method computes its range $[\min(x_i), \max(x_j)]$ and generates values for the n points uniformly from the interval min to max. The gap statistic for a given k is defined as $Gap_n(k) = E_n^* \log(W_k^*) - \log(W_k)$, where E_n^* is the expectation under a sample n from the reference distribution. E_n^* is expressed via bootstrapping by producing B copies of the reference datasets and, by computing the average $\log(W_k^*)$. The gap statistic calculates the deviation of the observed W_k value from its expected value, and the estimate of the optimal number of clusters is the value that maximizes $Gap_n(k)$. In this way, the algorithm was executed in RStudio and involved the following steps: (i). Cluster the data, changing the number of clusters from $k = 1, \dots, k_{max}$, and calculate W_k; (ii). Produce B copies of the reference datasets and cluster each of them for $k = 1, \dots, k_{max}$; (iii). Let $\overline{w} = (1/B)\sum_b \log(W_{kb}^*)$, calculate the standard deviation as: $sd(k) = \sqrt{(1/B)\sum_b(\log(W_{kb}^*) - \overline{w})^2}$ and define $s_k = sd_k \times \sqrt{1 + 1/B}$; and (iv). Select the number of clusters as the smallest k such that $Gap(k) \geq Gap(k+1) - s_{k+1}$. According to Fig. 6, this criterion does not cause any of the gap statistics to stand out, resulting in $k = 1$. However, among the set, $k = 2$ is evidently preferred in the gap statistics plot: it is the first local maximum and the stats for smaller k (that is, $k = 1$) are significantly lower. Larger values of k are likely to overfit for such a small dataset, and none are appreciably better than $k = 2$.

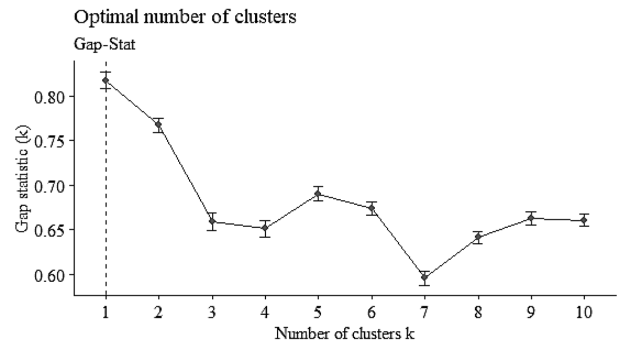

Fig. 6. Optimal number of cluster: Gap Statistic method.

In addition to the above methods, the NbClust package, published by Charrad, Ghazzali, Boiteau, & Niknafs (2014) [44], provides 30 indices for computing the relevant number of clusters by changing all combinations of the number of clusters, distance measures, clustering methods, etc. The results showed that most of these approaches suggested 2 as the number of optimal clusters. Consequently, the final analysis was performed (Tab.1), and the results were extracted using 2 clusters sizes of 65 and 32.

Table 1. Cluster analysis results

Cluster	A vector of integers (from 1:k) indicating the cluster to which each point is allocated.	: int [1:97] 1 2 2 1 1 2 1 1 1 1 ...
Centers	Cluster centers.	: num [1:2, 1:24] 1171 4911 1107 4683 1327 ...
Totss	The total sum of squares.	: num 1.7e+10
Withinss	Vector of within-cluster sum of squares, one component per cluster.	: num [1:2] 2.58e+09 4.55e+09
Tot.withinss	Total within-cluster sum of squares.	: num 7.13e+09
Betweenss	The between-cluster sum of squares.	: num 9.84e+09
Size	The number of points in each cluster.	: int [1:2] 65 32
Iter	The number of (outer) iterations.	: int 1
Ifault	integer: indicator of a possible algorithm problem – for experts.	: int 0

4.2. Managing multiple items

Following the proposed method, a replenishment schedule was set up for the 97 SKUs that were grouped into the two previous clusters. These products have an average ordering cost (C_r) of USD 8.45 (Cluster 1) and USD 7.0 (Cluster 2) per transaction, and a holding charge (h) of 0.24. Considering that impulse purchase products have a short lead-time since they are usually placed through frequent orders, with a shorter and less risky forecast horizon, the company has a 1-week ordering period in a traditional scenario. In addition, candidate or proposed ordering periods to be used in the two grouping techniques explained in Section 3, were the following: p_1 = 1 week, p_2 = 2 weeks, p_3 = 4 weeks, p_4 = 13 weeks, p_5 = 26 weeks, and p_6 = 52 weeks. The grouping technique 1 identify items according to their value and it is possible to establish decision rules for aggregating SKUs and use similar inventory policies: The concept here is to identify those items with higher value, which was defined as the annual demand times the cost per item D_iC_i, and they will be replenished at a faster rate, and the lower value ones will be ordered in bigger quantities, lower speed. Carry on the cluster analysis before this step was important because this method is recommended to be used for situations where products share a similar demand behavior, therefore, this method will be applied in the two clusters previously identified. As seen in Eq.3, it was necessary to find those annual values, that D_i times C_i value, where the total relevant cost ordering at one frequency $\delta(p_j)$ is equal to the total relevant cost of ordering at that next highest interval period $\delta(p_{j+1})$; All of those SKUs that fall in the common annual value segments will be grouped and managed commonly with the same ordering frequency.

Q_{ij}, which is the quantity ordered for item i at time period j, is going to be equal to the annual volume, D_i, times p_j, whatever candidate time period that was picked. Then, considering that the base period was one week, $Q_{ij} = D_i\left({}^{p_j}/_{52}\right)$:

$$C_rD_i\left(\frac{52}{D_ip_j}\right)+\frac{C_ih}{2}\left(\frac{D_ip_j}{52}\right)=C_rD_i\left(\frac{52}{D_ip_{j+1}}\right)+\frac{C_ih}{2}\left(\frac{D_ip_{j+1}}{52}\right) \quad (24)$$

Therefore, (24) reduces to,

$$\frac{52C_r}{p_j}+\frac{C_ihD_ip_j}{104}=\frac{52C_r}{p_{j+1}}+\frac{C_ihD_ip_{j+1}}{104} \quad (25)$$

Re-arranging all the terms,

$$\frac{C_ihD_ip_j}{104}-\frac{C_ihD_ip_{j+1}}{104}=\frac{52C_r}{p_{j+1}}-\frac{52C_r}{p_j} \quad (26)$$

(26) can be written as,

$$\frac{C_ihD_i}{104}\left(p_j-p_{j+1}\right)= 52C_r\left(\frac{1}{p_{j+1}}-\frac{1}{p_j}\right) \quad (27)$$

Thus, D_iC_i can be expressed as:

$$D_iC_i=\frac{5408C_r}{h(p_j-p_{j+1})}\left(\frac{1}{p_{j+1}}-\frac{1}{p_j}\right)=\frac{5408C_r}{hp_jp_{j+1}} \quad (28)$$

Regarding the decision rules, if the annual value is greater than a value φ_1, a one week's worth of product every week will be ordered - That is a real high-value item-. The next decision rule is going to be, if it is not greater than φ_1 as long as it is greater than φ_2, a two weeks' worth every two weeks will be ordered, and so on with each decision rule, as follows:

Decision rule for selecting between 1 week or 2 weeks is:

$$D_iC_i = {}^{5408C_r}/_{hp_1p_2} = \varphi_1;$$

Order every 1-week if $D_iC_i \geq \varphi_1$ (29)

Decision rule for selecting between 2 weeks or 4 weeks is:

$$D_iC_i = {}^{5408C_r}/_{hp_2p_3} = \varphi_2;$$

Order every 2 weeks if $\varphi_1 > D_iC_i \geq \varphi_2$ (30)

Decision rule for selecting between 4 weeks or 13 weeks is:

$$D_iC_i = {}^{5408C_r}/_{hp_3p_4} = \varphi_3;$$

Order every 4 weeks if $\varphi_2 > D_iC_i \geq \varphi_3$ (31)

Decision rule for selecting between 13 weeks or 26 weeks is:

$$D_iC_i = {}^{5408C_r}/_{hp_4p_5} = \varphi_4;$$

Order every 13 weeks if $\varphi_3 > D_iC_i \geq \varphi_4$ (32)

Decision rule for selecting between 26 weeks or 52 weeks is:

$$D_iC_i = {}^{5408C_r}/_{hp_5p_6} = \varphi_5;$$

Order every 26 weeks if $\varphi_4 > D_iC_i \geq \varphi_5$, order every 52 weeks otherwise. (33)

In brief, these SKUs, the real high values (decision rule = 1 week) will be ordered once a week - frequent in small quantities. The next group (decision rule = 2 weeks) will be ordered two weeks' worth of product every two weeks, and so on. Then what the proposed method is doing here is just segmenting the SKUs and, hence a company can have common ordering practices and inventory management for a cluster instead of doing them each individually and independently. Tab.2 presents ordering decision rules for each cluster.

Table 2. Ordering decision rules

Final ordering decision rules:	**Number of SKU**
	Cluster 1
Order every 1-week If $D_iC_i \geq$ USD 95,203.33	1 (1.54%)
Order every 2 weeks if USD 95,203.33 $> D_iC_i \geq$ USD 23,800.83	4 (6.15%)
Order every 4 weeks if USD 23,800.83 $> D_iC_i \geq$ USD 3,661.66	41 (63.08%)
Order every 13 weeks if USD 3,661.66 $> D_iC_i \geq$ USD 563.33	17 (26.15%)
Order every 26 weeks if USD 563.33 $> D_iC_i \geq$ USD 140.83	2 (3.08%)
Order every 52 weeks otherwise	0 (0.00%)
Total	65 (100.00%)
	Cluster 2
Order every 1-week If $D_iC_i \geq$ USD 78,866.67	1 (3.13%)
Order every 2 weeks if USD 78,866.67 $> D_iC_i \geq$ USD 19,716.67	12 (37.50%)
Order every 4 weeks if USD 19,716.67 $> D_iC_i \geq$ USD 3,033.33	19 (59.38%)

Order every 13 weeks if USD $3{,}033.33 > D_iC_i \geq$ USD 466.67	0 (0.00%)
Order every 26 weeks if USD $466.67 > D_iC_i \geq$ USD 116.67	0 (0.00%)
Order every 52 weeks otherwise	0 (0.00%)
Total	32 (100.00%)

In general, most of the products in cluster 1 will be ordered every 4 (63.08%) and 13 (26.15%) weeks, and only a small percent will be ordered once a week (1.54%), every 2 weeks (6.15%), and every 26 weeks (3.08%). Regarding cluster 2, 59.38% of the products will be ordered every 4 weeks, 37.50% every 2 weeks, and 3.13% once a week; Then, it is clear that cluster 2 includes higher-value products that cluster 1 because products in cluster 2 will be ordered more frequently than those in cluster 1. Figures 7 and 8 show the final ordering decision rules for both clusters.

Now, it is necessary to look at another approach to group SKUs, but this time using the grouping technique 2: if p is used instead of p^* (which is the optimal ordering frequency, or the duration of the cycle) because sometimes would be more reasonable to use an inventory policy that actually gets executed than something that might be theoretically optimal, SKUs would be ordered according to a time interval where $p^*/\sqrt{2} \leq 2^\alpha \leq \sqrt{2}p^*$. Besides, this grouping technique guarantees that δ will be within 6% of optimal, and also, according to equation (20), the range of the errors would be bounded by the p^*divided by the square root of two, and it will be less than the square root of two times that p^*. Solving for α out of that just by taking natural logs, it is possible to find the bounded terms for determining what α is going to be: $\ln\left(\frac{p^*}{\sqrt{2}}\right)/\ln(2) \leq \alpha \leq \ln\left(p^*\sqrt{2}\right)/\ln(2)$. Thus, p^* was calculated for each product i . Indeed, p_i^* is simply the economic order quantity over the annual demand: $p_i^* = Q_i^*/D_i = \sqrt{2C_rD_i/c_ih}/D_i = \sqrt{2C_r/D_ic_ih}$. Then, a value $p_{practical}$, which is the practical cycle length, was calculated for each SKU using Eq.23.

This grouping technique was applied for the exact same data set that was looked at for grouping by decision rules. Tab.3 presents the practical cycle length results. In addition, Figures 9 and 10 show a graphical representation for both clusters. In brief, most of the SKUs or products in cluster 1 will be ordered every 8 (44.62%) and 4 (40.00%) weeks, and a small percent will be ordered once a week (1.54%), every 2 weeks (6.15%), every 16 weeks (6.15%), and every 32 weeks (1.54%). Regarding cluster 2, 56.25% of the products will be ordered every 4 weeks, 37.50% every 2 weeks, 3.13% every 8 weeks, and 3.13% once a week. In this way, what the grouping technique 2 is doing essentially here is finding where there are breakpoints based on the $p_{practical}$ and where it would lump the SKUs. Companies would manage all these SKUs and clusters similarly based on the $p_{practical}$.

Table 3. Practical cycle length (weeks)

Replenishment Cycle Length	Number of SKU	
	Cluster 1	Cluster 2
Order every 1-week, $p_{practical} = 1$	1 (1.54%)	1 (3.13%)
Order every 2 weeks, $p_{practical} = 2$	4 (6.15%)	12 (37.50%)
Order every 4 weeks, $p_{practical} = 4$	26 (40.00%)	18 (56.25%)
Order every 8 week, $p_{practical} = 8$	29 (44.62%)	1 (3.13%)
Order every 16 weeks, $p_{practical} = 16$	4 (6.15%)	0 (0.00%)
Order every 32 weeks, $p_{practical} = 32$	1 (1.54%)	0 (0.00%)
Total	65 (100.00%)	32 (100.00%)

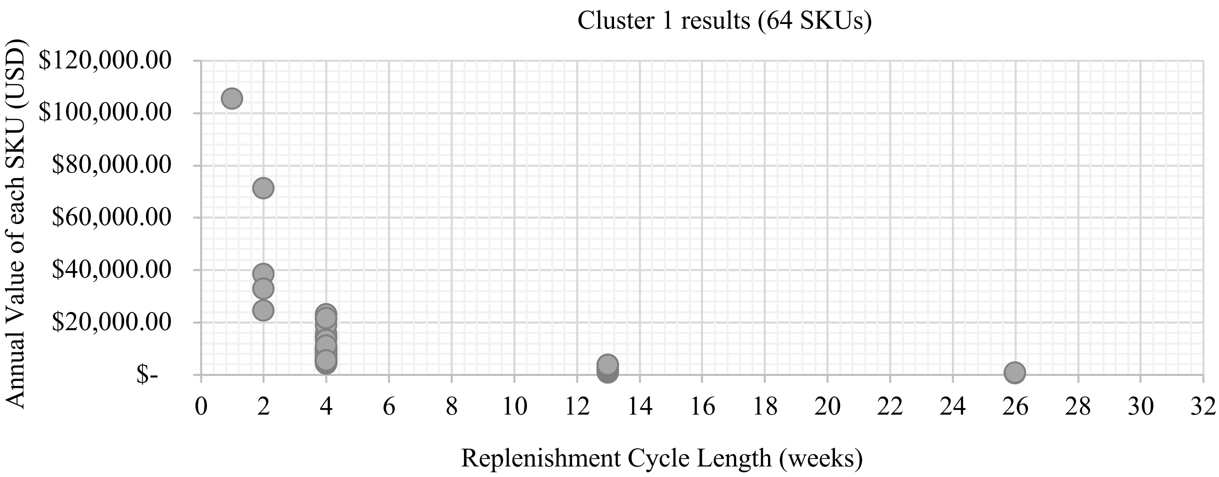

Fig. 7. Grouping technique 1 - Cluster 1 results.

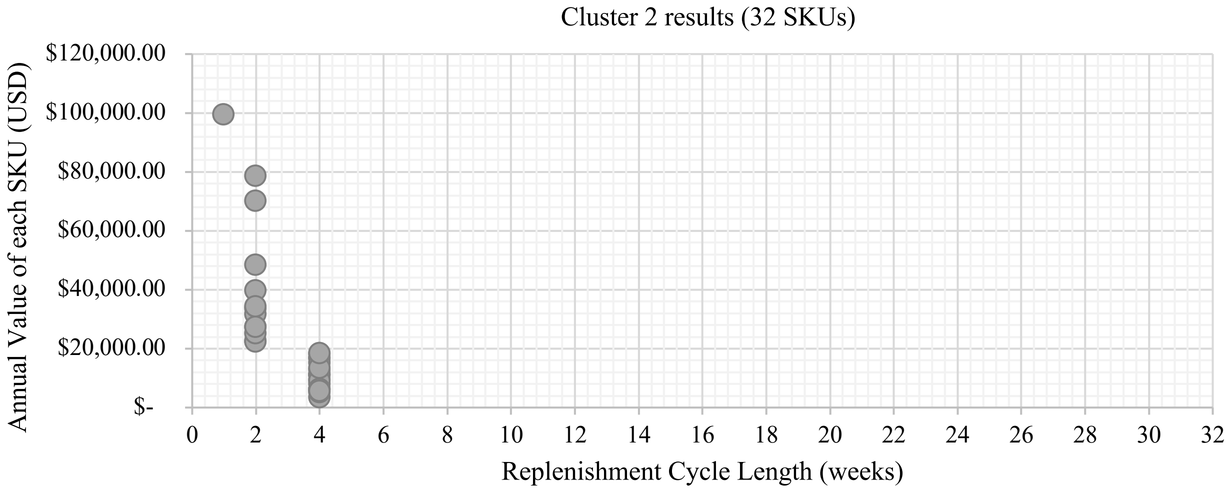

Fig. 8. Grouping technique 1 - Cluster 2 results.

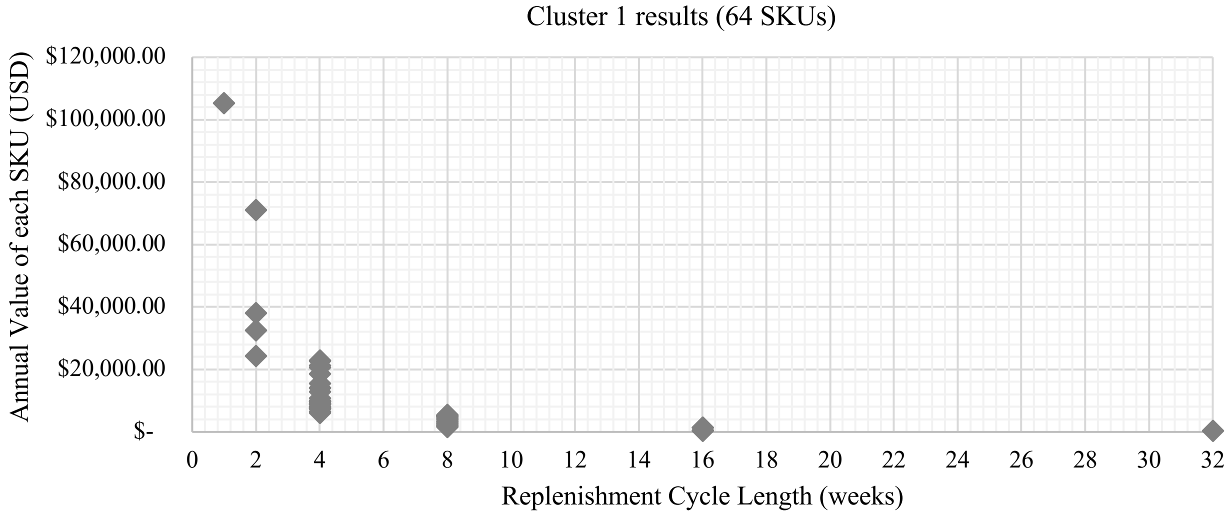

Fig. 9. Grouping technique 2 - Cluster 1 results.

If the two grouping techniques are compared, it is possible to see that there is a lot of similarities. Consider the scenarios given in figures shown, the replenishment cycle length is showed on the horizontal axis, and on the vertical is the annual value of each SKU. In general, the products would be aggregated slightly differently. For both techniques, there were no SKUs that would be ordered more than every 32 weeks, but many of them would be ordered every 2 and 4 weeks. The reason why some of these appear at 13, 16, 26, or 32 weeks was that these replenishment cycle lengths were given as an option. If instead, for the grouping technique 1, they would be given the option of a shorter number of weeks, which is what the grouping technique naturally falls into, then these SKUs would probably be ordered in that replenishment cycle length. But again, the grouping technique 1 assigns the SKUs based on the ordering frequency that was given to them, and the grouping technique 2 uses a value $p_{practical}$.

Finally, an analysis of the total relevant cost (δ) was carried out. It is important to mention that a first scenario based on the grouping technique 1 was compared with a second scenario based on the grouping technique 2 previously applied. Beyond having as it is purpose to execute the method in a real-life scenario, the main objective was to learn the economic impact as a key indicator of its implementation. These results were calculated for one year and they can be found in Tab.4. In cluster 1, it was noted that the greatest economic benefit is achieved in scenario 2, with a savings of 0.88% compared to scenario 1. As for cluster 2's total relevant cost, scenario 1 showed beneficial results, with a savings of 3.73% compared to scenario 2. Comparing the total value, the results showed a reduction in cost of only 1.06% in scenario 1 compared to scenario 2. This demonstrated that the method proposed in this case study obtained similar results in each of the scenarios for each of the clusters, as well as for the total relevant cost.

5. Conclusion and future work

The results show that the proposed method can be used to cluster impulse purchase products more effectively and the grouping techniques applied were efficient in terms of solution quality. The aim of the proposed unsupervised clustering-based method was not only to provide a classification of SKUs free of subjectivity processes but also to provide an approach to apply more efficient inventory policies for impulse purchase products. Therefore, the relevance of this research is related to solving problems concerning ordering these types of products independently or making an EOQ for each item. One of these is if all of the individual SKUs are ordered by themselves, and there are thousands of them, there will be a lack of coordination, and also, by doing them individually, it cannot be taken into any common constraints. As second, if these products are looked independently, some opportunities for consolidation will be missed as well as shared costs. And the final and perhaps most important problem is that individualized treatment is a waste of management time. Companies or product managers could spend a lot of time managing these items independently, and time is a scarce resource. Then, by applying a method as the one presented in this paper, they could find ways where clusters of SKUs can be treated together and commonly with the same inventory policies. In this way, the method brings economic and administrative benefits related to the management of impulse purchase products, minimizing total costs, and optimizing logistics operations

Table 4. Total Relevant cost: a comparison analysis

δ	**Scenario**	
	Scenario 1: Grouping technique 1	**Scenario 2: Grouping technique 2**
Cluster 1	USD 12,193.22	USD 12,085.61
Cluster 1	USD 8,502.14	USD 8,831.96
Total	USD 20,695.36	USD 20,917.57

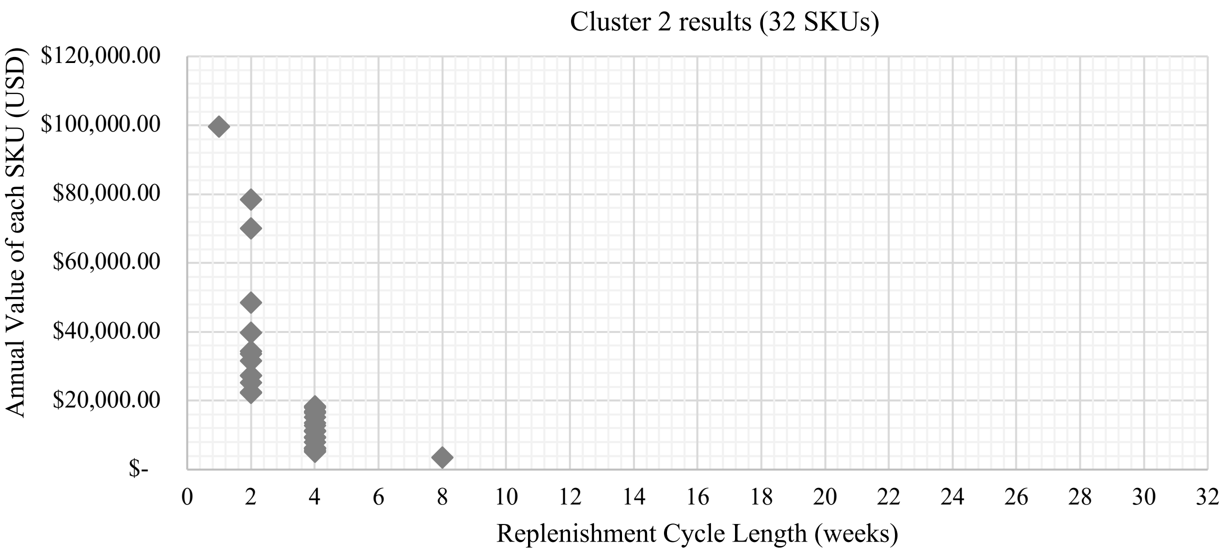

Fig. 10. Grouping technique 2 - Cluster 2 results.

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In sum, the proposed method mathematically represents a scheme that achieves a coordinated solution managing multiple products. It should also be noted that impulse buying products are characterized as products that cost little and are quickly consumed. Additionally, in general terms, an assumption related to constant lead time, it is an appropriate simplification of reality. Usually, a short lead time is a relatively common occurrence in the impulse purchase products market, as information distortion is magnified if replenishment lead times between stages are volatile or long. Then, by decreasing the replenishment lead time, companies and managers can minimize the uncertainty of demand during the lead time. Another critical aspect of these products is the way to calculate holding costs in practice. Commonly, managers consider the cost of holding stock in the storage facilities that are incurred before serving a set of buyer's stores, and the cost of holding stock in the display shelf defined as the multiplication of the product volume and the shelf space cost (remember that generally, these items are strategically displayed in hot spots), which depends on the shelf space to display the product in the store. Last but not least, firms in this sector may order in large lots because the presence of fixed costs associated with ordering, quantity discounts in product pricing, and short-term promotions, encourages different stages of a supply chain to exploit economies of scale and order in large lots.

Finally, the validation phase can be complemented with additional real cases. This is an aspect that would provide an enormous benefit to ensure greater credibility. In addition, it is possible to use this research as a starting point to move towards much more elaborate methods, which reflect greater complexity of the real systems, and better representation of logistics operations. As far as future research is concerned, this method presents several aspects to consider, mainly its implementation, since it would be interesting to see its development, impact, and response relating to other business environments. One of the limitations of this method is that the demand for the products within each cluster could diverge over time and the decision-makers should apply a new grouping strategy. For this reason, for future work, a new version of this method should provide diagnostic signals to indicate that clustering solutions performance is degrading and that there is a need to form new clusters. Also, a future research opportunity would be to consider a larger dataset to validate the results obtained in both grouping techniques or compare to other relevant unsupervised machine learning applications since a disadvantage of k-means is that it is sensitive to outliers and different results may occur if the order of the data changes.

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