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Modified Modal Strain Energy Method for Analyzing the Dynamic Damping Behavior of Constrained Viscoelastic Structures

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Abstract

Modal strain energy (MSE) method is an efficient approximation approach for kinetics parameter calculation of constrained viscoelastic structures. MSE fails to determine the precise dynamic behavior of viscoelastic structures when the stiffness matrix is a complex one. To address this issue, this study proposes a new modified MSE method to calculate the loss factor and natural frequency of a constrained viscoelastic structure on the basis of the correlation analysis of the current modal strain method. The modifying factor changed with the amplitude of the modal order loss factor. A prototype system with four parameters, which was equivalent to a viscoelastic sandwich beam or plate, was used to analyze the error between the new modified method and current methods. The proposed method was applied to a viscoelastic suspension. Results show that the proposed method obtains minimum relative errors of 1.2% and 2.3% for structural loss factor and natural frequency as compared with existing methods. This study provides a certain reference for the performance analysis, structural design, and improvement of constrained viscoelastic structures.

Keywords: Constrained viscoelastic structure, modal strain energy method, loss factor, natural frequency

1. Introduction

Damping reduction has been widely used as a vibration control technology in various fields, such as military, agriculture, aviation, and aerospace. This technology fully utilizes the principle of damping dissipation energy and enhances the kinetics stability of the mechanical system [1]. A constrained viscoelastic structure is composed of lower and upper layers made of elastic material (such as steel and aluminum) with high strength and the middle layer made of viscoelastic material (such as rubber and plastic). This structure can increase loss factor and avoid strength–stiffness loss, thereby leading to efficient vibration damping effect and load function [2].

The vibration damping research on viscoelastic structures involves damping material preparation and damping parameter test kinetics modeling of the composite structure. Kinetics modeling, including loss factor and nature frequency, is a key issue [3]. In recent years, many researchers have investigated the kinetics parameters of damping structures [4-8]. However, viscoelastic dynamic performance is difficult to precisely predict owing to the complex stiffness matrix of viscoelastic structures.

Modal strain energy (MSE) method has been widely applied in many engineering fields to address the above-mentioned problem. In this study, a modified MSE method based on previous modifying strategies and basic MSE is proposed by involving a changing factor that varies with loss factor to precisely evaluate viscoelastic dynamic performance.

Based on the analysis above, a problem using the modified MSE method is examined. The proposed method is applied to an engineering case to validate its accuracy.

2. State of the art

MSE is widely used in viscoelastic engineering because of its simple form, relative precision, and less calculation cost compared with traditional methods [9, 10]. The structural loss factor in MSE can be obtained through the ratio of material loss factor to strain energy. Subsequently, complex stiffness is acquired without calculating complex eigenvalues. However, the calculation error in MSE increases with the increase in viscous component, thereby negatively affecting the structural loss factor [10].

Recently, considerable literature reports the development of various related methods, from theoretical analysis to finite element method (FEM) verification, several reliable conclusions on damping loss factor and natural frequency have been provided [11]. Wang [12] adopted a complex eigenvalue method to solve the motion equation with multiple eigenvalues and eigenvectors. This method is extremely complicated and lacks the characteristics of kinetics stress and strain needed for a constrained viscoelastic structure. Jaber [13] first proposed the theoretical method for solving natural frequency and loss factor of complex laminated beams under various constrain conditions. This method employs five hypotheses that depend on the application objects. Alfouneh et al. [14-16] established the super element method for complex laminated

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beams and used complex modulus to describe viscoelastic layer. FEM has also been adopted to calculate natural frequency and modal loss factor. Copetti [17-19] utilized the advantages of FEM and system perturbation method to solve the damping characteristic of viscoelastic structures through ASKA software. Ren et al. [20] used complex modulus to describe the frequency correlation of a laminated viscoelastic material and proposed an iterative method of MSE and eigenvalue to solve the dynamic parameters of viscoelastic laminated structures.

The aforementioned proposed methods present difficulty in dealing with large viscoelastic structures associated with obvious errors. The precision of MSE has been improved using modifying strategies to meet the engineering requirement [21, 22]. Lv et al. [23] used absolute value method of strain energy (AVMSE) to analyze the damping characteristics of structures. In this method, the modulus of viscoelastic material is replaced by the absolute modulus. Structural loss factor is accurately approximated when large viscoelastic proportion in the complex structure is considered. Reference [24] improved AVMSE and proposed a new MSE method (RMSE). This method needs the strain energy of viscoelastic material to multiply a corrected factor. The comparison showed that RMSE improves accuracy of prediction for the loss factor of viscoelastic material.

Although AVMSE and RMSE are modified MSE methods, their modal correction factors of different orders for viscoelastic material are the same and may cause excessive or insufficient corrections [25]. Considering that different order modals possess significantly different loss factors for viscoelastic material, this study introduces a new correction factor that changes with the loss factor amplitude and proposes a new method called ACMSE.

The remainder of the study is organized as follows. Section 3 proposes ACMSE by analyzing the inner connection between RMSE and AVMSE. Subsequently, modal loss factor and natural frequency are derived using ACMSE. Section 4 analyzes the error distribution on a four-parameter viscoelastic model to validate the effectiveness of ACMSE. Section 5 conducts an engineering application of ACMSE on a viscoelastic suspension to validate the feasibility of ACMSE. Section 6 elaborates the conclusions of the study.

3. Methodology

3.1 Constrained viscoelastic structure

A complex stiffness matrix is applied on the constrained viscoelastic structure shown in Fig. 1; the motion equation is [25]:

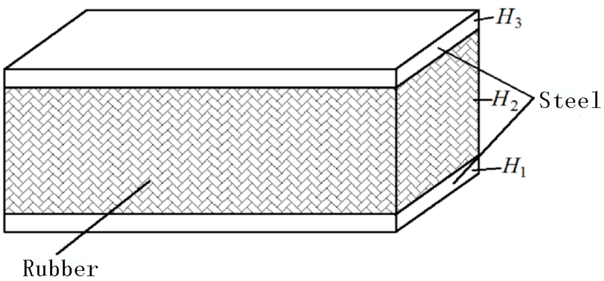


Fig. 1. Constrained viscoelastic structure

$$M\ddot{X} + (K_R + jK_{VI})X = 0 \tag{1}$$

$$K_R = K_{eR} + K_{VR} \tag{2}$$

where K_R and K_{VI} are the real and image parts of the complex stiffness matrix, respectively, K_{eR} is the stiffness matrix outside the viscoelastic material, K_{VR} is the stiffness matrix's real part of the viscoelastic material, M is the mass matrix, X is the displacement vector, j is the imaginary unit.

If the constrained viscoelastic structure only contains a type of viscoelastic material and the loss factor of viscoelastic material is β_V, then the following relation can be yield:

$$K_{VI} = \beta_V K_{VR} \tag{3}$$

The solution form of Eq. (1) is:

$$X = \Phi_r^* e^{j\omega_r^* t} \tag{4}$$

Where Φ_r^* and ω_r^* are the complex eigenvector and complex circle frequency of the i-th order, respectively.

According to the analysis of Rao [8]:

$$\omega_r^{*2} = \omega_r^2 + j\eta_r\omega_r^2 \tag{5}$$

Where ω_r is the real part of ω_r^*, η_r is the modal loss factor of the i-th order.

When the Rayleigh quotient and complex circle frequency are calculated, the following equation is obtained [8]:

$$\omega_r^{*T} = \frac{\Phi_r^{*T}(K_{eR} + K_{VR} + jK_{VI})\Phi_r^*}{\Phi_r^{*T} M \Phi_r^*} \tag{6}$$

3.2 Brief recall of MSE methods

3.2.1 MSE

In MSE, the real eigenvector Φ_r approximately replaces the complex eigenvector Φ_r^*, the approximate complex circle frequency $\omega_{r\ MSE}^{*2}$ of the i-th order is:

$$\omega_{r\ MSE}^{*2} = \frac{\Phi_r^T(K_{eR} + K_{VR})\Phi_r}{\Phi_r^T M \Phi_r} + j\frac{\Phi_r^T \beta_V K_{VR} \Phi_r}{\Phi_r^T M \Phi_r} \tag{7}$$

When the real parts of Eqs. (5) and (7) are compared and introduced into Eqs. (2) and (3), the approximate modal loss factor of the i-th order is [25]:

$$\eta_{rMSE} = \beta_V \frac{\Phi_r^T K_{VR} \Phi_r}{\Phi_r^T (K_{VR} + K_{eR})\Phi_r} \tag{8}$$

3.2.2 AVMSE

In AVMSE, the structural complex eigenvector is replaced by the eigenvector calculated by the real part K_R of structural stiffness matrix in Eq. (4) without considering the lag influence of viscoelasticity on structural mode shape. The error increases along with the increase of the image stiffness in the structural matrix. The influence of image stiffness is considered and modified by taking its absolute value in AVMSE. Real mode Φ_r is obtained by modified stiffness matrix K_α and approximately replaces composite structure mode Φ_r^* [26].

$$K_\alpha = \sum_{j=1}^{NE} K_{R_j}\sqrt{1+\beta_j^2} = K_{eR} + \sum_{j=1}^{NVE} K_{VR_j}\sqrt{1+\beta_j^2} \tag{9}$$

where NE is the element number, β_j is the material loss factor of jth element, NVE is the viscoelastic element number, K_{R_j} is the stiffness matrix of the j-th element, and K_{VR_j} is the real part of stiffness matrix of the j-th element.

The corresponding eigenvalue problem in this method is:

$$\left(-M\bar{\omega}_r + K_\alpha\right)\Phi_r = 0 \tag{10}$$

If Φ_r approximately replaces complex eigenvector Φ_r^* and only one type of damping material is present in the structure, then the approximate circle frequency of the r-th order in AVMSE is:

$$\begin{aligned}\omega_{r\ AVMSE}^{*2} = & \frac{\Phi_r^T(K_{eR}+\sqrt{1+\beta_V^2}K_{VR})\Phi_r}{\Phi_r^T M\Phi_r} + \\ & j\frac{\Phi_r^T\beta_V\sqrt{1+\beta_V^2}K_{VR}\Phi_r}{\Phi_r^T M\Phi_r}\end{aligned} \tag{11}$$

The approximate mode loss factor $\eta_{r\,AVMSE}$ of the r-th order is:

$$\eta_{r\,AVMSE} = \beta_V\frac{\Phi_r^T\sqrt{1+\beta_V^2}K_{VR}\Phi_r}{\Phi_r^T(\sqrt{1+\beta_V^2}K_{VR}+K_{eR})\Phi_r} \tag{12}$$

3.2.3 RMSE

In AVMSE, the calculation of modal loss factor is modified by stiffness matrix and eigenvector and it may cause excessive modification. Rongong [24] proposed a modified method in which Eq. (12) is divided by a modifying factor $\sqrt{1+\beta_V^2}$ and the approximate mode loss factor $\eta_{r\,RMSE}$ of the r-th order is:

$$\eta_{r\,RMSE} = \beta_V\frac{\Phi_r^T K_{VR}\Phi_r}{\Phi_r^T(\sqrt{1+\beta_V^2}K_{VR}+K_{eR})\Phi_r} \tag{13}$$

The MSE of viscoelastic material cannot be obtained directly in the finite element analysis in accordance with Eq. (13). $\eta_{r\,RMSE}$ can be expressed as:

$$\begin{cases}\eta_{r\,RMSE} = \dfrac{1}{\dfrac{1}{\beta_V}+\alpha_{RMSE}(\dfrac{1}{\eta_{r\,AVMSE}}-\dfrac{1}{\beta_V})} \\ \alpha_{RMSE} = \sqrt{1+\beta_V^2}\end{cases} \tag{14}$$

where α_{RMSE} is the modifying factor.

Considering that frequency square is proportional to stored energy [12], the approximate circle frequency $\omega_{r\,RMSE}$ of the r-th order is:

$$\omega_{r\,RMSE} = \lambda_{r\,AVMSE}\sqrt{\frac{\Phi_r^T\left(K_{VR}+K_{eR}\right)\Phi_r}{\Phi_r^T(\sqrt{1+\beta_V^2}K_{VR}+K_{eR})\Phi_r}} \tag{15}$$

Eq. 15 can be rewritten as:

$$\omega_{r\,RMSE} = \lambda_{r\,AVMSE}\sqrt{\frac{\beta_V+\eta_{r\,AVMSE}\left(1/\alpha_{RMSE}-1\right)}{\beta_V}} \tag{16}$$

3.3 ACMSE

The modifying factors in AVMSE and RMSE are the same for the viscoelastic composite structure. However, different loss factors theoretically exist for different modal orders, thereby improving the calculation accuracy. Thus, a new modifying factor is proposed in this study on the basis of $\alpha_{r\,RMSE}$ in RMSE and $0 \le \eta_r \le \beta_V$.

$$\begin{cases}\alpha_{r\,ACMSE} = \sqrt{1+\beta_V^2-\left(\eta_{r\,RMSE}+\beta_V\right)^2/4} \\ \mu_r = 1-\dfrac{\eta_{r\,RMSE}}{\beta_V}\end{cases} \tag{17}$$

Accordingly, $0 \le \mu_r \le 1$.

The approximate modal loss factor $\eta_{r\,ACMSE}$ of the r-th order in ACMSE is:

$$\eta_{r\,ACMSE} = \frac{1}{\dfrac{1}{\beta_V}+\alpha_{r\,ACMSE}(\dfrac{1}{\eta_{r\,AVMSE}}-\dfrac{1}{\beta_V})} \tag{18}$$

Similar to the process in RMSE, the approximate circle frequency $\omega_{r\,ACMSE}$ of the r-th order in ACMSE is:

$$\omega_{r\,ACMSE} = \lambda_{r\,AVMSE}\sqrt{\frac{\beta_V+\eta_{r\,AVMSE}\left(1/\alpha_{r\,ACMSE}-1\right)}{\beta_V}} \tag{19}$$

4 Result analysis and discussion

4.1 Prototype system with four parameters

The errors of four methods, i.e. MSE, AVMSE, RMSE and ACMSE are analyzed using a prototype system with four parameters, which is equivalent to a viscoelastic sandwich beam (plate), to validate the rationality and accuracy of ACMSE.

Torvik et al. [26]. proposed a prototype system with four parameters (Fig. 2) to analyze MSE.

The complex stiffness of the system can be obtained by the viscoelastic-elastic principle as follows:

$$K^* = \frac{1}{1/k_1+1/\left(k_2+j\beta k_2\right)}+k_3 \tag{20}$$

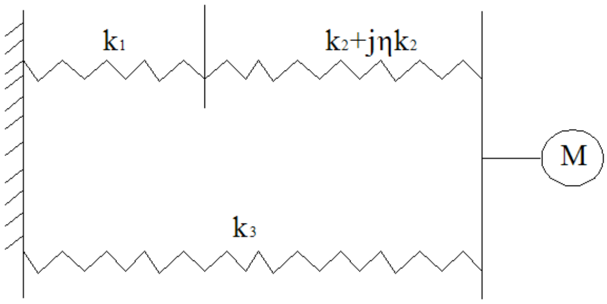


Fig. 2. Prototype system with four parameters

The complex stiffness of the system can be obtained by the viscoelastic-elastic principle as follows:

$$K^{*}=\frac{1}{1/k_1+1/\left(k_2+j\beta k_2\right)}+k_3 \tag{20}$$

System loss factor η_s can be obtained by the ratio of imaginary and real parts of system complex stiffness as follows:

$$\eta_s=\frac{\beta\cdot x}{x^2\left(y+1\right)\beta^2+x^2+yx^2+x+y+2xy} \tag{21}$$

Where $x=k_2/k_1$, $y=k_3/k_1$, and β is the loss factor of the complex stiffness spring.

From the system homogeneous solution, the natural frequency ω_s is obtained as:

$$\omega_s=\mathrm{Re}\left(\sqrt{K^{*}/M}\right) \tag{22}$$

Complex stiffness elasticity modulus is replaced by the real part in MSE and by the absolute value in AVSME. The modifying factor is improved in RMSE on the basis of AVMSE, and the structural loss factors and natural circle frequency of these MSE methods are obtained as follows:

$$\begin{cases}\eta_{MSE}=\dfrac{\beta\cdot x}{\left(y+xy+x\right)\left(1+x\right)}\\ \omega_{MSE}=\sqrt{\dfrac{z\left(y+xy+x\right)}{1+x}}\end{cases} \tag{23}$$

$$\begin{cases}\eta_{AVMSE}=\dfrac{\beta\cdot\alpha\cdot x}{\left(y+\alpha xy+\alpha x\right)\left(1+\alpha x\right)}\\ \omega_{AVMSE}=\sqrt{\dfrac{z\left(y+\alpha xy+\alpha x\right)}{1+\alpha x}}\end{cases} \tag{24}$$

$$\begin{cases}\eta_{RMSE}=\dfrac{1}{\dfrac{1}{\beta}+\alpha\left(\dfrac{1}{\eta_{AVMSE}}-\dfrac{1}{\beta}\right)}\\ \omega_{RMSE}=\omega_{AVMSE}\sqrt{\dfrac{\beta+\eta_{AVMSE}\left(1/\alpha-1\right)}{\beta}}\end{cases} \tag{25}$$

Where $z=k_1/M$ and $\alpha=\sqrt{1+\beta^2}$.

Similar to the modifying factor improvement functions in RMSE, modifying factor α' varies with the system efficient loss factor in ACMSE. The structural loss factor and natural circle frequency are inferred as:

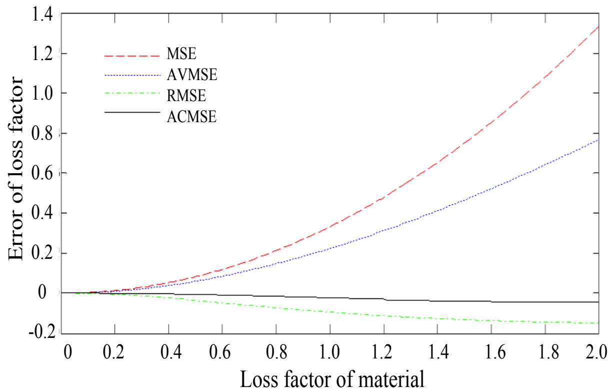
$$\begin{cases}\eta_{ACMSE}=\dfrac{1}{\dfrac{1}{\beta}+\alpha'\left(\dfrac{1}{\eta_{AVMSE}}-\dfrac{1}{\beta}\right)}\\ \omega_{ACMSE}=\omega_{AVMSE}\sqrt{\dfrac{\beta+\eta_{AVMSE}\left(1/\alpha'-1\right)}{\beta}}\end{cases} \tag{26}$$

where $\alpha'=\sqrt{1+\left(1-\eta_{RMSE}/\beta\right)\beta^2}$.

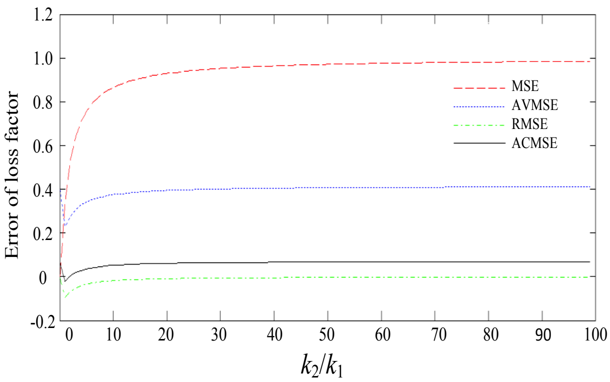
From the above-mentioned inference on structural loss factor η and natural frequency ω in these MSE methods, η is found related to stiffness ratio x, y and β. Natural frequency ω presents a function relationship with stiffness ratio x, y, complex spring loss factor β, and z, for convenience of analysis, z is assumed to be constant and negligible [25].

4.2 Error analysis for loss factor

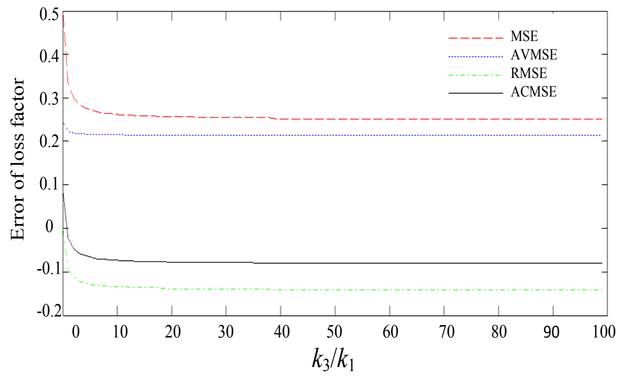
The four above-mentioned MSE errors are explored. x, y is in the range of $0.01–100$, indicating four-order change in stiffness ratio; β is in the range of 0–2, representing the loss factors of common viscoelastic material. The relative error curves of structural loss factor and natural frequency in MSE, AVMSE, RMSE, and ACMSE are shown in Figs. 3 and 4.



(a) $k_2/k_1=1, k_3/k_1=1, \beta\in\left[0,2\right]$



(b) $\beta=1, k_3/k_1=1, k_2/k_1\in\left[0.01,100\right]$



(c) $\beta = 1, k_2/k_1 = 1, k_3/k_1 \in [0.01,100]$

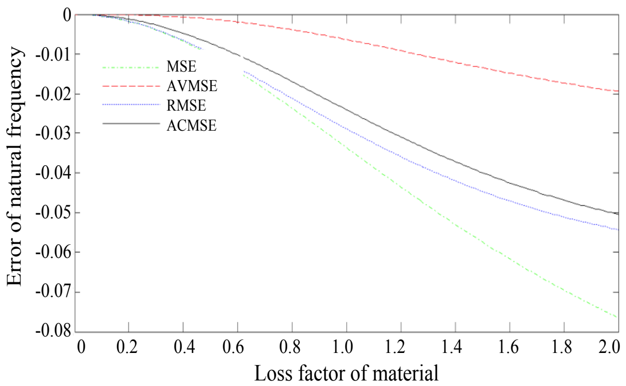
Fig. 3. Relative error calculation for various loss factors

The following conclusions can be obtained from the analysis above.

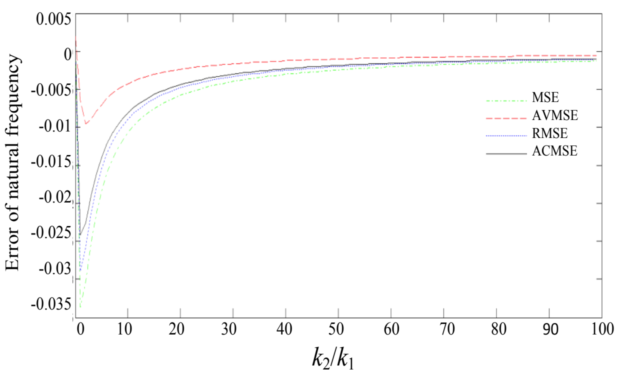
When stiffness ratio x , y is invariable, the errors of structural loss factor in the four methods increase with the increase in material loss factor. The error in MSE is particularly obvious. Thus, the method needs to be improved.

Compared with AVMSE and RMSE, ACMSE obtains minimum structural loss factor error with a lower bound.

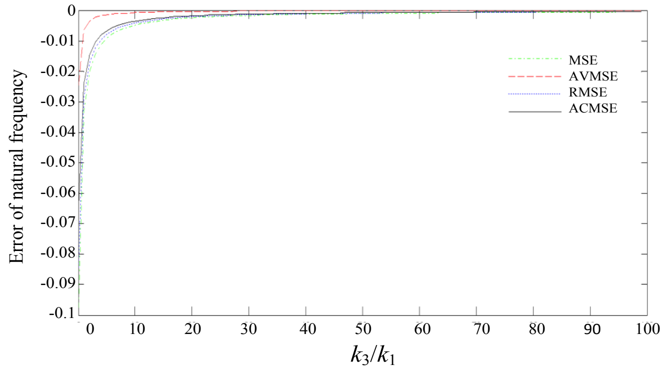
When β and stiffness ratio k_3/k_1 , k_2/k_1 are invariable, the structural loss factor error in ACMSE is the minimum. With the increase in stiffness ratio, the error stabilizes.



(a) $k_2/k_1 = 1, k_3/k_1 = 1, \beta \in [0,2]$



(b) $\beta = 1, k_3/k_1 = 1, k_2/k_1 \in [0.01,100]$



(c) $\beta = 1, k_2/k_1 = 1, k_3/k_1 \in [0.01,100]$

Fig. 4. Relative error calculation for natural frequency by various MSE

4.3 Error analysis for natural frequency

Fig. 4 shows the relative error calculation for natural frequency by various MSE methods.

From the relative error curves for natural frequency by various MSE methods, the following conclusions are drawn.

When stiffness ratio x , y is invariable, the errors of structural loss factor in the four methods increase with the increase in material loss factor. The error in MSE is the maximum and increases rapidly.

The natural frequency error in AVMSE is the minimum. The error in ACMSE is only slightly larger than that in AVMSE whereas smaller than the errors in MSE and RMSE.

When material loss factor β and stiffness ratio are invariable and k_2/k_1 is in a very small area, the errors of the four methods increase with k_2/k_1 . The error in ACMSE is larger than that in AVMSE only. With the increase in k_2/k_1 , the errors in these methods decrease and stabilize.

When material loss factor β and stiffness ratio are invariable, the errors in these methods decrease with the increase in k_3/k_1 . The error in AVMSE is the smallest, and that of ACMSE is smaller than the errors in MSE and RMSE.

With the increase in k_3/k_1 , the errors in the four methods are very close and stable in general.

In summary, the proposed ACMSE can more efficiently calculate structural loss factor and natural frequency among all the compared methods.

4.4 Example verification

A constrained viscoelastic structure is used to validate ACMSE. The structure is mounted on a crawler bulldozer to absorb vibration and shock. The structure uses damping technology and thus possesses good buffer damping performance [27].

Fig. 5. Picture of viscoelastic suspension

The loss factor and natural frequency of a viscoelastic suspension installed in a bulldozer with 410 horsepower (Fig. 5) are calculated using the four MSE methods. The precision of ACMSE compared with that of other MSE methods is validated using ANSYS software by FEM.

The mechanical behavior of steel and rubber is simulated by 3D element Solid 185. The steel and rubber materials are regarded as isotropic. The model and physical parameters used in the FEM analysis are shown in Tab. 1.

Table 1. Main geometric and physical parameters of viscoelastic suspension's rubber pad

Geometrical parameters of rubber	Parameter	Structural layer	Density ρ	Elasticity modulus, shear	Poisson's
base diameter D /mm	305				
Total height h_t /mm	45				
ceiling height h_u /mm	10.4	Restraint Layer	7800	2.100 × 1011	0.300
central height h_m /mm	30.6	Damping Layer	1130	0.896 × 106	0.499
Bottom height h_b /mm	4.0	Basement Layer	7800	2.100 × 1011	0.300
Central support angle θ /(°)	55				
Total radius of circular arct R /mm	950				

The loss factor entity model of the damping structure in Fig. 5, FEM, stresses, and strain image under 3 Hz of a typical working condition are shown in Figs. 6, 7, 8, and 9, respectively.

The results of loss factor and natural frequency of the damping structure analyzed by ANSYS using ACMSE and three other MSE methods are shown in Table 2.

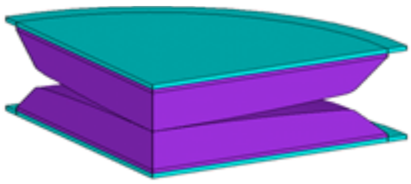


Fig. 6. One-fourth symmetric 3D model of viscoelastic suspension

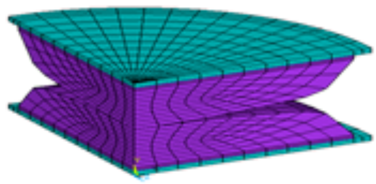


Fig. 7. One-fourth symmetric 3D FEM model of viscoelastic suspension

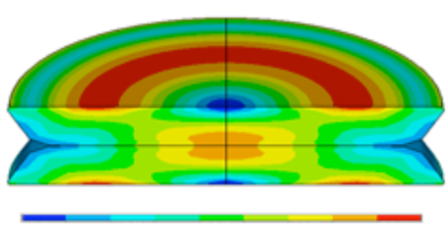


Fig. 8. Stress contour picture for viscoelastic damping structure at 3 Hz

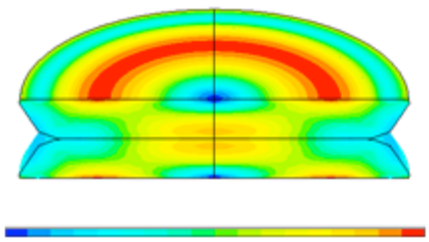


Fig. 9. Strain contour picture for viscoelastic damping structure at 3 Hz

Table 2. Loss factor and natural frequency calculation of MSE methods

Modal	MSE result		AVMSE result		RMSE result		ACMSE result		ANSYS result	
	Loss factor	Natural	Loss factor	Natural	Loss factor	Natural	Loss factor	Natural	Loss factor	Natural
1	0.198	267.35	0.274	283.25	0.275	282.21	0.296	283.15	0.298	285.35
2	0.203	267.83	0.269	283.69	0.279	282.53	0.306	283.35	0.314	285.83
3	0.221	271.96	0.280	284.36	0.281	282.94	0.324	284.06	0.325	286.46
4	0.216	273.52	0.291	284.68	0.289	283.31	0.318	284.18	0.319	286.52
5	0.231	275.76	0.289	285.26	0.294	283.25	0.329	285.23	0.333	291.96

The loss factor error in ACMSE is the smallest compared with the errors in ACMSE and MSE. The natural frequency errors in the three other MSE methods are smaller than the error in MSE, and the error in ACMSE is only smaller than that in AVMSE. This conclusion is consistent with the further error analysis and proves the rationality and accuracy of ACMSE. Therefore, the proposed method can meet the requirement of a few engineering designs.

5 Conclusions

To evaluate the dynamic behavior of viscoelastic structure more precisely, this study analyzed the characteristics and relations of MSE, AVMSE, RMSE, and proposed an ACMSE with loss factor varies. The following conclusions could be drawn..

(1) The modifying factor changes with the corresponding modal loss factor; thus, the calculation accuracy of ACMSE for viscoelastic damping features is better than the accuracies of MSE, AVMSE, and RMSE.

(2) The error analysis shows that the natural frequency and modal loss factor errors are 1.2% and 2.3%, respectively, for ACMSE. This finding also proves the accuracy of ACMSE.

(3) ACMSE can be applied not only to the kinetics analysis of viscoelastic constrained damping structure with two elastic layers but also to complex constrained damping structures with multiple layers.

This research proposed a new MSE method namely ACMSE for analyzing dynamic response of multiple-layer constrained viscoealastic damping structure. On the other hand, it provides a certain reference for the performance analysis, structural design, and improvement of constrained viscoelastic structures. The material loss factor presents a certain non-linear variation for large strain condition. Therefore, the future study will comprehensively explore the application of MSE into the non-linear kinetics parameter analysis of damping structures with multiple constraints.

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