Distinctive Approach to Design Tree in Wavelet Packet based OFDM System

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Received 5 February 2017; Accepted 12 May 2017

Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is the effective modulation technique for transmitting large amount of digital data over radio waves. Peak to Average Power Ratio (PAPR) is the main problem of OFDM. The objective of this paper is to reduce PAPR in Wavelet Packet Tree (WPT) based OFDM using Cost Function Algorithm. The output of proposed system is measured with two parameters- Complementary Cumulative Distribution Function (CCDF) for PAPR and Bit Error Rate (BER). The proposed system achieves PAPR improvement over Mallat technique using different Wavelet Packets.

Keywords: OFDM, PAPR, QPSK, Wavelet Packet Tree and BER

1. Introduction

OFDM is considered as speedily growing candidate of future generation wireless communication system. It is an effective technique for high data rate and transmission capacity. Due to this benefit, OFDM is used as main technology behind different applications such as Asymmetric Digital Subscriber Line (ADSL), HIPERLAN/2, Digital Audio Broadcasting/Digital Video Broadcasting (DAB/DVB). A major drawback of OFDM at the transmitter is high Peak-to-Average Power Ratio (PAPR). This results from the superposition of numerous statistically autonomous sub-channels which can beneficially sum up to high peaks. The peak of the signal increases with the number of sub-carriers. Moreover, high values of PAPR increases the complexity of analog-to-digital (A/D) and digital-to-analog (D/A) converters and reduces the efficiency of amplifier.

Various techniques in [1-9], have already been proposed in literature to reduce PAPR of original OFDM system such as Partial Transmit Sequence (PTS) [2], Selective Mapping (SLM) [3], Clipping [4], Coding [5], Hadamard Transform [6], Tone Reservation and Tone Injection [7], Interleaving [8], and Nonlinear Companding Transform[9] etc. All these systems use Fast Fourier Transform (FFT) as signal transformation with different modulation techniques. Wavelet Packet Modulation (WPM) uses wavelets which offer flexibility that can be created to fulfill an engineering demand.

WPM was first proposed by Lindsey [10] as an alternative to OFDM. As WPM has analogous drawbacks of OFDM due to multicarrier modulation (MCM), large numbers of techniques have been proposed to decrease the PAPR in WPM. A Threshold Control method is presented to decrease the PAPR [11]. Different Modulations with clipping are also compared to decrease the PAPR in OFDM System [12-13]. The PAPR is improved at the cost of BER degradation. A Wavelet Packet Tree Pruning technique with different nodes and number of pass processing is proposed for the reduction of PAPR in WOFDM systems [14]. Better Wavelet Packet Tree Search is presented in [15]. Selection of the optimal wavelet tree based on Hungarian Algorithm has been discussed in [16]. Analysis of PAPR using FFT and Wavelet based OFDM is discussed in [17]. Wavelet Packet Modulation for mobile Communication is discussed in [18]. Bit Error Rate performance of Discrete Wavelet Transform using time dispersive channel is discussed in [19]. The wavelet based OFDM consisting clipping and companding is reported in [20]. The PAPR reduction using genetic algorithm in lifting based wavelet packet modulation is described in [21]. A new Cost Function based Search Algorithm to design a new Wavelet Packet Tree is presented in this paper for reduction in PAPR with better BER performance and is also compared to conventional Mallat tree structure.

2. OFDM System and its PAPR

The block diagram of transmitter and receiver is shown in Fig 1. The transmitter performs Inverse Discrete Wavelet Packet Transform (IDWPT) using Wavelet Packet basis which is generated by the filter bank. The modulated signal $y[k]$ can be given as

$$y[k] = \sum_p \sum_q M^{-1} q[p][k \rightarrow k - M]$$

(1)

where $X(pq)$ shows the constellation encode $p$th data symbol modulating the $q$ th waveform. M shows the different channels in the system and $q[p][k]$ is a Wavelet Packet Basis function [14]. The orthogonal property of the subcarriers is given by
\[ < \varphi_{q_1}[k], \varphi_{q_2}[k] > = \delta[q_1 - q_2] \]  

(2)

Where \(< >\) is the inner product operator and \(\delta[\cdot]\) is the Dirac-Delta function.

At the receiver side, the received signal is demodulated by Discrete Wavelet Packet Transform. The demodulated data are decoded using Constellation Mapping and Parallel to Serial conversion. Due to Gaussian channel characteristics, the received symbol \(c[k]\) is expressed as

\[ c[k] = y[k] + g[k] \]  

for the \(k\)th subcarrier,

Here \(g[k]\) is the noise function.

To form the different wavelet packet tree structures, the wavelet modulator filters are now given as

\[ T_{\text{cost}} = \min_j \frac{1}{L} \sum_{d=1}^{M} \text{PAPR}_d \]  

(9)

Where \(j\) is the order of wavelet packet decomposition, \(M\) is the number of subchannels at each node of different levels \((L)\).

3. Cost Function based Search algorithm

The improved search algorithm is given as follows:

1. Start with root node and find the best basis by initiation on each level.
2. Decompose the signals with two perfect reconstruction orthogonal filters given by \(j\).
3. Each set of nodes is used to create the random wavelet modulator filter, until each wavelet modulator filter has a different tree structure.

The wavelet modulator filters corresponding to different wavelet packet tree structure are defined as

\[ V_i^t(n) = \prod_{n=1}^{d_i} (v_i^t(n))_{12} \]  

(10)\

\[
\begin{align*}
&v_i^t = h(n)w_i^t \geq 0.5 \\
&v_i^t = g(n)w_i^t < 0.5
\end{align*}
\]

(11)

where \(V_i^t(n)\) is the \(i\)th filter of the \(t\)th wavelet packet tree structure. The \(i\)th filter of the \(L\)th layer of \(s\)th wavelet tree structure is \(v_i^t\) and \(w_i^t\) is the random number within \([0,1]\). It is either low pass filter \(h(n)\) or high pass filter \(g(n)\). If the system meets requirement of \(\frac{1}{n_i} = 1\), then continue, else go to step (2).

4. Find the minimum cost value with the next level of child nodes. If

\[ T_{\text{cost}}[\text{node}] < \sum_{\text{child nodes}} T_{\text{cost}} \]  

(12)

then merge and set \(T_{\text{cost}}[\text{node}]\) and find other node with next cost function else split \(T_{\text{cost}}[\text{node}]\) and algorithm is repeated with \(T_{\text{cost}}[\text{node}]\).

5. Increase the count number by 1, if the number of
subchannels is smaller than required then go to (4) otherwise save this structure as improved tree structure.

Figure 2 shows the Mallat Tree Structure using 5 subchannels. The new improved Tree Structure using Daubechies Wavelet-1 (db-1), Daubechies Wavelet-4(db-4), Daubechies Wavelet-10 (db-10), Daubechies Wavelet-2 (db-2) and Daubechies Wavelet-8 (db-8) has been designed in Fig. 3, Fig. 4 and Fig 5 respectively.

![Fig. 3. Improved Tree Structure for 5 Subchannels using db-1(Haar)](image)

![Fig. 4. Improved Tree Structure for 5 Subchannels using db-4 and db-10](image)

![Fig. 5. Improved Tree Structure for 5 subchannels using db-2 and db-8](image)

4. Numerical and simulation results

In this paper, PAPR has been computed with Daubechies-1(db-1), Daubechies-2(db-2), Daubechies-4(db-4), Daubechies-8(db-8) and Daubechies-10 (db-10) for Wavelet Packet based OFDM using 5 sub-channels with QPSK modulation and compared with Mallat Tree Structure in Fig.6, Fig. 7, Fig. 8, Fig. 9 and Fig. 10 respectively. Due to search complexity, we have taken small number of sub-channels. However it is possible to implement it by large number of sub-channels.

![Fig. 6. Comparison of PAPR of Mallat tree and New Improved technique using Haar Wave Packet (db-1)](image)

![Fig. 7. Comparison of PAPR of Mallat tree and New Improved technique using db-2 Wavelet Packet](image)

![Fig. 8. Comparison of PAPR of Mallat tree and New Improved technique using db-4 Wavelet Packet](image)
Wavelet Packet Tree Structure and its comparison with Mallat Tree Structure using db-1, db-2, db-4, db-8 and db-10. It is observed that with new Wavelet Packet Tree Structure Technique, PAPR is 7.53 dB and has an improvement of 0.74 dB over Mallat Technique for Haar Wavelet (db-1) as shown in Tab. 1. Also with new Wavelet Packet Tree Structure Technique, BER is $2.579 \times 10^{-4}$ and has an improvement of $7.37 \times 10^{-5}$ over Mallat Technique for Haar Wavelet as shown in Tab. 2.

Table 1. PAPR for different Daubechies Wavelet using Mallat and Improved Technique

<table>
<thead>
<tr>
<th>Daubechies wavelet used</th>
<th>Mallat Technique PAPR (dB)</th>
<th>Improved Technique PAPR (dB)</th>
<th>Improvement of PAPR with new technique over Mallat Technique (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>db-1</td>
<td>8.27</td>
<td>7.53</td>
<td>0.74</td>
</tr>
<tr>
<td>db-2</td>
<td>12.68</td>
<td>9.92</td>
<td>2.76</td>
</tr>
<tr>
<td>db-4</td>
<td>11.63</td>
<td>10.32</td>
<td>1.31</td>
</tr>
<tr>
<td>db-8</td>
<td>12.55</td>
<td>10.84</td>
<td>1.71</td>
</tr>
<tr>
<td>db-10</td>
<td>12.41</td>
<td>11.31</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Table 2. BER for different Daubechies Wavelet using Mallat and Improved Techniques

<table>
<thead>
<tr>
<th>Technique used</th>
<th>Bit Error Rate (BER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mallat with db-1</td>
<td>0.0003316</td>
</tr>
<tr>
<td>Improved db-1</td>
<td>0.0002579</td>
</tr>
<tr>
<td>Improved db-2</td>
<td>0.02275</td>
</tr>
<tr>
<td>Improved db-4</td>
<td>0.01849</td>
</tr>
<tr>
<td>Improved db-8</td>
<td>0.003227</td>
</tr>
<tr>
<td>Improved db-10</td>
<td>0.02051</td>
</tr>
</tbody>
</table>

5. Conclusion

A new Cost Function based Algorithm presented here searches the 5-subchannels for different Daubechies wavelet family. Results show that PAPR is reduced as compared to Mallat tree structure. Haar Wavelet (db-1) gives the best PAPR reduction as compared to others. BER of Mallat is also in close agreement with new tree structure using Haar wavelet.

Acknowledgement

We extend our sincere thanks to anonymous reviewer for their suggestions and valuable comments.

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References