

Lecture Note

Wheatstone Bridge and Bioengineering

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Abstract

Wheatstone bridge is a circuits that has been used in many applications. In these lecture notes, a brief history, the basic circuits and mathematical expressions, a side wide bridge as well as the connection of Wheatstone bridge with biomechanics are presented. A representative application for hand movement and a variation in microfluidics are described.

Keywords: Wheatstone bridge, bioengineering, Christie.

1. Introduction

In 1833 S. Hunter Christie published the basic application of the later Wheatstone bridge in the Philosophical Transactions of the Royal Society of London [1]. Through a series of experiments, S.H. Christie managed to balance a device by studying the conduction power of the conductors, which depends directly on the cross section of the wire [2]. Ten years later Charles Wheatstone in his work "An account of several instruments and processes for determining the constants of a voltaic circuits" describes for the first time the Wheatstone bridge in the form we know it today [3].

The Wheatstone bridge has many applications in everyday life and, as expected, in the field of bioengineering [4]. The definition of bioengineering was given only in 1998 during a symposium and is: "Bioengineering integrates physical, chemical, or mathematical sciences and engineering principles for the study of biology, medicine, behavior or health. It advances fundamental concepts, creates knowledge from the molecular to the organ systems level, and develops innovative biologics materials, processes, implants, devices, and informatics approaches for the prevention, diagnosis, and treatment of disease, for patient rehabilitation, and for improving health [4]. As the definition suggests, this is an area that is by definition multidisciplinary, which is experiencing continuous prosperity.

The Wheatstone bridge has a number of applications that are distinguished in three main areas: a) the measurement of precision, very small changes in resistances, b) the measurement of physical parameters (mechanical forces, light, temperature, etc.) and c) the measurement of inductance, capacitance and impedance. Indicative applications of the Wheatstone bridge are smoke sensors, temperature sensors, light sensors, the measurement of mechanical loads, etc.

The basic variants of the Wheatstone bridge are shown in Figure 1.

In case Fig. 1. a) one of the resistors is variable and the three are constant, in case b) the two opposites are constant while the other two are variable and in case c) all four resistors are variable. A typical variant of the Wheatstone bridge is the replacement of two of the four resistors with a straight wire section, whereby solving it, length can be calculated instead of resistance value as illustrated in figure 2.

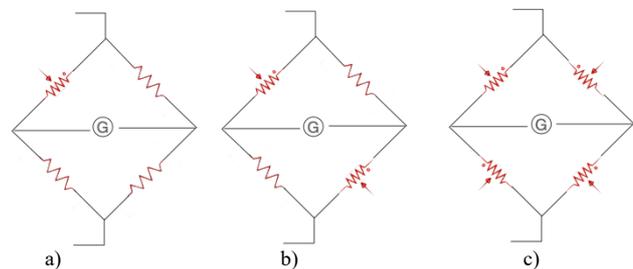


Fig. 1. Basic variants of Wheatstone bridge a) 1/4, b) 2/4 and c) full bridge [5], [6].

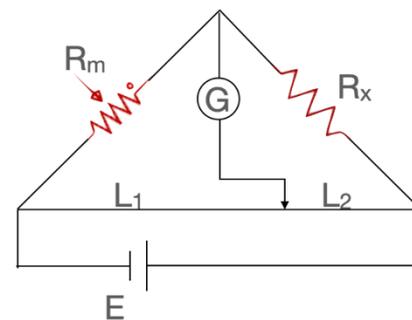


Fig. 2. Side wide bridge

2. Wheatstone Bridge Equations

Figure 3 shows the case of the Wheatstone bridge 1/4 where only one resistor is variable and one is unknown R_x . A galvanometer is placed between the BD, which is used to measure the current. It is an instrument for measuring low electrical voltages, depending on the existence of current and the mechanical deviation (rotation) of a coil. Resistors R_1 και

R_3 are constant while R_2 is variable. By varying the value of resistor R_2 we manage to make the galvanometer read zero which shows that no current is leaking between the BD. In this condition the system is balanced and the equation 1 is valid.

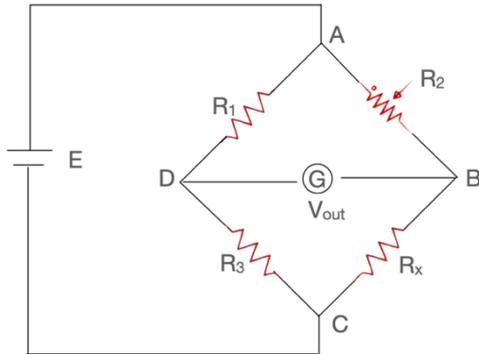


Fig. 3. 1/4 Wheatstone bridge

$$\frac{R_1}{R_3} = \frac{R_2}{R_x} \quad (1)$$

In fact, what happens is that the potential difference between points D and B is zero ($V_{BD}=0$) and is described by the equation (2):

$$V_G = V_{out} = V_D - V_B = V_{R_3} - V_{R_x} = 0 \quad (2)$$

Due to equation (2) the following applies

$$R_D = \frac{R_3}{R_1 + R_3} \quad (3)$$

$$R_B = \frac{R_x}{R_2 + R_x} \quad (4)$$

In equilibrium, the following equation applies:

$$R_D = R_B \quad (5)$$

Thus

$$\frac{R_3}{R_1 + R_3} = \frac{R_x}{R_2 + R_x} \quad (6)$$

$$R_3(R_2 + R_x) = R_x(R_1 + R_3) \quad (7)$$

$$R_3R_2 + R_3R_x = R_xR_1 + R_3R_x \quad (8)$$

$$R_3R_2 = R_xR_1 \quad (9)$$

$$R_x = \frac{R_3R_2}{R_1} \quad (10)$$

3. Side Wide Bridge

One of the main applications of the Wheatstone bridge is the side wide bridge, in which, as already mentioned, the two

resistors are replaced by a conductor of length l , cross section S and resistor ρ , and as is known

$$R = \rho \frac{L}{S} \quad (11)$$

From figure 3 it can be concluded that $l = L_1 + L_2$

As a result $R_1 = \rho \frac{L_1}{S_1}$ and $R_2 = \rho \frac{L_2}{S_2}$ which due to the fact that the resistivity and the cross section of the conductor are of the same material are the same regardless of its length.

Therefore:

$$\frac{R_{L_1}}{R_{L_2}} = \frac{L_1}{L_2} \quad (12)$$

In figure 3 when the galvanometer resets so the system will be in equilibrium the same case as in 1/4 bridge will apply, i.e.:

$$R_x = \frac{R_{L_2}}{R_{L_1}} R_m = \frac{L_2}{L_1} R_m \quad (13)$$

Due to $L = L_1 + L_2$ (14)

$$R_x = \frac{L - L_1}{L_1} R_m \quad (15)$$

If we knew from the beginning the value of the unknown resistance we could calculate the length of one of the two conductors (L_1 or L_2).

4. Bioengineering Applications

The use of the Wheatstone bridge in bioengineering occurs in combination with strain gauges used to measure bone strain, muscle movement or even the calibration of medical equipment [7]–[9]. Other applications include X-ray energy measurement, robotic surgery, dialysis machines and [10]–[12].

The Wheatstone Bridge and the Strain Gauges are also based on the application of M Van der Kamp et al. to measure grip force during precision grip tasks [13]. In this work 2 bridges are used to determine each position resulting from the application of pressure at 2 points (two-point load) thus giving the exact movement of the hand. The device used is in Figure 4 where the output voltage of bridge 1 is given by equation 16.

$$V_{out_1} = \left(\frac{R_1}{R_1 + R_7} - \frac{R_3}{R_3 + R_5} \right) V_{in} \quad (16)$$

Of particular interest is its use in Lab on Chip applications [14]. In this application, a Lab on Chip was developed for the analysis of liquid samples, and its operation is based on the principle of operation of the Lab on a Chip. The sample to be analyzed (figure 5) is in the middle of the bridge and is defined by the change of the "resistors" R_1R_3 and R_2R_4 . By modifying the values of these resistances, the bridge loses its

balance and a new amount of liquid to be tested is introduced at the corresponding point Q which corresponds to the galvanometer. The flow can be found using the equation 17.

$$Q_{bridge} = Q \frac{R_1 R_3 - R_2 R_v}{(R_1 + R_v)(R_2 + R_3) + R_{BR}(R_1 + R_2 + R_3 + R_v)} \quad (17)$$

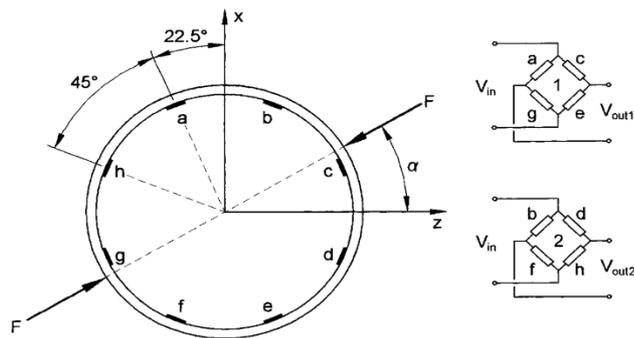


Fig. 4. M Van der Kamp et al proposed circuit

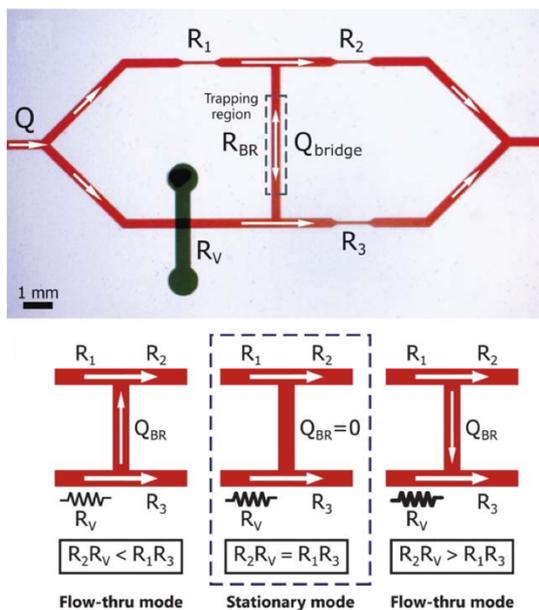


Fig. 5. A microfluidic device developed based in Wheatstone bridge [14]

5. Conclusions

In this Lecture Note, the 3 basic variants of the Wheatstone bridge were presented, for the calculation of the value of the unknown resistors as well as the methodology for the calculation of the conductor length using it. Finally, its basic applications in the field of bioengineering were presented for the calculation of the position of the hand as well as a lab on a chip.

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