Bifurcation and Chaos in Converter Interfaces in Solar PV Systems-A Review

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Abstract

The paper attempts to present a detailed study of the nonlinear phenomena that includes bifurcations, sub-harmonics and chaos in an effort to arrive at the appropriate choice of converter interface suitable for specific PV systems and evolve criterions for their design and control. The resurgence of switching dc-dc converters as power processing units in renewable energy applications appear to open up new perceptions for harnessing energy efficiency. The focus orient towards review the modelling techniques used for nonlinear analysis in the different operating modes of the converter and identity the gap in the related domain with special relevance to the converter interfaces in solar PV systems. The exercise incites to explain the forms of bifurcations, the routes to chaos and the associated control techniques for arbitrating its significance in this perspective. The benefits of analysis of nonlinear phenomena in regulatory applications claim a new space in energy processing applications and forge to explore their role as maximum power point (MPP) trackers. The emphasis endeavours to enhance the scope of use of solar energy and offer a fresh dimension to support the clean energy act and perpetuate the desire to enliven a greener world.

Keywords: Bifurcations, Chaos, Sub-harmonics, Nonlinearity, DC-DC converter, Solar PV

1. Introduction

The global energy-induced CO₂ emissions continue to increase from around 35.6 billion metric tons in 2020 to 43.2 billion metric tons in 2040 [1]. The challenge envisages higher penetrations of renewable energy in an effort to preserve the ecosystems and gather a biodiversity with higher environmental and socio-economic benefits in the process of transitioning to a Green Economy [2]. The path foresees a co-ordinated growth of renewable energy and augurs the demands of the national economy through effective industrial restructuring and formulation or adaption of appropriate policies [3].

The nation appears to experience a massive growth especially in solar energy capacity from 5GW in 2006 to 5.167GW in 2015[4]. The solar policy initiatives that include accelerated depreciation, generation based incentives, tax holidays, net metering, diverse feed-in tariffs and emphasis on manufacturing and R&D in solar PV components forge to perpetuate the generation of solar PV [5]. The technological upgradation in PV industry augurs a fresh perspective to comply the commercial availability of the solar distributed power generation for a sustainable market growth [6].

The lower conversion efficiency of PV remains as a main barrier against successful implementation of the solar projects. The commercialized silicon solar cells offer an average efficiency ranging from 14% to 17%. However solar energy can be directly converted into direct current (dc) electricity through photovoltaic (PV) array, in which case it becomes important to match the array voltage corresponding to the maximum dc power produced by PV panels to that of the load under real-world operating conditions [7-8].

The switching power converters that form part of an integral solar PV system turn out to be the prime source of nonlinearity, nonlinear components such as diodes and feedback controls adds to the richness of nonlinearity. It assumes significance to investigate the nonlinear phenomena including bifurcations, coexisting attractors and chaos in order to design reliable power electronic interfaces [9].

The maximum power point tracker (MPPT) relies on moving the operating PV voltage or current to extract the maximum power. The tracker usually requires a high efficiency power processing circuits called dc-dc converter to sweep the PV power [10]. The type of converter and its parameters sizing significantly affects the optimum operation of the PV systems. A judicious choice of the converter in accordance with the requirements of the load allows the MPPT to extract the maximum converter efficiency [11].

The current-voltage (I-V) properties of PV array exhibit a nonlinear characteristic that varies under environmental conditions create a phenomenon called chaos. It incites to be a field of emerging cross-disciplinary science of nonlinear dynamics that inherit to be highly sensitive to initial conditions [12]. A large variety of strange behaviours emanate in nonlinear systems such as sub-harmonics, quasi-periodic oscillation, intermittency, and apparently random motion [13].

Besides the converter composes of a set of reactive components and switching devices sandwiched to deliver the energy proficiently from the source to the load when driven, controlled and modulated by appropriate techniques. It brings in nonlinear behaviours and on its part serve to introduce chaos. Chaos research opens up the possibility of the application of nonlinear characteristics in power...
electronic systems and might require fresh approaches to improve the performance of the solar PV systems [14].

Thus the different dynamic behaviours exhibited both by the PV array and the switching power converter invites a detailed study of the available stability models to predict the boundary between such dynamic behaviours. The entire system might exhibit what relates to as bifurcation phenomena under some supply and load conditions. The suppression of chaos and sub harmonic oscillations for a wide range of input voltage and loads thus form to be a prime design constraint in the PV system control.

2. Modelling Techniques for Nonlinear Analysis of DC-DC Converter

The two major traditional approaches to model the switching dc-dc converters rely on small signal analysis based on linearization and state space averaging using PWM switch model. The inability to predict the fast scale dynamics by conventional approaches follows it up with sampled data modelling technique which become naturally suitable for power converters owing to its recurrent way operation and control [15].

The stacked solution to each of the state equation for each circuit configuration over one switching cycle provides a large signal discrete-time nonlinear difference equation showing the total increment acquired during a cycle. The approximation by piecewise linear segments gives an approximated large signal discrete time model with reduced complexity in analysing the nonlinear behaviour while retaining the fundamental nonlinear properties of the system [16].

The discrete-time maps find use in the boost converters to represent the assignment rule, which assigns the actual state vector (inductor current and capacitor voltage) to the next state [17]. The derivation of an iterative map with the general form \( x_{n+1} = f(x_n) \) produces maps with dimensions higher or equal than two which makes the analysis difficult.

The output voltage essentially remains constant due to the presence of the capacitor in the buck converter particularly with sufficiently smaller clock periods compared to the time constant. The system reduces to the 1-D form and the inductor current waveform becomes piecewise linear, allowing simplicity in modelling [18].

A new sampled-data model for the current-mode controlled buck converter including the effects of delay in the pulse-width-modulator (PWM) determines the correct method of measuring the current loop gain [19]. The poincare map obtained by sampling the system trajectory at the rate of switching frequency enables to study the stability of the dual channel resonant buck converter systems by analysing the local behaviour of the map near periodic state denoted by a fixed point in it [20].

The Filippov’s method of differential inclusions, in the absence of a nonlinear map obtains the linearization around a periodic orbit for a voltage-mode-controlled buck converter. A monodromy matrix composed of the state transition matrices for the pieces of the orbit that lie in the individual subsystems and the saltation matrix that connects the perturbation just before a switching to that just after [21].

The literature reports the exhaustive scope of the Fourier series expansion of the steady-state feedback signal, time domain analysis of steady-state feedback signal and the introduction of auxiliary state vector for the process of modelling the converters [22-24]. A summary of contributions of modelling techniques of the nonlinearities in various DC-DC converter with remarkable elucidation of consequences in the study of stability of such systems are reported in Tab.1.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Ref. No.</th>
<th>Converter analysed</th>
<th>Modelling technique</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>Voltage fed series resonant converter</td>
<td>Sampled data dynamic model</td>
<td>Described both large signal dynamics and the dynamics of small perturbations about cyclic steady state operation. Model used to design state feedback and periodic output feedback control system for the converter.</td>
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<td>2</td>
<td>16</td>
<td>Voltage mode PWM buck converter</td>
<td>Iterated nonlinear mappings</td>
<td>Predicted the behaviour of real converters such as multiple pulsing, skipped cycles, sub-harmonics and chaos outside the province of conventional linearized modelling. Involved complexity like tedious analytical derivation and numerical computation</td>
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<td>3</td>
<td>17</td>
<td>Current controlled SEPIC and boost converter in CCM</td>
<td>Discrete time model built by stroboscopic mapping</td>
<td>Stroboscopic map obtained by uniform sampling of the system state at the switching frequency. Model used to investigate the nonlinearities of converter through characteristic multipliers and computation of Lyapunov exponent</td>
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<tr>
<td>4</td>
<td>18</td>
<td>Current controlled buck converter</td>
<td>One dimensional mapping</td>
<td>Model developed for ideal switching, missed switching caused by delay and missed switching caused by switching transient. Changes seen in bifurcation diagram developed from model of ideal conditions and model with switching delay and transients.</td>
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<tr>
<td>5</td>
<td>19</td>
<td>Peak Current controlled Boost converter</td>
<td>One dimensional discrete time map</td>
<td>Model obtained from inductor current waveform and the solution of ODE describing boost converter with duty cycle as state variable. Model allowed for studying the nonlinearity of converter with any of its parameter as a bifurcation parameter. Model used to confirm period doubling, intermittency and chaotic behaviour.</td>
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<td>6</td>
<td>20</td>
<td>Dual channel resonant buck converter in</td>
<td>Poincare map function</td>
<td>Model used to test the stability of voltage feedback control loop by the Eigenvalues of Jacobian matrix of the Poincare map. Model identified Hopf bifurcation with a large increase in</td>
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3. Types of Bifurcations in DC-DC Converters

The bifurcation diagram constitutes to be a powerful tool to investigate the nonlinear phenomena, where in the periodic steady state of the system exists either as a signal point or several points equal to the periodicity of the system for a fixed parameter. It necessitates plotting numerous points in the diagram because chaos means period infinity and ensures that the points do not fall at the same position [25].

The saddle node bifurcation shown in Fig.1 occurs at a critical parameter value where on one side of it, there occurs multiple solutions (attractors) and on the other side of it, no solution exists. With the voltage loop closed and a large feedback gain in the voltage loop, the converter does not result in saddle node as in Fig.1 [26]. It derives the critical conditions of bifurcations of the valley current mode control in the boost converter at light loading [27]. The saddle node bifurcation occurs in the boost converter if it exercises a control of the output voltage. However it does not occur if the system does not consider the parasitic inductor resistance [28].

![Fig. 1. Occurrence of Saddle node bifurcation for converter operation as in [26]](image)

A stable periodic orbit may bifurcate to a stable torus at a certain point of the parameter space as in Fig. 2. It refers to a kind of bifurcation is called the Neimarck–Sacker bifurcation, or the Hopf bifurcation of the associated discrete-time dynamics and occurs on the period of the free oscillations of the RLC circuit (the natural period) being larger than the pulse width modulation (PWM) period. The behaviour of a closed loop boost converter, with a variation of the PWM period exhibits that Hopf bifurcation occurs at a certain value of the parameters [29].

![Fig. 2. Occurrence of Hopf bifurcation through torus breakdown as in [29]](image)

A fourth-order current-controlled Cuk converter operates chaotically even in free-running (autonomous) mode and the computer simulations of the circuits reveal the bifurcation from the stable equilibrium state (fixed point), through limit cycles and quasi-periodic orbits, and eventually to chaos [30].

A continuous time model explains the bifurcations in current controlled Luo topology operating in the continuous conduction mode. The dynamics obtained by varying the
load and reference voltage, with the remaining parameters constant show a fundamental operation initially and eventually to enter into the chaotic regime through a quasi periodic operation [31].

A condition for period doubling bifurcation expressed in terms of the solvability of a pair of algebraic equations requires the design of a feed forward control scheme used to adjust the compensating ramp and stabilizing the buck converter [32].The evolution process of period-doubling bifurcation from period-1 to period-2, then to period-4 appears in a voltage mode controlled fly-back converter operating in discontinuous conduction mode (DCM) with feedback gain k as bifurcation parameter [33].

The averaged model of a boost converter functioning in the continuous current mode (CCM) with slope compensation as bifurcation parameter allows estimating the period-doubling bifurcation point and the diagram as seen from Fig.3. The corresponding values of the slope compensation at the loss of the asymptotic stability of the proposed average model leading to creation of a 2T periodic orbit turns out to be consistent with those at the onset of the first period-doubling bifurcation obtained by simulations [34].

![Fig. 3. Occurrence of period doubling bifurcation as in [34]](image)

The Table 2 summarizes the types of bifurcations seen in various dc-dc converter and their operation and control insights.

<table>
<thead>
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<th>Table 2. Bifurcation types observed in DC-DC converter operation</th>
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4. Study of Mechanism of Bifurcations and Transition to Chaos

The study of mechanism relating to the loss of stability of period-1 orbit and transition to chaos may provide sufficient understanding in the design of converters that are reliable and predictable in their performance when operating in unstable modes or even chaotically. It can also help in designing control methods for sustaining the stability of the systems. The thoughtfulness of the routes to chaos in converter systems allows engineering the systems with a higher degree of confidence as stable systems through quasi periodic route to chaos, period doubling route chaos and intermittency route to chaos.

The investigations reveal the quasi periodic route to chaos in boost, buck-boost and load resonant converters under various operating conditions [35] – [37] and Fig.4 illustrates that of a boost converter. The dynamics of output voltage of boost converter exhibited period doubling route to chaos by increasing the feedback control gain of proportional controller in [38]. The numerical and experimental results of period doubling route to chaos in voltage controlled buck converter in CCM with input voltage as bifurcation parameter reported in [39] illustrated through phase portraits in Fig.5. [40] show the period doubling route to chaos through period-1,2,4 and 8 and finally exhibits the chaotic attractor.

The intermittency route to chaos occurs in switching power supplies, not protected against the intrusion of spurious signals or in the presence of parasitic inductances and capacitances [41]-[43] for single and interleaved buck converter with the addition of the intrusion of interference into the input and reference voltages. The Fig.6 exhibits the intermittency route to chaos as the strength of the intrusion signal increases in the inductor current waveform.

5. Control of Chaos

The idea of control over chaos represents a fundamental break from the previous approaches since, while the system remains in a chaotic state, it stabilizes on one of the unstable periodic orbits. Through small perturbations, the unpredictable (undesirable) dynamics transforms into predictable (desirable) behaviour without major changes to the functioning conditions [44].

A host of control methods to control chaos in various dc-dc converters namely OGY method, Nonlinear time delayed feedback controller, Control of triangular wave used in PWM, sliding mode control, resonant parametric perturbation, adaptive ramp technique and washout filter find spread use [45]-[50]. The phenomenon of bifurcation, sub-harmonics and chaos emanates in dc-dc converters used in energy applications employing PV panels [51]-[54]. The criteria of occurrence of such nonlinearities and its consequences on the performance of the overall PV system operation urge attention and the methods to govern the incidence of nonlinearity and control its effects on system process with a vision of achieving desired energy efficiency poses challenges.
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7. Criteria for Topology Selection for PV System

The topology study in [55] reports that the boost converter to be advantageous in PV MPPT applications compared to buck converters in terms of continuous input current, dynamic response and cost. It allows elimination of high side driver for the switch and additional blocking diode as advantages of boost converter over the buck counterpart. The buck–boost converters offer to be a suitable topology so as to provide perfect MPPT since there exists no dependencies on loading or environmental conditions. In spite of its voltage flexibility the buck–boost converter turns out to be always efficiency, or price disadvantaged [56] and [57].

In order to increase the voltage flexibility, the scenario of selection of converter topologies experience a transition to cuk, SEPIC and Luo converters, suitable for standalone systems. The cuk converter shows an inverted output and comparatively higher diode power loss than the SEPIC. The SEPIC converter exhibits better dynamics of MPPT and found more stable at MPP than Cuk converter. A significant step up in the voltage required as in 340-V minimum dc bus voltage for 240-V transformer-less ac grid connection can be achieved by positive output super-lift Luo converter which operates in first quadrant with large voltage amplification and high voltage transfer gain. It enjoys a higher power density and efficiency at higher power ratings [58] – [60].

8. Conclusion

The dc–dc converters have been designed conventionally to work under a constant voltage source and therefore their behaviours deem not to comply the desired requirements when connected to a variable current source like a photovoltaic panel. The operating conditions of dc–dc converters in such applications has been seen to be widely different than in the aforementioned studies and hence demands a great need for further research. The boost converter has been on focus for low rated PV MPPT applications and demands perception of nonlinear analysis and control of chaos to efficiently transfer the available power from panel to load.

The SEPIC converter has been slated for such medium rated PV with battery charging applications and augurs a study for nonlinearities and its control in various modes. The positive-output super-lift Luo, owing to its high efficiency and low switching loss has been seen to suit high power industrial applications where large voltage boost becomes a necessity from the available low-voltage sources. The survey has been forayed to suggest that boost, SEPIC and positive output super-lift Luo converter as potential topologies to be used in solar PV MPPT applications. The detailed analysis of their dynamical behaviour and its influence on the overall PV system performance will provide useful guidelines for choice and design of converters and deeper insights in devising control strategies for various modes that will ensure the reliability and predictability of the complete system.

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[1] https://www.eia.gov/forecasts/ieo/emissions.cfm


