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Towards the Number of Errors that the Code Detects for Sure

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Abstract

In this paper we consider an error-detecting code with a fixed length of redundancy. The analysed code is defined using the algebraic structure quasigroup. We consider the case when a linear quasigroups of order 16 is used for coding. Using simulations, we obtain the number of errors that the code surely detects when the length of the redundancy is 8, 12 and 16 bits. At the end, we summarize the results and make some conclusions.

Keywords: Error-detecting code; Linear quasigroup; Coded block; Binary-symmetric channel.

1. Introduction

Nowadays when the transmission and storage of data is greater than ever, it is of utmost importance to ensure accurate transmission and data storage. When a message is transmitted through the channel, under the influence of the noises in the cannel, it can be incorrectly transmitted. For that reason, control of errors is of exceptional importance. The codes for error control (error-detecting and error-correcting codes) play a major role in the communication systems, allowing digital data to be protected from errors. They are used in satellite, network and mobile communication and in any other form of digital data transmission. But, they are also used in the magnetic and optical storage devices.

In our previous work, we have defined and analyzed several error-detecting codes based on the algebraic structure quasigroup ([1], [2], [3], [4], [5]). These codes have rate ½ which means that an input block of length n is extended into a block of length $2n$, i.e., the number of redundant characters is equal to the number of information characters. But, for practical implementations of particular interest are the codes with a fixed length of redundancy, i.e., codes that always add fixed number of redundant characters regardless of the length of the input block. For that reason, in [6] we have defined such a code, which is the subject of this paper.

For every error-detecting code it is important to know its ability to detect errors. For that reason, two parameters are important for every error-detecting code. The first one is the probability of undetected errors, which is the probability that there will be errors in transmission that the code will not detect. The second parameter is the number of errors that the code surely detects, which is the maximum number of incorrectly transmitted bits up to which the code will detect the errors for sure. In our previous research ([6] and [7]), we have investigated the defined code from the aspect of the probability of undetected errors. In order to have complete analysis of the code, in this paper we will be focused on the number of errors that the code detects for sure. In [7] we investigated the cases when a linear quasigroup of order 4, order 8 and order 16 is used for coding. Since the quasigroup of order 16 gives the smallest probability of undetected errors, in this paper we will investigate the number of errors that the code detects for sure when this quasigroup is used for coding.

2. The fixed length redundancy code

2.1 Definition of the code

The code that we analyze in the paper is defined using quasigroups.

Definition 1: Quasigroup is algebraic structure $(Q,*)$ such that

$$(\forall\, u, v\in Q)(\exists!\, x, y\in Q)(x*u=v \wedge u*y=v) \qquad (1)$$

Definition 2: The quasigroup $(Q,*)$ of order 2^q is linear if there are non-singular binary matrices A and B of order $q\times q$ and a binary matrix C of order $1\times q$, such that

$$(\forall x, y\in Q)\ x*y=z \Leftrightarrow \mathbf{z}=\mathbf{x}A+\mathbf{y}B+C \qquad (2)$$

where $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are binary representations of x, y and z as vectors of order $1\times q$ and + is a binary addition.

For a finite quasigroups, order of the quasigroup is the number of elements in the quasigroup. In this paper we will take that the elements of a quasigroup of order 16 are {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}.

The error-detecting code that we analyse in this paper is defined in a following way: Let $(Q,*)$ be a linear quasigroup and let the input blocks of length n characters from the quasigroup Q are coded. Let k be an integer such that $1\le k\le n-1$. Each input block $a_0a_1\ldots a_{n-1}$, where all $a_i\in Q$, is extended into a block $a_0a_1\ldots a_{n-1}d_0d_1\ldots d_k$, where the redundant characters are defined with:

$$d_i=a_i*a_{i+1}*a_{i+2}*\ldots*a_{i+n-2},\ \ i=0, 1, \ldots, k \qquad (3)$$

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where all operations in indexes are modulo n. The coded block $a_0a_1\dots a_{n-1}d_0d_1\dots d_k$, converted into binary form, is transmitted through the binary symmetric channel.

From the definition of the model follows that this code always adds fixed number of redundant characters, regardless of the length of the input block. Namely, in the beginning, we choose the value of k in the model. The model will always add k+1 redundant character on the input block, regardless of its length. From the constraint $k \le n-1$ follows that $k+1 \le n$, which means that the length of the input block must be greater than or equal to the length of the redundancy.

Due to the noises in the channel, some of the characters may be incorrectly transmitted. In order to check whether the received block is correctly transmitted, the receiver checks whether equations (3) are satisfied for the received block. If there is some i for which the equation (3) is not satisfied, the receiver concludes that there are errors in transmission, and it asks the sender to send the block once again. Otherwise, it accepts the block as correctly transmitted. But, since the redundant characters are also transmitted through the binary symmetric channel, it is possible that they are incorrectly transmitted, too. This can result with a situation in which (3) is satisfied for all $i \in \{0, 1, \dots, k\}$, although some of the information characters a_0, a_1, …, a_{n-1} are incorrectly transmitted. This means that it is possible to have undetected errors in transmission. For this reason, it is important to know the error-detecting capability of the code. The probability of undetected errors of this code is analyzed in [6] and [7]. Now, using simulations we will obtain the number of errors that the code surely detects when a quasigroup of order 16 is used for coding. This number depends on the length of the input block n, the parameter k, but also it depends on the order of the quasigroup used for coding.

The simulation process that we used is described in detail in [8]. In short, in the simulation process we generate a large number of input blocks over the alphabet Q, where Q is the quasigroup used for coding. Then, these input blocks are coded and transmitted through a simulated binary-symmetric channel. For each i from 1 to the length of the coded input blocks (expressed in bits), we count the number of transmitted coded blocks with i incorrectly transmitted bits and the number of these coded blocks in which the error is not detected. The number of errors that the code detects for sure is the largest integer s such that there are not incorrectly transmitted coded blocks with i incorrectly transmitted bits in which the error is not detected, for all i from 1 to s. In order to obtain accurate results, we chose the probability of bit-error in the binary-symmetric channel such that the number of incorrectly transmitted coded blocks with i incorrectly transmitted bits to be large number (order 10^4) for small values of i, i.e., for values of i for which the number of incorrectly transmitted coded blocks with at most i incorrectly bits in which the error is not detected is equal to zero.

In the next example we will demonstrate the coding procedure and how the receiver checks whether there are errors in transmission.

Example 1: We will use a quasigroup of order 16 for coding. When a quasigroup of order 16 is used for coding, the input messages are over the alphabet Q={0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}. Let the parameter k in the model of the code is equal to 5. This means that the code will add k+1=6 characters to each input block, regardless of its length. Let suppose that the input block is 3E458F21C. This means that the input block has length n=9 characters from the quasigroup and the information characters are a_0=3, a_1=E, a_2=4, a_3=5, a_4=8, a_5=F, a_6=2, a_7=1 and a_8=C.
Let for coding be used the following quasigroup of order 16:

*	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	7	F	8	D	A	2	5	B	C	4	3	6	1	9	E
1	F	8	0	7	2	5	D	A	4	3	B	C	9	E	6	1
2	D	A	2	5	0	7	F	8	6	1	9	E	B	C	4	3
3	2	5	D	A	F	8	0	7	9	E	6	1	4	3	B	C
4	B	C	4	3	6	1	9	E	0	7	F	8	D	A	2	5
5	4	3	B	C	9	E	6	1	F	8	0	7	2	5	D	A
6	6	1	9	E	B	C	4	3	D	A	2	5	0	7	F	8
7	9	E	6	1	4	3	B	C	2	5	D	A	F	8	0	7
8	7	0	8	F	A	D	5	2	C	B	3	4	1	6	E	9
9	8	F	7	0	5	2	A	D	3	4	C	B	E	9	1	6
A	A	D	5	2	7	0	8	F	1	6	E	9	C	B	3	4
B	5	2	A	D	8	F	7	0	E	9	1	6	3	4	C	B
C	C	B	3	4	1	6	E	9	7	0	8	F	A	D	5	2
D	3	4	C	B	E	9	1	6	8	F	7	0	5	2	A	D
E	1	6	E	9	C	B	3	4	A	D	5	2	7	0	8	F
F	E	9	1	6	3	4	C	B	5	2	A	D	8	F	7	0

The input block is coded using equation (3). We obtain the redundant characters:

$d_0=a_0*a_1*a_2*a_3*a_4*a_5*a_6*a_7$=3*E*4*5*8*F*2*1=E
$d_1=a_1*a_2*a_3*a_4*a_5*a_6*a_7*a_8$=E*4*5*8*F*2*1*C=3
$d_2=a_2*a_3*a_4*a_5*a_6*a_7*a_8*a_0$=4*5*8*F*2*1*C*3=D
$d_3=a_3*a_4*a_5*a_6*a_7*a_8*a_0*a_1$=5*8*F*2*1*C*3*E=1
$d_4=a_4*a_5*a_6*a_7*a_8*a_0*a_1*a_2$=8*F*2*1*C*3*E*4=B
$d_5=a_5*a_6*a_7*a_8*a_0*a_1*a_2*a_3$=F*2*1*C*3*E*4*5=9

With this we obtained the coded block $a_0a_1a_2a_3a_4a_5a_6a_7a_8$ $d_0d_1d_2d_3d_4d_5$=3E458F21CE3D1B9. In order to be transmitted through a binary symmetric channel, the coded block must be converted into binary form. This is done by representing each character from the quasigroup Q with 4 bits in the binary representation. In this way is obtained the coded block into binary form:

001111100100010110001111001000011100111000111101
000110111001.

This block is transmitted through the binary symmetric channel. But, let suppose that due to the noises in the channel, the second bit is incorrectly transmitted, and the receiver receives:

011111100100010110001111001000011100111000111101
000110111001.

This output block, converted over the alphabet Q is: $a_0'a_1'a_2'a_3'a_4'a_5'a_6'a_7'a_8'd_0'd_1'd_2'd_3'd_4'd_5'$= 7E458F21CE3D1B9. Now, in order to check whether there are errors in transmission, the receiver checks whether the received output block satisfies the equation (3) for all $i \in \{0, 1, 2, 3, 4, 5\}$. If for some i the equation is not satisfied the receiver concludes that the block is incorrectly transmitted and asks for repeated transmission of that block. Otherwise, it accepts the block as correctly transmitted. Checking whether the equation (3) is satisfied for i=0:

$a_0'*a_1'*a_2'*a_3'*a_4'*a_5'*a_6'*a_7'$=7*E*4*5*8*F*2*1=5

But, the first redundant character in the output block is d_0'=E. Since $d_0' \ne a_0'*a_1'*a_2'*a_3'*a_4'*a_5'*a_6'*a_7$, the equation (3) is not satisfied for i=0. Therefore, the receiver concludes that the block is incorrectly transmitted. In the above example we had incorrectly transmitted coded block with 1 incorrectly transmitted bit and the code detected the error in transmission.

But, now suppose that not only the second bit is incorrectly transmitted, but also the 37th, 39th, 40th, 45th, 46th, 48th, 51th, 52th, 55th, 57th, 58th and 60th bits are incorrectly transmitted, too. This means that the receiver receives the block: 0111111001000101100011110010000 111000101001100000010100010100. The output block converted into string over the alphabet Q is: $a_0'a_1'a_2'a_3'a_4'a_5'a_6'a_7'a_8'd_0'd_1'd_2'd_3'd_4'd_5'$=7E458F21C530294. Now, the receiver checks whether this block satisfies the equation (3):

$a_0'*a_1'*a_2'*a_3'*a_4'*a_5'*a_6'*a_7'$=7*E*4*5*8*F*2*1=5=$d_0'$
$a_1'*a_2'*a_3'*a_4'*a_5'*a_6'*a_7'*a_8'$=E*4*5*8*F*2*1*C=3=$d_1'$
$a_2'*a_3'*a_4'*a_5'*a_6'*a_7'*a_8'*a_0'$=4*5*8*F*2*1*C*7=0=$d_2'$
$a_3'*a_4'*a_5'*a_6'*a_7'*a_8'*a_0'*a_1'$=5*8*F*2*1*C*7*E=2=$d_3'$
$a_4'*a_5'*a_6'*a_7'*a_8'*a_0'*a_1'*a_2'$=8*F*2*1*C*7*E*4=9=$d_4'$
$a_5'*a_6'*a_7'*a_8'*a_0'*a_1'*a_2'*a_3'$=F*2*1*C*7*E*4*5=4=$d_5'$

Since the equation (3) is satisfied for all $i\in\{0, 1, 2, 3, 4, 5\}$, the receiver concludes that the block is correctly transmitted, although some bits are incorrectly transmitted. This means that in this case the code does not detect the error in transmission.

In this situation beside the information character, five of the six redundant characters are incorrectly transmitted, too. The redundant characters are incorrectly transmitted in a way that the equation (3) is satisfied and therefore the code did not detect the errors in transmission. As we can see from the above example, in order the error in transmission to not be detected, a large number of redundant bits should be incorrectly transmitted, too. Since the probability of bit-error in the binary symmetric channels is very small, the chances for such a case are pretty small.

Although this scenario occurs rarely, it is still possible to have a situation in which the coded block is incorrectly transmitted in a way that the code does not detect the error. For this reason, it is important to know the number of incorrectly transmitted bits that the code will detect for sure.

3. Results and discussion when a linear quasigroup of order 16 is used for coding

In this subsection we consider the cases when the redundancy is 8, 12 and 16 bits and a quasigroup of order 16 is used for coding.

The linear quasigroup of order 16 used for coding is represented with the following binary matrices:
A={{0, 1, 1, 1}, {1, 0, 1, 1}, {1, 1, 0, 1}, {1, 1, 1, 1}}, B={{1, 0, 1, 1}, {1, 1, 0, 1}, {1, 1, 1, 1}, {0, 1, 1, 1}} and C={{0, 0, 0, 0}}

The matrices are presented as a list of lists, i.e., the matrix is a list in which the first list is the first row, the second list is the second row of the matrix etc.

Note that this is the same quasigroup from *Example 1*, only with another representation.

The obtained results from the simulations when the length of the redundancy is 8, 12 and 16 bits are given in Fig. 1, Fig. 2 and Fig. 3, respectively. In the first column of the Fig. s is given the length of the input block n expressed in characters from the quasigroup used for coding. In the next columns are given the number of incorrectly transmitted coded blocks with i incorrectly transmitted bits $e[i]$ and the number of incorrectly transmitted coded blocks with i incorrectly transmitted bits in which the error is not detected $ue[i]$, $i\leq 5$. For each value of the length of the input block n, the first value of $ue[i]$ which is not equal to zero (i.e., $i=\min\{j|e[j]\neq 0\}$) is bolded.

Since each character from the quasigroup of order 16 is represented with 4 bits in the binary representation, the redundancy of 8 bits is redundancy with length 2 characters from the quasigroup. This means that in the model defined with (3) the parameter k=1. Due to the constraint $k\leq n$-1 follows that $n\geq 2$. For this reason, the block length n in Fig. 1 is greater than or equal to 2.

Fig. 1. The number of incorrectly transmitted coded blocks with i incorrectly transmitted bits $e[i]$ and the number of such coded blocks in which the error is not detected $ue[i]$ when the linear quasigroup of order 16 is used for coding, the length of the input block is n characters from the quasigroup and the redundancy is 8 bits.

n	*e* [1]	*ue* [1]	*e* [2]	*ue* [2]	*e* [3]	*ue* [3]	*e* [4]	*ue* [4]	*e* [5]	*ue* [5]
2	32951	0	27397	**1849**	14264	0	5083	80	1330	0
3	27165	0	28411	0	18931	**155**	8978	52	3230	8
4	21279	0	27060	0	22236	**99**	12941	39	5698	24
5	35395	0	24879	**123**	11155	55	3671	13	901	8
6	32789	0	27074	0	13747	**97**	5270	20	1530	5
7	30256	0	27656	**188**	16176	129	6962	39	2429	11
8	32884	0	26725	**447**	13729	125	5165	27	1652	4
9	30767	0	27572	**375**	15562	176	6742	48	2230	12
10	28544	0	28017	**503**	17382	235	7963	65	2908	16
11	33258	0	26237	**549**	13122	214	4891	68	1534	10
12	31838	0	26859	**574**	14765	251	5880	72	1860	9
13	30050	0	27619	**608**	16097	258	7137	117	2498	34
14	28582	0	27528	**757**	17434	320	8062	112	3063	36
15	35471	0	24260	**642**	10613	197	3458	41	947	15
16	34711	0	24759	**788**	11685	280	4001	80	1169	18
17	33947	0	25497	**862**	12663	283	4709	88	1432	18
18	32779	0	25953	**869**	13931	330	5421	119	1669	30
19	31696	0	26673	**991**	14736	387	6101	143	1986	35
20	30479	0	26980	**1094**	15984	413	6743	170	2296	41
21	29438	0	27315	**1081**	16579	456	7369	191	2670	56
24	37381	0	19106	**787**	6451	186	1715	62	375	6
27	36434	0	21199	**984**	8201	263	2336	71	549	15
30	17957	0	11356	**556**	4908	189	1521	62	371	6
33	17519	0	12252	**672**	5536	189	1854	61	547	19
36	16757	0	12743	**708**	6311	241	2430	82	714	23
39	15935	0	13140	**740**	7211	250	2961	111	922	28
42	15229	0	13315	**733**	7803	327	3466	127	1171	48
45	14453	0	13666	**782**	8485	327	3867	156	1430	52
48	16237	0	13088	**791**	6798	253	2695	135	887	35
51	15578	0	13279	**802**	7580	318	3175	135	989	35
54	15004	0	13479	**855**	7935	321	3557	170	1208	44
57	14494	0	13466	**833**	8420	360	3923	180	1501	55
60	16928	0	12440	**772**	6171	245	2339	121	699	23
64	16440	0	12794	**804**	6724	311	2757	140	847	30

As we can see from Fig. 1, when the length of the information block is 2 characters from the quasigroup, the code detected all $e[1]$=32951 incorrectly transmitted coded blocks with only 1 incorrectly transmitted bit ($ue[1]$=0 when n=2). But, there are some incorrectly transmitted coded blocks with 2 incorrectly transmitted bits in which the error is not detected (i.e., $ue[2]$=1849≠0 when n=2). This means that in the case when the input block has length 2 characters from the quasigroup Q, the code detects for sure only 1 incorrectly transmitted bit. Similarly, when the length of the input blocks is n=3 characters from the quasigroup, the code detected all incorrectly transmitted coded blocks with less than 3 incorrectly transmitted bits, i.e., $ue[i]$=0 for all i<3. But, there are $ue[3]$=155 incorrectly transmitted coded blocks with 3 incorrectly transmitted bits in which the error in transmission is not detected. Therefore, when the length of the input block is 3 characters from the quasigroup, the code surely detects up to 2 incorrectly transmitted bits.

In a same way, we conclude that when the length of the input block is 4 or 6 characters, the code surely detects up to 2 incorrectly transmitted bits.

When the length of the input block is 5 characters and also in the case when the length of the input block is greater than or equal to 7 characters from the quasigroup used for coding, the code surely detects 1 incorrectly transmitted bit.

From the same reason as in Fig. 1, when the redundancy is 12 bits, which are 3 characters from the quasigroup of order 16 (which means k=2 in the model), the length of the input block n must be greater than or equal to 3 characters (Fig. 2).

From Fig. 2, follows that in the case when the redundancy is 12 bits and the quasigroup of order 16 is used for coding, the code surely detects up to 3 incorrectly transmitted bits when the length of the input block is 3 or 4 characters from the quasigroup, up to 2 incorrectly transmitted bits when the length of the input block is 5, 6 or 7 characters from the quasigroup and 1 incorrectly transmitted bit when the length of the input block is greater than or equal to 8 characters from the quasigroup.

As in the previous cases, when the redundancy is 16 bits, which is 4 characters from the quasigroup of order 16 (i.e., k=3 in the model), the length of the input block n must be greater than or equal to 4 characters from the quasigroup used for coding (Fig. 3). From Fig. 3, we can see that when the redundancy is 16 bits and the quasigroup of order 16 is used for coding, the code surely detects up to 3 incorrectly transmitted bits when the length of the input block is 4 or 6 characters from the quasigroup, up to 4 incorrectly transmitted bits when the length of the input block is 5 characters from the quasigroup, up to 2 incorrectly transmitted bits when the length of the input block is 7 or 8 characters from the quasigroup and 1 incorrectly transmitted bit when the length of the input block is greater than or equal to 9 characters from the quasigroup.

In Table 1 are given the summarized results about the number of errors that the code detects for sure in the three considered cases. Namely, in Table 1 is presented the number of incorrectly transmitted bits that the code surely detects when the quasigroup of order 16 is used for coding and the redundancy is 8, 12 and 16 bits: *sdb*_16_*r*8, *sdb*_16_*r*12 and *sdb*_16_*r*16, respectively. These results are presented for all possible values of the length of the input blocks n. The length of the input block is expressed in characters from the quasigroup used for coding.

Since according to the definition of the code, the length of the information block must be greater than or equal to the length of the redundancy, there are some empty fields in Table 1. For example, in the case when the length of the redundancy is 12 bits, which are 3 characters from the quasigroup, the length of the information block must be at least 3 characters. For this reason, the field that corresponds to *sdb*_16_*r*12 and n=2 is empty. For the same reason, the fields that correspond to *sdb*_16_*r*16 and n=2 and n=3 are empty.

Fig. 2. The number of incorrectly transmitted coded blocks with i incorrectly transmitted bits $e[i]$ and the number of such coded blocks in which the error is not detected $ue[i]$ when the linear quasigroup of order 16 is used for coding, the length of the input block is n characters from the quasigroup and the redundancy is 12 bits.

n	*e* [1]	*ue* [1]	*e* [2]	*ue* [2]	*e* [3]	*ue* [3]	*e* [4]	*ue* [4]	*e* [5]	*ue* [5]
3	21329	0	27163	0	22384	0	12849	**12**	5681	6
4	16381	0	24713	0	23419	0	16233	**2**	8512	0
5	23750	0	27794	0	20614	**11**	11095	3	4847	1
6	19962	0	26305	0	22489	**7**	13692	6	6609	0
7	16631	0	24443	0	23290	**61**	16036	14	8516	4
8	36500	0	23549	**100**	9921	35	3149	1	828	0
9	34695	0	25145	**190**	11679	48	3998	7	1153	0
10	33437	0	25953	**203**	13265	84	4903	15	1437	0
11	31806	0	26821	**338**	14841	107	6019	27	1937	5
12	30203	0	27363	**383**	16225	152	6981	34	2446	7
13	28384	0	27712	**435**	17351	170	8042	37	3071	11
14	35518	0	24129	**411**	10709	114	3439	36	918	3
15	34704	0	24775	**505**	11663	147	4124	33	1197	4
16	33580	0	25803	**541**	12588	189	4706	60	1386	9
17	32646	0	26337	**612**	13689	219	5412	68	1601	9
18	31599	0	26686	**679**	14815	259	6080	78	1997	13
19	30757	0	26977	**771**	15676	267	6597	92	2332	19
20	29417	0	27436	**807**	16516	324	7537	110	2693	33
21	28145	0	27545	**886**	17624	315	8094	137	3091	29
24	37156	0	19742	**730**	6959	153	1941	42	414	8
27	36573	0	21640	**853**	8465	178	2544	56	592	13
30	17728	0	11763	**498**	5020	133	1679	44	460	4
33	17125	0	12317	**544**	5910	181	2099	55	639	13
36	16504	0	12831	**619**	6725	186	2559	77	809	15
39	15668	0	13285	**689**	7343	226	3079	87	967	25
42	15170	0	13478	**722**	8095	256	3466	123	1259	34
45	14197	0	13754	**758**	8547	296	4016	148	1529	33
48	15992	0	13298	**707**	6999	254	2774	116	951	30
51	15533	0	13212	**702**	7756	282	3231	127	1142	37
54	14811	0	13390	**793**	8171	319	3687	135	1311	39
57	14001	0	13527	**808**	8720	321	4092	176	1569	60
60	16639	0	12656	**794**	6356	231	2427	110	752	28
64	16274	0	12990	**765**	6777	275	2770	118	922	37

As we can see from Table 1, in the case when the redundancy has length of 12 bits, when the length of the input block increases, the number of errors that the code surely detects decreases. But this property is not valid for the other two lengths of the redundancy.

From the aspect of the number of errors that the code surely detects, it is best to divide the input message into blocks of length 5 characters of the quasigroup and to code these blocks in a way that 16 bits are added to each block (i.e., to choose k=3 in the model). In this case the code will detect for sure every incorrectly transmitted coded block with up to 4 incorrectly transmitted bits. Since each character from the quasigroup of order 16 is presented with 4 bits in the binary representation, 16 bits are 4 characters from the quasigroup. These means that in this case on each input block of length 5 characters, 4 redundant characters are added. Therefore, in this case the code rate will be 5/9.

Fig. 3. The number of incorrectly transmitted coded blocks with i incorrectly transmitted bits $e[i]$ and the number of such coded blocks in which the error is not detected $ue[i]$ when the linear quasigroup of order 16 is used for coding, the length of the input block is n characters from the quasigroup and the redundancy is 16 bits.

n	*e* [1]	*ue* [1]	*e* [2]	*ue* [2]	*e* [3]	*ue* [3]	*e* [4]	*ue* [4]	*e* [5]	*ue* [5]
4	36843	0	63787	0	70558	0	56180	**7**	34822	0
5	59679	0	104381	0	117169	0	94377	0	58771	**3**
6	49731	0	73857	0	69746	0	47813	**3**	25563	1
7	55507	0	77002	0	68436	**11**	43879	10	21984	2
8	46982	0	72165	0	70044	**72**	48929	17	27438	1
9	100375	0	78199	**245**	39681	82	14977	8	4443	0
10	95265	0	80750	**433**	44246	106	17962	25	5799	3
11	90782	0	82165	**619**	48242	187	21051	41	7310	3
12	85654	0	83065	**794**	52176	246	24537	52	9023	10
13	80011	0	82956	**917**	56184	308	27807	88	11094	8
14	103779	0	74903	**957**	35221	250	12423	44	3580	7
15	101368	0	76666	**1042**	38374	283	14139	72	4324	9
16	98454	0	78727	**1305**	41227	385	15994	105	5030	17
17	95336	0	79837	**1404**	44416	423	18105	114	5998	24
18	92254	0	81258	**1648**	46833	555	20058	144	6850	31
19	88424	0	82478	**1825**	49927	634	22027	192	7917	41
20	74358	0	35283	**873**	11134	153	2680	32	525	7
21	74516	0	37020	**961**	12009	154	2951	32	633	4
24	73743	0	40950	**1239**	14936	254	4196	67	940	5
27	72277	0	44813	**1453**	18181	357	5477	84	1443	18
30	70284	0	47940	**1779**	21519	461	7123	151	1988	23
33	67448	0	50635	**2078**	24208	626	8873	175	2731	45
36	65244	0	51904	**2253**	27348	704	10900	253	3410	61
39	61904	0	53495	**2388**	30403	871	12823	317	4453	89
42	58578	0	54405	**2595**	32985	922	15045	414	5396	105
45	65824	0	51504	**2428**	26586	802	10383	270	3256	79
48	63375	0	53072	**2715**	28781	899	11918	370	4041	94
51	60598	0	53844	**2894**	31359	994	13689	411	4797	124
54	58286	0	54214	**3012**	33491	1081	15304	503	5525	130
57	55624	0	54670	**3075**	35390	1223	16925	575	6601	183
60	49609	0	38349	**2097**	19590	703	7403	253	2274	59
64	48309	0	39146	**2309**	21271	820	8431	257	2868	99

Table 1. The number of incorrectly transmitted bits in the coded block that the error-detecting code surely detects when the linear quasigroup of order 16 is used for coding and the redundancy is 8, 12 and 16 bits: *sdb*_16_*r*8, *sdb*_16_*r*12 and *sdb*_16_*r*16. The length of the input block is *n* characters from the quasigroup used for coding.

n	*sdb*_16_*r*8	*sdb*_16_*r*12	*sdb*_16_*r*16
2	1		
3	2	3	
4	2	3	3
5	1	2	4
6	2	2	3
7	1	2	2
8	1	1	2
>=9	1	1	1

If we want to have larger code rate, we can divide the input message into longer blocks. When long input blocks are coded (i.e., n>50), then it is better to choose shorter redundancy. Namely, for longer input blocks the code surely detects equal number of incorrectly transmitted bits (i.e., it surely detects 1 incorrectly transmitted bit) and the probabilities of undetected errors are equal regardless of the length of the redundancy ([7]). Therefore, since in this case the error-detecting capabilities are equal, regardless of the length of the redundancy, it is better to have shorter redundancy since in that way we have less calculations and faster coding and transmission of the coded blocks.

Also, from Table 1 we can see that for fixed length of the input block, if longer redundancy is added to the input block, then the number of incorrectly transmitted bits that the code detected for sure is greater or at least equal to the number of surely detected incorrectly transmitted bits when a shorter redundancy is added.

4. Conclusions

The obtained experimental results suggest that in the case when the redundancy is 8 bits and the quasigroup of order 16 is used for coding, the code surely detects up to 2 incorrectly transmitted bits when the length of the input block is 3, 4 or 6 characters from the quasigroup. When the input block has length 2 or 5 characters, but also when the input block has length greater than or equal to 7 characters, the code detects for sure 1 incorrectly transmitted bit.

When the redundancy is 12 bits, the code surely detects up to 3 incorrectly transmitted bits when the length of the input block is 3 or 4 characters from the quasigroup. For input blocks with length from 5 to 7 characters, the code surely detects up to 2 incorrectly transmitted bits. When the length of the input block is greater than 7 characters, the code surely detects 1 incorrectly transmitted bit.

In the case when the redundancy is 16 bits, the code surely detects up to 4 incorrectly transmitted bits when the length of the input block is 5 characters from the quasigroup, up to 3 incorrectly transmitted bits when the input block has length 4 or 6 characters, up to 2 incorrectly transmitted bits when the input block has length 7 or 8 characters and 1 incorrectly transmitted bit when the input block has length greater than 8 characters. When the length of the input blocks is fixed, longer redundancy implies greater or equal number of surely detected incorrectly transmitted bits.

The largest number of surely detected incorrectly transmitted bits in the coded blocks is achieved when the redundancy is 16 bits and input blocks of length 5 characters from the quasigroup are coded. This means that from the aspect of the number of errors that the code detects for sure, it is best to divide the input message into blocks of length 5 characters and to code each of them separately such that the redundancy has length 16 bits (i.e., to choose k=3 in the model of the error-detecting code).

In the case when the goal is to have a large code rate, the input message should be divided into longer blocks. In a case when the blocks are long enough such that the error-detecting capabilities of the code are equal, regardless of the length of the redundancy, it is best to add as short as possible redundancy, i.e., a redundancy of length 8 bits.

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